FAST COLOR QUANTIZATION USING MACQUEEN'S K-MEANS ALGORITHM

by

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Skyler Thompson

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ABSTRACT

Although not strictly necessary in all color image processing applications today, color quantization still plays an important role in certain, typically hardware constrained, applications. In this thesis, a novel color quantization method based on MacQueen's k-means algorithm, is proposed and compared to the more popular batch k-means algorithm. The proposed method uses the maximin initialization method and quasi-random sampling to achieve high quality, fast, and deterministic results. In comparison to other well-known color quantization methods, the proposed method achieves very competitive results while being much faster.

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CHAPTER 1: INTRODUCTION

Pixels in an RGB image are comprised of three-color values: red, green, and blue. RGB color images are then simply a rectangularly arranged collection of RGB triplets. Because the red, green, and blue values can each range from 0 to 255, the total number distinct colors that can be represented in a conventional 24-bit color image is 16,777,216. This can be quite a lot of information, especially with larger, more complex images. Therefore, the technique of color quantization (CQ) was conceived to help alleviate some of the storage related constraints (Heckbert, 1982). Modern hardware can easily handle these images, however there are still a large number of applications where CQ is used and where advances in this field are quite useful.

Data Clustering

Data Clustering is a type of unsupervised learning (Celebi and Aydin, 2015) in which data points are grouped into "clusters," each of which is typically represented by a center. Each cluster's data points are typically similar to each other in some sense. Various algorithms have been invented over the years for data clustering, with one of the most popular algorithms being k-means and its numerous variants. K-means is a partitional clustering method (Celebi, 2014) in which n observations are partitioned into k clusters with each observation belonging to a single cluster.

In the context of CQ, clustering methods are typically divided into two different categories: pre-clustering and post-clustering. Pre-clustering methods recursively find nested clusters in either a divisive (top-down) or agglomerative (bottom-up) fashion. Pre-clustering algorithms are typically faster than post-clustering algorithms and sacrifice accuracy for speed. Post-clustering, on the other hand, find all k clusters simultaneously.

Color Quantization

CQ is a technique in which the number of colors in an input image is significantly reduced by picking a predetermined number of colors from a palette and applying them to all similarly colored pixels in the image (Braudaway, 1987). For example, if an original image has around 100,000 colors, one may desire to reduce the number of colors to 64 in order to achieve a faster and better compression. There have been several methods devised for quantizing images over the past four decades (Heckbert, 1982) and among the best of them is k-means. K-means clustering (Celebi, 2009; Celebi, 2011) is a natural candidate because CQ can be viewed conveniently as a three-dimensional clustering problem. Like nearly all post-clustering algorithms, this technique is typically performed in two steps. The first is to choose the color palette, and the second is to map each of the original pixels to the closest color in the palette.

CHAPTER 2: RELATED WORK

Several CQ techniques are used in the color image processing literature (Brun, 2002). Among them are: popularity (POP), median-cut (MC), modified popularity (MPOP), octree (OCT), variance-based method (WAN), greedy orthogonal bipartitioning (WU), center-cut (CC), self-organizing map (SOM), radius-weighted mean-cut (RWM), modified maximin (MMM), pairwise clustering (PWC), split and merge (SAM), Cheng and Yang (CY), fuzzy c-means (FCM), adaptive distributing units (ADU), and variance-cut (VC).

Heckbert (1982) proposed the popularity method which builds a 16 x 16 x 16 color histogram using 4 bits per channel, and then takes the K most frequent colors in the histogram as the color palette. He also proposed the median-cut method that builds a 32 x 32 x 32 color histogram containing pixel values reduced by uniform quantization to 5 bits per channel. The histogram is split recursively into smaller boxes until K boxes are obtained.

Braudaway (1987) introduced the modified popularity method that starts by building a 2^R x 2^R x 2^R histogram using R bits per channel. It chooses the most frequent color as the first palette color and then reduces the frequency of each color. The remaining colors are chosen in a similar fashion.

Gervautz and Purgathofer (1988) proposed the octree method which builds an octree (tree data structure in which each internal node has up to eight children) that is a representation of the input image color distribution. Then, it prunes the tree starting from the bottom by merging nodes until K colors are obtained.

Wan, Prusinkiewicz, and Wong (1990) proposed the variance based method, which is similar to Median-cut and starts the same but at each step, the box with the largest square error is split along the principal axis at the point that minimizes marginal weighted variance.

Wu (1991) proposed the greedy orthogonal bipartitioning procedure that is similar to the variance-based method with the exception that at each step, the box with the largest square error is split along the axis that minimizes the sum of the variance on both sides.

Joy and Xian (1993) proposed the center-cut method which is very similar to median-cut except at each step, the box with the greatest range on any coordinate axis is split along its longest axis at the mean point.

Dekker (1994) applied a one-dimensional self-organizing map (Kohonen, 1990) to color quantization. A random subset of pixels is used in the training phase and the final weights of the centers are taken as the color palette.

Yang and Lin (1996) proposed the radius-weighted mean-cut method that is similar to the variance-based method, except the box is split along the vector from the origin to the radius-weight mean at that point.

Xiang (1997) introduced the modified maximin method (Gonzalez, 1985) which chooses the first center arbitrarily from the data set and the rest of the centers are chosen to be the points with the largest minimum distance to the previously selected centers. Each of the initially chosen centers are then recalculated as the mean of the points assigned to them.

Velho, Gomez, and Sobreiro (1997) introduced the pairwise clustering method that builds a 2^R x 2^R x 2^R color histogram and controls a Q x Q joint quantization error matrix where Q is the number of colors in the reduced color histogram. The clustering procedure starts with Q singleton clusters, each of which contains one image color. In each iteration, the pair of clusters with the least joint quantization error is merged (Ward, 1963). This process is repeated until K clusters remain.

Brun and Mokhtari (2000) created the split and merge method which first partitions the color space into *B* partitions uniformly. This initial set is represented as an adjacency graph. In the second (last) phase, *B* – K merge operations are performed to obtain the final clusters (K). The pair of clusters with the minimum joint quantization error are merged during the second phase.

Cheng and Yang (2001) created the method named after themselves, that is similar to WAN, except at each step the box is split along a specially chosen line defined by the mean color and the color that is farthest away at the mean point.

Wen and Celebi (2011) conducted a comparative study among several variants of k-means and fuzzy c-means algorithms. They demonstrated that fuzzy c-means is substantially slower than k-means and, in terms of quantization effectiveness, the former algorithm is neither objectively nor subjectively superior to the latter.

