Problems of the Month

March 2020

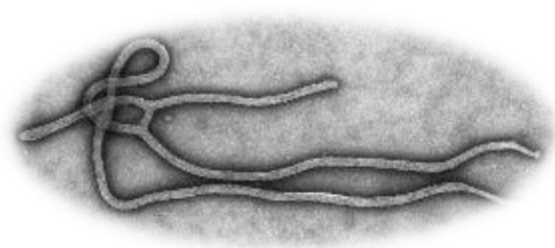
General Problem:

In epidemiology, the spread of a virus can be characterized by its *basic reproduction number*, called "R_0". Each person that is infected, infects R_0 other people. For example, the seasonal flu has an R_0 or approximately 2. Assume an infection lasts 1 week and infects 2 people at the end of the week. If 3 people are initially infected, how many people are infected at the end of 5 weeks?

Calculus Problem:

In epidemiology, a more involved model that can work with finite populations is called the "SIR" model. S stands for the number of *susceptible* individuals. I for the number *infected*, and R for the number *recovered*. For this question, assume we have a disease that follows the following equations:

$$\frac{dS}{dt} = -0.8\frac{SI}{N}$$
$$\frac{dI}{dt} = 0.8\frac{SI}{N} - \frac{I}{5}$$
$$\frac{dR}{dt} = \frac{I}{5}$$



If in a population of $N = 10{,}000$ people, new cases are breaking out at a rate of $\frac{dI}{dt} = 40$ per day and recovering at a rate of $\frac{dR}{dt} = 25$ per day, how many people are still susceptible to the virus?

Challenge Problem:

Consider the graph $y = 16 - x^4$ and the points $P(-1,15)$ and $Q(2,0)$. Find the x-coordinate of a point R on the graph so that the total area bounded by the graph and the line segments PR and RQ is as small as possible.