Celebi (2014) introduced a CQ methods based the adaptive distributing units algorithm (Uchiyama and Arbib, 1994). This algorithm is an online clustering algorithm based on the competitive learning paradigm. What makes this postclustering method interesting is that it does not require initialization.

Celebi, Wen, and Hwang (2015) developed the variance-cut method that is similar to MC with the exception that, at each step, the box with the greatest SSE is split along the coordinate axis with the greatest variance at the mean point. They also proposed the variance-cut with Lloyd iterations method that is like VC except it locally optimizes the two subpartitions resulting from each split using 10 Lloyd iterations.

CHAPTER 3: PROPOSED COLOR QUANTIZATION METHOD

This chapter serves to explain the proposed CQ method. The proposed method starts by converting an image to a one-dimensional array of pixels (RGB values). Then, it will initialize cluster centers using an appropriate adaptive method. After this, it will begin to cluster the image by presenting pixels using a modified version of MacQueen's k-means algorithm. For each presented pixel, once it finds the closest center to the pixel, it will immediately update this center. It will continue to do this until the cluster centers do not move, or until a predetermined number of iterations is reached. Finally, once MacQueen's algorithm converges, each pixel in the input image is then mapped to its nearest center.

Initialization

The simplest type of initialization is random selection. While random selection is very efficient, it simply is not reliable enough to be used for this work (Celebi *et al.*, 2013). A common alternative to random selection is the maximin method of initialization (Gonzalez, 1985). In the maximin method, the first center is chosen arbitrarily, and the successive centers are chosen to be the point with the greatest minimum Euclidean distance to the previously selected center. Euclidean distance between two RGB pixels (R_1, G_1, B_1) and (R_2, G_2, B_2) is given by the equation $\sqrt{(R_2 - R_1)^2 + (G_2 - G_1)^2 + (B_2 - B_1)^2}$. While it is typical to choose the first center randomly, this essentially makes the entire method randomized. Instead, picking the mean data point in the image as the starting point is a convenient and deterministic approach, and the one used here. The pseudocode for maximin is given in Table 1.

Table 1. Maximin Initialization Method Pseudocode

Step	Description
1	Take one center arbitrarily from the dataset and make it the first center.
2	In iteration (i = 2, 3,…, K), the ith center is chosen to be the one with the greatest minimum Euclidean distance to the nearest previously selected (i−1) centers.

Lloyd's Algorithm

Lloyd's algorithm (Lloyd, 1982; Lindie *et al*., 1980) is one of the most common clustering algorithms in scientific and engineering applications (Celebi et al., 2013). Lloyd's algorithm is commonly referred to as batch k-means and starts with a data set $X = \{x_1,\ldots, x_N\} \subseteq R^D$ and a positive integer value K, that is, the desired number of. For color quantization purposes, N, D, and K correspond, respectively, to the number of pixels in the input image, number of color channels (three in our case: RGB), and the number of colors desired. The algorithm then assigns each data point to the closest cluster, thereby minimizing the sum of error (SE) given by SE = $\sum_{x\in X} d(\mathrm{x}, \{c_1, \ldots, c_K\})$, where $d(\mathrm{x}, \{c_1, \ldots, c_K\})$ denotes the Bregman divergence of x to the nearest center in $\{c_1,\ldots,c_K\}$. Bregman divergences are a family of nonmetric dissimilarity functions including Mahalanobis distance, Kullback-Leibler divergence, and Itakura-Saito divergence (Banerjee *et al.*, 2005). The most popular, and the one used in this case, is the squared Euclidean distance. Because the squared Euclidean distance is used, the squared error SE becomes the sum of squared error (SSE).

MacQueen's Algorithm

An online formulation of the batch k-means algorithm was proposed by MacQueen (MacQueen, 1967). It is similar to Lloyd's algorithm in that each point is assigned to the cluster that has the nearest center to that point (Thompson, 2020). The two algorithms differ, however, in the way points are recomputed. Unlike the batch algorithm, that updates all centers after the presentation of the entire set of points, the online algorithm updates the nearest center after the presentation of each point. The online algorithm can be viewed as an instance of the competitive learning paradigm (Rumelhart and Zipser, 1985).

In a basic competitive learning algorithm, a randomly distributed set of units compete for the right to respond to a subset of inputs (Rumelhart and Zipser, 1985). After the presentation of each input, the unit that most closely resembles the input is the winner and moves toward the input. This is termed "hard competitive learning" because only the winner unit is adapted. Let $x^{(t)}$ be the input at time t (t = 1,2,…) and $c^{(t)}$ be the corresponding nearest unit with respect to the ℓ_2 distance. The adaptation equation for $c^{(t)}$ is given by

$$c^{(t+1)} = c^{(t)} + \eta^{(t)}\left(x^{(t)} - c^{(t)}\right),$$

where $\eta \in [0,1]$ is the learning rate, which is typically a monotonically decreasing function of time. The larger the η value, the more emphasis given to new input and hence the faster the learning. However, very large values of η may prevent the algorithm from converging. In general, η is chosen to satisfy the Robbins-Monro conditions (Robbins-Monro(1951):

$$\lim_{t \to \infty} \eta(t) = 0,$$

$$\sum_{t=1}^{\infty} \eta(\mathrm{t}) = \infty,$$

$\sum_{t=1}^{\infty} \eta(t)^2 < \infty.$

The conditions ensure that the learning rate decreases enough to suppress noise but not too fast to avoid premature convergence. The pseudocodes for the batch and online algorithms are given in Tables 2 and 3 respectively.

Table 2. Batch K-Means Algorithm Pseudocode

Step	Description
1	Let $\{c_1,\ldots,c_K\}$ be the initial set of centers.
2	For each i ∈ $\{1,\ldots, K\}$, set cluster C_i to be the set of points in X that are closer in terms of d to c_i than they are to any other center.
3	For each i ∈ $\{1,\ldots, K\}$, set the center c_i of cluster C_i to be the centroid of all points in C_i.
4	Repeat Lloyd iterations (steps 2 and 3) until convergence.

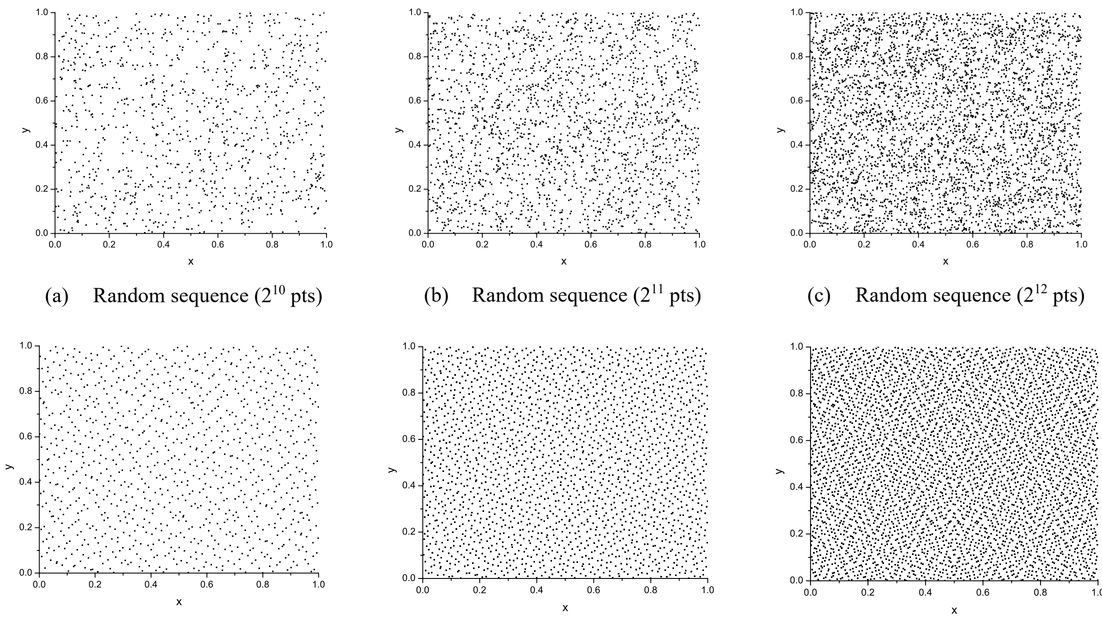
Table 3. MacQueen's K-Means Algorithm Pseudocode

Step	Description
1	Let $\{c_1,\ldots, c_K\}$ be the initial set of centers and $n_1 = \ldots = n_K = 1$.
2	Select a random point x_r from X and find the nearest center c_i to this point.
3	Update the nearest center and the cardinality of the corresponding cluster $c_i \leftarrow (n_i c_i + x_r)/(n_i + 1)$, $n_i \leftarrow n_i + 1$. This ensures that the nearest center c_i now accurately represents the mean of all points in C_i.
4	Repeat steps 2 and 3 until convergence.

Presentation

Batch clustering algorithms, such as Lloyd's algorithm, are less likely than online algorithms such as MacQueen's to escape poor local minima (Celebi, 2014). However, online algorithms have two major drawbacks. First, stochastic selection of random input data could potentially make each run generate different results, effectively randomizing the output of the algorithm. Second, the presentation order matters and different orders of presentation will result in different partitions. To solve this problem, quasi-random sampling is used instead of pseudo-random sampling for the proposed method. A quasi-random sequence differs from a pseudo-random sequence in that it fills the D-dimensional Euclidean space R^D more uniformly. Specifically, in this work, Sobol' quasi-random sampling (Bratley and Fox, 1988) is used. Fig. 1 depicts three pseudo-random sequences generated by the popular MT19937 generator (Matsumoto and Nishimura, 1998) (top row) and three corresponding Sobol' sequences (bottom row). Because quasi-random sampling is deterministic, this method of sampling now makes the modified MacQueen's algorithm also deterministic.

Figure 1. Comparison of pseudo-random (a-c) and quasi-random (d-f) sampling



(a) Random sequence (2^{10} pts)
(b) Random sequence (2^{11} pts)
(c) Random sequence (2^{12} pts)
(d) Sobol' sequence (2^{10} pts)
(e) Sobol' sequence (2^{11} pts)
(f) Sobol' sequence (2^{12} pts)

Color Quantization

After the method converges, we have a color palette (also known as a "color map"). With the color palette at hand, we can start the actual CQ process. Every pixel will be cycled through and compared against all of the cluster centers in order to find the closest center to this pixel. After this, the pixels' color values in the output image are replaced by those of the closest center.

CHAPTER 4: EXPERIMENTAL RESULTS AND DISCUSSION

This chapter explains the experimental procedure, the results of the experiments, and a discussion of the results. First, the test dataset used in the experiments is presented. Next, the procedure used for the experiment is explained and the results are presented. Finally, a detailed discussion of these results is provided.

Input Data

Table 4 presents the images used in the experiment, their name, file size, and number of colors. These images are common in the CQ literature and present a diverse amount of test data for the experiment. Note that both file size and the number of colors is given because there is not a direct relationship between the two (i.e., a large file may not necessarily have more colors). Baboon, Lenna, Peppers are from the USC-SIPI Image Database; Motocross and Parrots are from the Kodak Lossless True Color Image Suite; Goldhill, Fish, and Pills are by Lee Crocker, Luiz Velho, and Karel de Gendre. It is worth noting that the images used in the experiment are binary PPM files (P6) and the program developed for this thesis only accepts this specific format for reasons of programmer convenience and portability.

Table 4. Test Images

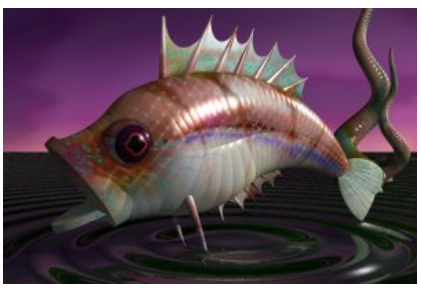
Image	Image Name	File Size	Num Colors
	Baboon	997KB	230,427
	Fish	228KB	28,170
	Goldhill	1.6MB	90,966

Image	Image Name	File Size	Num Colors
	Lenna	997KB	148,279
	Motocross	433KB	63,558
	Parrots	324KB	72,079
	Peppers	997KB	183,525

Image	Image Name	File Size	Num Colors
	Pills	1.6MB	206,609

Experimental Setup

There are three parameters that are tested in the presented CQ method. The first is K, which is present in nearly all CQ methods. The second and third are the learning rate ($p \in (0.5,1]$) and sampling fraction ($f \in (0, 1]$) respectively. The learning rate controls the rate of adaptation and the sampling fraction controls the proportion of input pixels that participate in the learning phase of MacQueen's algorithm. Determining the best possible parameter combination for this CQ method involved considering 24 distinct possibilities: $p \in \{0.5, 0.6, 0.7, 0.8, 0.9, 1\} \times f \in \{0.25, 0.5, 0.75, 1\}$. For each input image and $K \in \{32, 64, 128, 256\}$ value, each image was quantized using the CQ method separately with each of the 24 combinations of (p, f) values. The computed Mean Squared Error (MSE) between the input and output images was then taken. The MSE is given by the mathematical formulation

$$\mathrm{MSE}(\mathrm{X}, \widehat{\mathrm{X}}) = \frac{1}{HW} \sum_{h=1}^{H} \sum_{w=1}^{W} \left|\left| X(h,w) - \hat{X}(h,w) \right|\right|_2^2,$$

where X and $\widehat{\mathrm{X}}$ denote, respectively, the H x W original and quantized images in the RGB color space. These 24 MSE values were then ranked from best to worst, with the lowest value being the best and the highest being the worst. Finally, the mean and standard deviation of the MSE ranked values were calculated. These results are given in Table 5.

Table 5. Mean and Standard Deviation Ranks for Various Parameter Combinations

p	*f*	Mean	Std
0.5	0.25	9.5	2.8
	0.5	4.8	2.1
	0.75	2.9	1.3
	1.00	**1.4**	**0.8**
0.6	0.25	10.6	2.8
	0.5	6.6	1.7
	0.75	5.4	2.4
	1.00	3.4	2.3
0.7	0.25	14.1	3.1
	0.5	9.8	2.1
	0.75	7.8	2.2
	1.00	7.4	3.3
0.8	0.25	17.5	2.9
	0.5	14.3	1.9
	0.75	13.0	2.5
	1.00	11.6	2.7
0.9	0.25	20.7	1.7
	0.5	17.7	1.8
	0.75	17.3	1.9
	1.00	16.5	2.6
1.0	0.25	23.4	1.2
	0.5	22.0	1.5
	0.75	21.6	1.5
	1.00	20.7	1.7

Comparison with Other CQ Methods

The proposed method is compared to the 15 well-known color quantization methods described above: popularity (POP), median-cut (MC), modified popularity (MPOP), octree (OCT), variance-based method (WAN), greedy orthogonal bipartitioning (WU), center-cut (CC), radius-weighted mean-cut (RWM), pairwise clustering (PWC), split and merge (SAM), Cheng and Yang (CY), variance-cut (VC), variance-cut with Lloyd iterations (VCL), self-organizing map (SOM), and modified maximin (MMM).

Two variations of MacQueen's and two variants of Lloyd's (batch) k-means algorithms were implemented for comparison. The two variations of batch (BKM) were developed as an adaptation of Lloyd's k-means clustering algorithm: One-pass BKM (denoted by BKM_1) and convergent BKM (denoted by BKM). One-pass BKM is nothing but the batch version of MacQueen's algorithm where the set of image pixels is presented to the algorithm exactly once. Convergent BKM, on the other hand, performs Lloyd iterations until cluster memberships of points no longer change. The two variations of MacQueen's algorithm are: one with pseudo-random sampling (denoted by MKM_p) and one with quasi-random sampling (denoted by MKM_q). The pseudo-random sampling variant uses the MT19937 (Mersenne Twister) generator. In each iteration, a pseudo-random data point is presented to the algorithm by generating an unbiased pseudo-random integer using a recent algorithm due to Lemire (2019). The quasi-random variant of MacQueen's algorithm uses a quasi-random sampling using a Sobol' sequence. All four k-means variant were initialized using the maximin method.

Table 6 compares the effectiveness, or quality, of the CQ methods. The best (lowest) values are bolded for emphasis. For the only randomized color quantization method, MKM_q, the values are given in the format m_s, where m and s are the mean and standard deviation, respectively, over 100 independent runs.

Table 6. Comparison of Quantization Effectiveness

CQ	K				K				K				K			
	32	64	128	256	32	64	128	256	32	64	128	256	32	64	128	256
	Baboon				Fish				Goldhill				Lenna			
POP	1679.5	849.5	330.7	170.4	2827.6	482.5	105.2	69.8	576.7	199.3	101.8	73.1	347.2	199.5	84.5	65.3
MC	643.0	445.6	307.4	213.0	282.3	189.4	121.2	75.9	293.9	188.8	132.3	86.5	214.0	146.1	112.4	80.3
MPOP	453.1	290.4	195.0	109.3	198.4	145.5	66.2	47.7	200.2	140.7	66.7	48.6	194.5	138.9	60.0	47.8
OCT	530.2	306.6	203.6	125.0	218.4	125.1	77.8	44.3	230.3	130.3	79.0	45.7	186.7	110.0	66.0	40.6
WAN	528.3	385.7	266.0	178.0	311.6	209.0	124.5	77.1	229.0	141.2	94.5	64.4	216.5	140.8	87.6	56.7
WU	468.3	288.3	186.5	118.6	187.6	111.6	69.0	43.8	196.0	114.2	71.4	45.2	158.2	99.1	61.7	39.4
CC	473.1	299.7	202.5	144.7	189.8	127.3	82.3	56.5	202.0	134.9	87.9	57.9	189.1	125.5	80.6	52.2
RWM	459.0	301.6	188.1	120.2	176.7	109.0	68.9	44.4	179.8	118.3	71.0	44.5	161.2	94.6	60.1	39.2
PWC	469.4	308.8	206.7	128.8	201.5	130.9	93.1	69.4	193.8	125.1	88.9	70.9	186.9	108.0	78.8	65.0
SAM	464.9	293.9	188.8	119.8	198.5	120.1	74.0	48.5	179.3	111.2	70.4	46.7	158.0	102.0	65.0	45.4
CY	465.9	280.9	187.3	117.7	193.8	112.5	72.0	44.8	186.3	121.6	72.2	46.4	166.4	97.6	62.5	41.9
VC	450.6	273.5	179.9	117.6	168.1	106.5	67.4	43.4	174.8	109.5	68.3	42.4	145.6	91.7	60.7	38.9
VCL	425.6	264.0	173.1	115.3	169.9	102.5	65.1	43.1	169.3	104.3	66.2	42.0	146.3	89.2	59.2	38.6
SOM	433.6	268.9	163.9	108.2	180.4	114.1	60.4	45.1	182.1	104.2	59.5	38.4	140.2	87.4	50.5	33.9
MMM	510.0	368.4	230.4	147.5	223.4	144.2	81.7	53.7	239.9	143.1	95.4	61.0	183.3	114.2	73.5	48.5
BKM_1	505.0	341.7	218.2	138.2	242.5	139.9	87.3	48.9	250.9	149.3	90.6	61.5	192.9	124.1	72.2	48.1
BKM	**374.2**	**234.3**	**149.3**	**95.6**	**142.6**	**90.2**	**57.3**	**34.8**	**143.8**	**83.0**	**52.0**	**34.2**	**130.8**	**74.7**	**46.8**	**30.3**
MKM_p	375.3$_{1.4}$	236.4$_{.6}$	152.2$_{.3}$	98.2$_{.2}$	147.9$_{3.1}$	93.3$_{1.1}$	59.5$_{.5}$	37.0$_{.3}$	144.4$_{.7}$	84.1$_{.5}$	53.3$_{.2}$	35.6$_{.2}$	131.3$_{.5}$	75.2$_{.3}$	47.7$_{.2}$	31.4$_{.1}$
MKM_q	375.6	236.2	152.0	97.6	148.7	92.8	59.2	36.2	144.3	83.3	53.3	35.7	131.4	75.4	47.9	31.3
	Motocross				Parrots				Peppers				Pills			
POP	1288.6	474.3	201.6	93.5	4086.8	371.7	180.6	104.0	1389.3	367.7	218.3	129.1	788.2	222.9	124.0	85.3
MC	437.6	254.0	169.4	114.3	441.0	265.1	153.6	112.3	377.6	238.9	173.8	121.9	324.2	233.8	159.5	100.4
MPOP	287.5	177.9	84.1	53.3	379.8	212.1	104.7	59.4	338.7	204.9	112.1	69.3	277.5	175.2	88.4	55.1
OCT	300.5	158.9	96.2	54.2	342.4	191.2	111.2	63.8	317.4	193.1	113.9	68.9	281.9	159.8	99.1	56.9
WAN	445.6	292.1	168.7	92.4	376.0	233.4	153.4	92.2	348.1	225.7	157.2	106.4	294.9	197.7	133.1	87.7
WU	268.1	147.2	86.7	51.0	299.2	167.3	95.4	58.3	278.9	165.5	102.2	66.1	261.2	150.1	89.5	55.0
CC	335.1	202.0	122.6	74.9	398.8	246.5	148.7	78.9	418.4	256.8	160.7	107.9	285.9	171.7	111.9	77.4
RWM	251.4	150.1	83.7	51.0	296.5	171.0	99.8	60.6	295.6	178.8	107.1	69.2	260.4	149.7	88.8	55.6
PWC	243.2	161.2	101.5	78.0	349.4	205.1	125.8	86.0	344.8	183.7	121.1	80.0	283.4	169.3	110.5	75.6
SAM	238.1	138.5	81.8	53.5	282.4	157.5	92.4	58.8	275.7	159.2	100.8	65.9	246.2	141.2	85.0	53.7
CY	248.0	146.6	89.3	53.0	313.2	178.6	106.7	64.5	317.3	186.1	114.1	72.6	237.8	157.9	96.4	58.8
VC	253.2	144.5	79.6	48.8	290.6	166.4	98.0	58.5	294.8	169.3	108.0	69.5	234.4	146.6	90.2	54.2
VCL	240.6	131.5	77.1	47.9	263.7	157.5	96.6	57.2	261.1	160.3	103.8	68.4	229.8	141.4	85.7	53.8
SOM	301.7	134.7	70.3	44.2	279.4	151.5	82.2	47.7	270.9	160.5	89.9	69.1	226.4	137.8	72.4	46.0
MMM	407.9	276.9	138.2	85.6	352.1	194.8	128.7	68.5	341.5	213.3	136.5	85.2	276.2	174.9	117.2	75.6
BKM_1	389.4	237.8	166.2	85.7	363.7	202.1	121.3	71.6	363.1	232.2	138.7	92.8	307.5	188.3	121.5	72.0
BKM	197.5	**115.0**	**68.0**	**42.9**	**230.7**	129.5	**73.2**	**44.3**	**248.7**	**148.1**	**87.7**	**55.0**	**198.4**	**111.1**	**66.3**	**41.0**
MKM_p	200.2$_{4.2}$	115.9$_{1.6}$	71.9$_{.9}$	45.0$_{.4}$	236.6$_{3.6}$	129.5$_{1.2}$	75.9$_{.7}$	45.2$_{.3}$	257.5$_{3.3}$	148.6$_{1.1}$	89.6$_{.4}$	57.6$_{.2}$	199.7$_{1.4}$	112.3$_{.6}$	67.3$_{.4}$	42.5$_{.2}$
$MKMq_q$	**194.3**	116.7	72.7	44.6	241.2	**127.2**	75.9	**44.3**	258.3	148.9	89.4	57.6	199.0	112.0	67.3	42.4

Table 7 compares the CPU time for the k-means based CQ methods on three of the eight images. Because the three most effective CQ methods are consistently superior to the others, they are the only ones compared in this experiment. They are compared on the three most well-known images in CQ literature (Lenna, Baboon, and Peppers), with each having the same resolution of 512 x 512 pixels. The remaining methods are left out because they are either pre-clustering methods that sacrifice effectiveness for speed, or post-clustering methods that do not perform particularly well. "Init" is the initialization time (msec), "Clust" is the clustering time (msec), "cr" is the clustering time for BKM divided by that for BKM, MKM_p, or MKM_q, and "tr" is the total time for BKM divided by that for BKM, MKM_p, or MKM_q. All methods were implemented using the C language with the gcc v8.2.0 compiler and the Intel Core i7-6700 (3.4GHz) processor. The time figures were averaged over 100 independent runs.

Table 7. Comparison of CPU Time

Image	CQ	32				64				128				256			
		init	clust	cr	tr	init	clust	cr	tr	init	clust	cr	tr	init	clust	cr	tr
Baboon	BKM	39	4413	1	1	76	14249	1	1	146	27196	1	1	283	40378	1	1
	MKMp	37	54	82	49	73	73	195	98	144	111	246	107	283	200	202	84
	MKMq	37	64	69	44	73	83	171	92	144	122	223	103	283	200	202	84
Lenna	BKM	38	4319	1	1	75	9259	1	1	147	26378	1	1	283	32952	1	1
	MKMp	36	52	82	49	72	73	127	64	143	109	241	105	283	200	165	69
	MKMq	36	63	68	44	72	84	111	60	143	122	217	100	283	200	165	69
Peppers	BKM	36	2610	1	1	74	6259	1	1	147	33127	1	1	284	30275	1	1
	MKMp	37	54	48	29	71	72	87	44	143	110	300	131	284	198	153	63
	MKMq	36	63	41	27	72	83	76	41	143	122	272	126	284	198	153	63

Discussion of Results

For the presented CQ method, Table 3 $(p,f) = (0.5, 1)$ is the best combination attaining mean and standard deviation ranks of 1.4 and 0.8 respectively. This parameter combination almost always generates the lowest distortion. Despite the fact that $p = 1$ is the most common choice in the statistical literature, the experiments demonstrated that $p = 0.5$ may be a better choice for finite data sets and $K > 1$. Table 3 shows that the mean MSE ranks constantly decrease with increasing f. Given that typically the more input pixels that are used, the better the learning for the dataset, this is not surprising. From a theoretical and empirical perspective, the best parameter combination for the examined CQ method is $(p,f) = (0.5, 1)$.

Table 6, which compares the MSE of the different CQ methods shows several interesting results:

- Interestingly, MKM performs either best or close to the best every time and BKM_1 performs among the worst, despite the fact that both make a single pass over the image using almost the same operations. Both are also initialized with maximin. The only explanation for this drastic performance difference is the online nature of MKM, which helps it to easier escape from poor local minima and learn faster.
- Post-clustering methods are more effective than pre-clustering methods in almost all results, with VC and VCL performing better than the other pre-clustering methods (VCL is not purely pre-clustering because it performs refinement using Lloyd iterations after every split).

- Both the pseudo-random and quasi-random versions of MacQueen's k-means (MKM_p and MKM_q) have very similar effectiveness and in most cases, BKM is slightly more effective than either variant. The differences in MSE are negligible, however, and BKM only outperforms MKM by a few units.

Table 7, which compares the CPU time for each of the top three clustering methods (BKM, MKM_p, and MKM_q) shows the following observations:

- By observing the total times ratios, one can see that MKM_p is ever so slightly faster than MKM_q, because pseudo-random sampling is more efficient than quasi-random sampling. This is a very small trade-off for a deterministic scheme, however. Both methods are faster than BKM and the larger K becomes, the faster the methods become in relation.
- Maximin exhibits linear behavior with respect to K. When K is increased, the initialization time for MKM increases at the same rate (doubling each time K does).
- BKM does not scale predictably. In some cases when doubling K, the clustering time is increased by a factor of more than 5, whereas in other cases, doubling K actually decreases clustering time. This is because that, for a given image, the number of iterations for Lloyd's algorithm cannot be determined in advance and varies based on several factors including initial centers and distribution of colors in the image.
- In BKM, initialization time is very small compared to clustering time. The same cannot be said for MKM, which sometimes has the initialization phase take longer than the clustering phase. A faster initialization method than

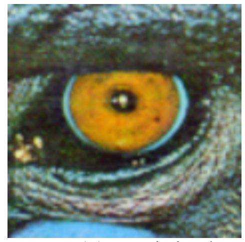
maximin can be used to help compensate for this but implementation convenience and portability is sacrificed as a result.

- The image used makes a huge impact on the execution time of BKM. For example, for K = 64, BKM takes ≈ 14.2s on Baboon, whereas clustering for Peppers takes ≈ 6.3s. This is a huge difference and can be explained by the amount of colors and the complexity of the image (higher complexity means higher execution time).

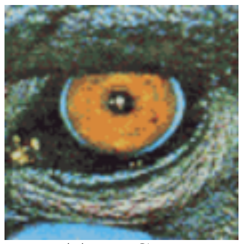
Images Quantized Using Various CQ Methods

Figures 2 and 4 below show sample quantization results for close-up parts of Baboon, Peppers, and Pills images. Figures 3 and 5 show the full-scale error images for these images. The error image for a particular CQ method was obtained by taking the pixelwise absolute difference between the original and quantized images. For better visualization, pixel values of the error images were multiplied by 4 and then negated.

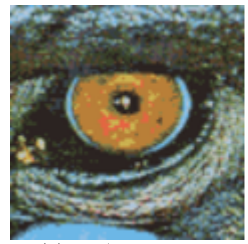
Figure 2. Baboon output images (K = 32)



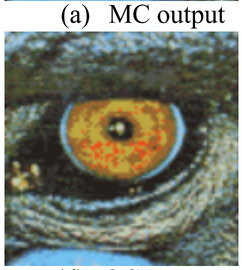
(a) Original



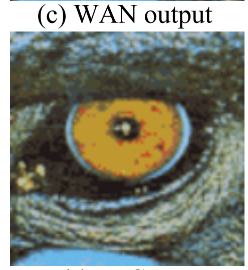
(a) MC output



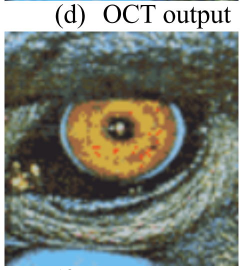
(c) WAN output



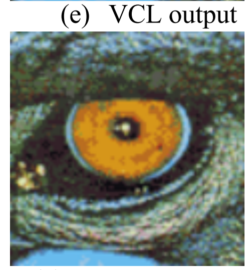
(d) OCT output



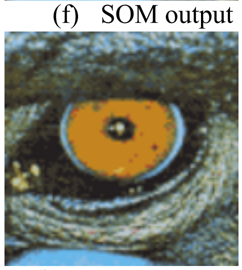
(e) VCL output



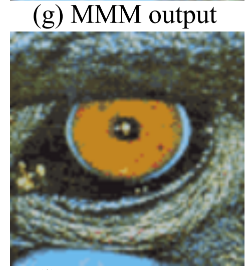
(f) SOM output



(g) MMM output



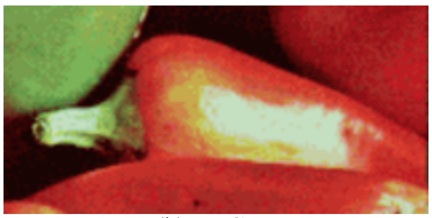
(h) BKM output



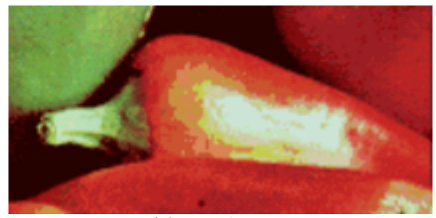
(i) MKM_q output

Figure 3. Peppers output images (K = 64)

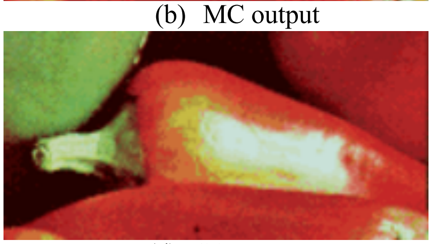
(a) Original



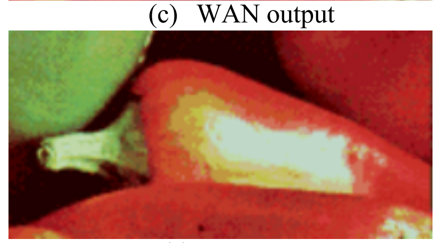
(b) MC output



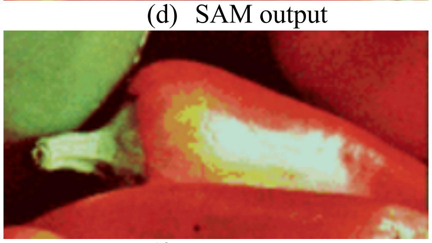
(c) WAN output



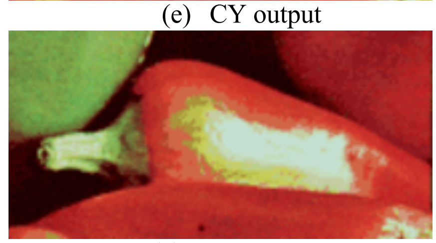
(d) SAM output



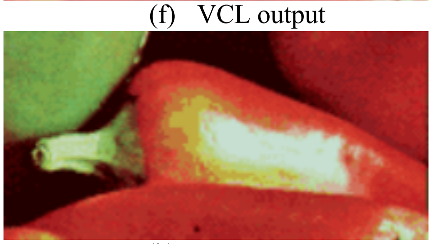
(e) CY output



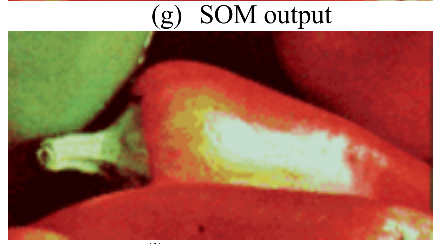
(f) VCL output



(g) SOM output

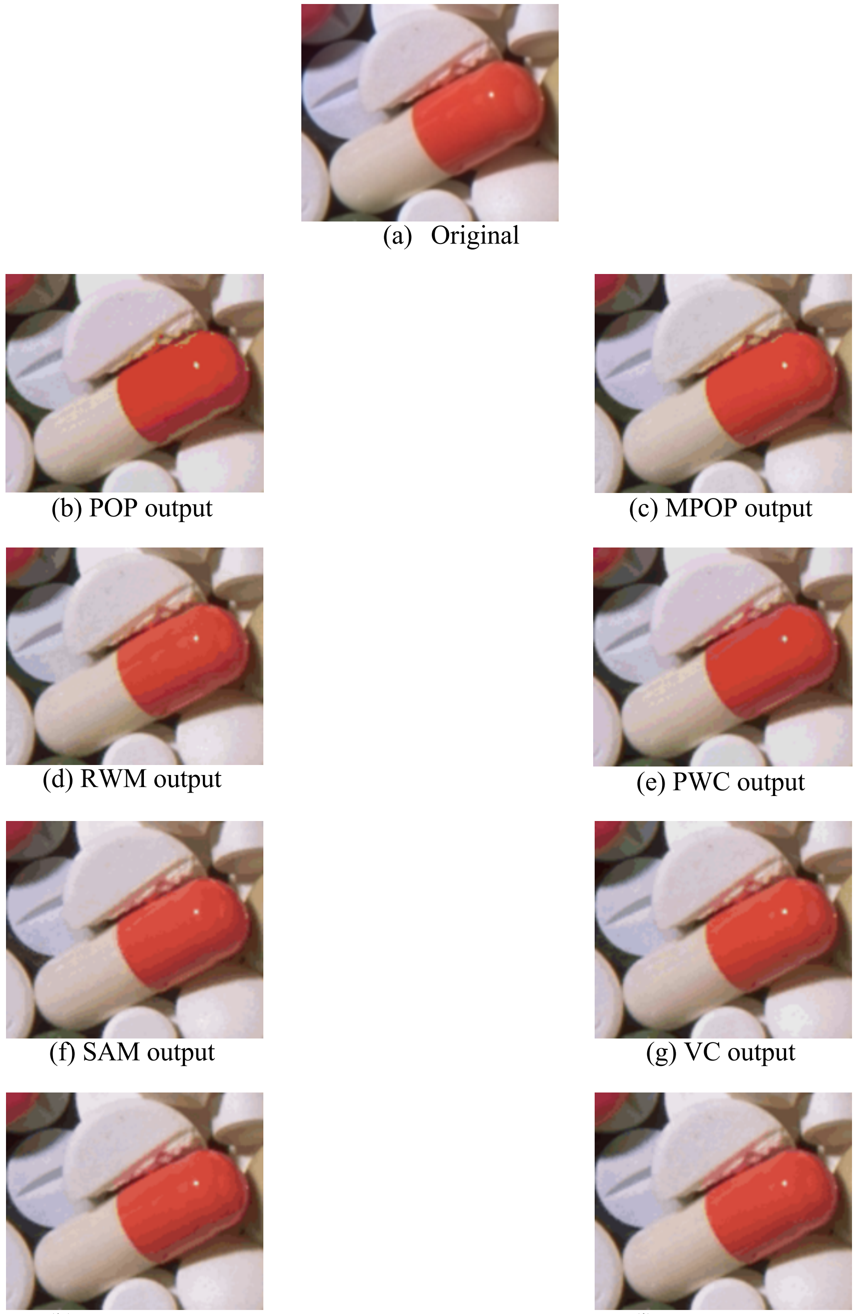


(h) BKM output



(i) MKM_q output

Figure 4. Pills output images (K=128)



(a) Original

(b) POP output

(c) MPOP output

(d) RWM output

(e) PWC output

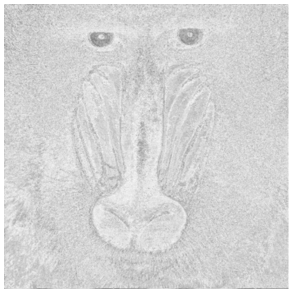
(f) SAM output

(g) VC output

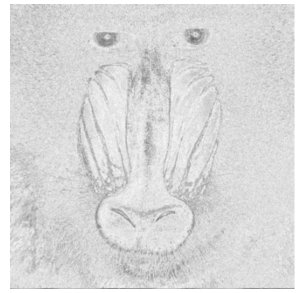
(h) BKM output

(i) MKM_q output

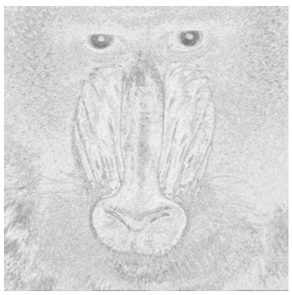
Figure 5. Baboon error images (K=32)



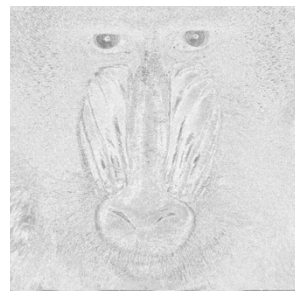
(a) MC error



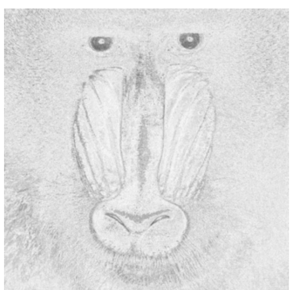
(b) WAN error



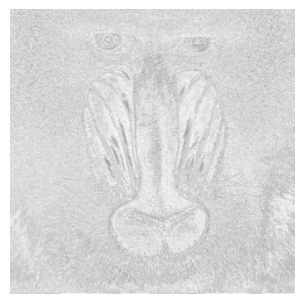
(c) OCT error



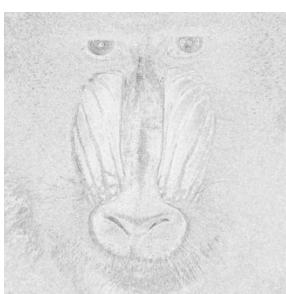
(d) VCL error



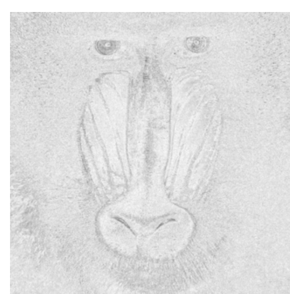
(e) SOM error



(f) MMM error

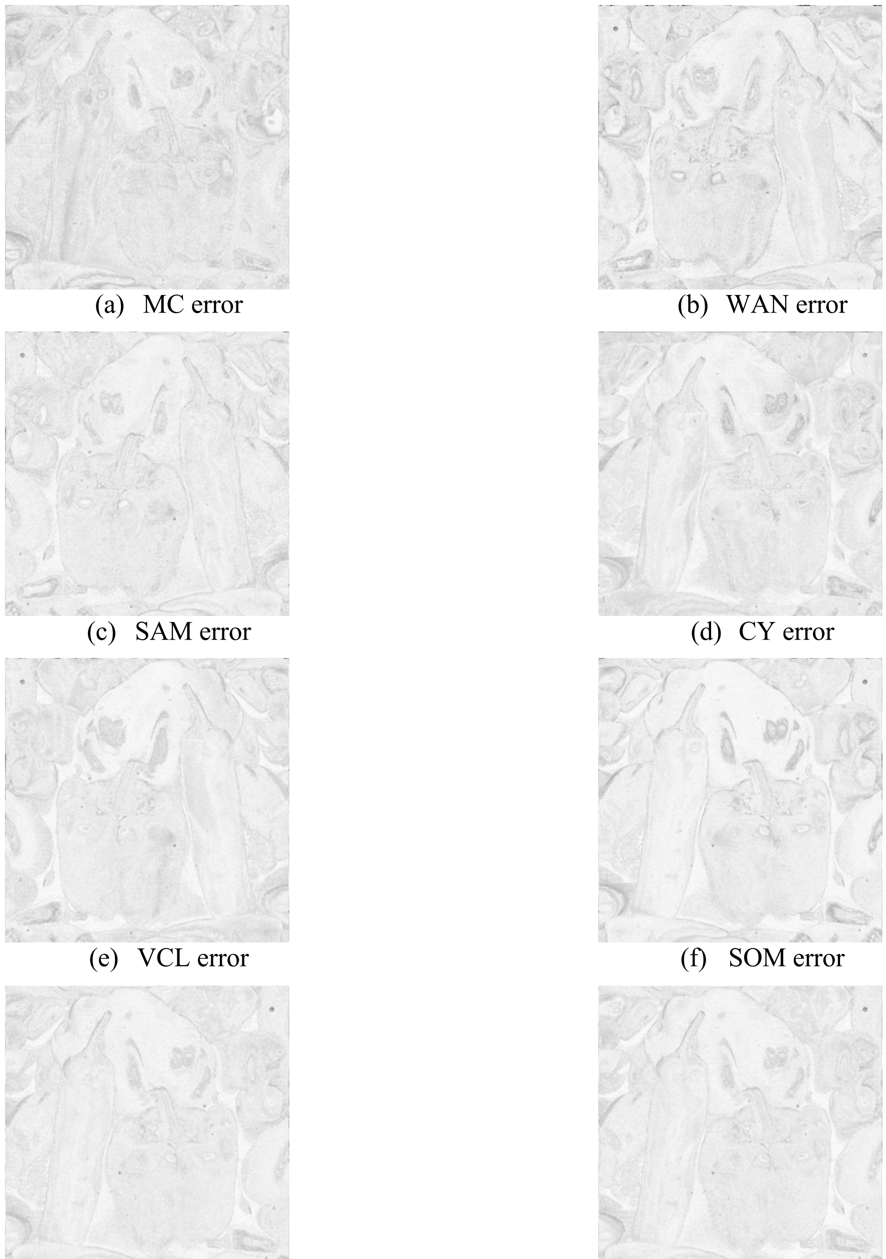


(g) BKM error



(h) MKM_q error

Figure 6. Peppers error images (K=64)



(a) MC error

(b) WAN error

(c) SAM error

(d) CY error

(e) VCL error

(f) SOM error

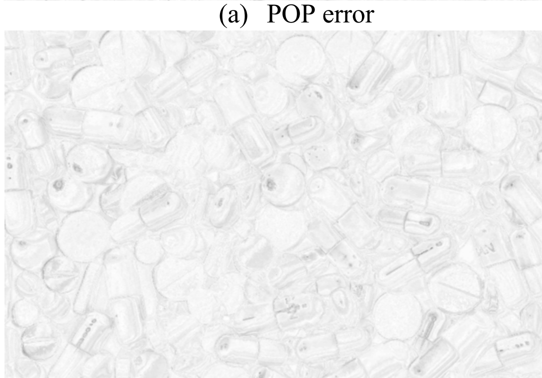
(g) BKM error

(h) MKM_q error

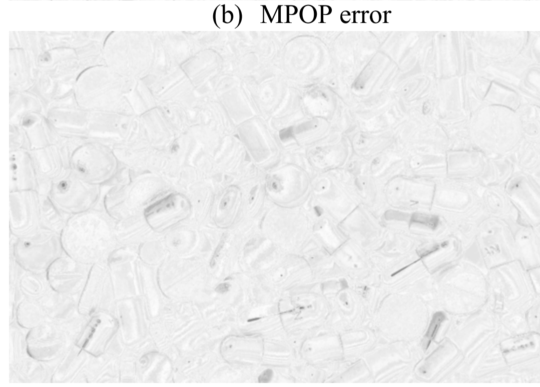
Figure 7. Pills error images (K = 128)

(a) POP error

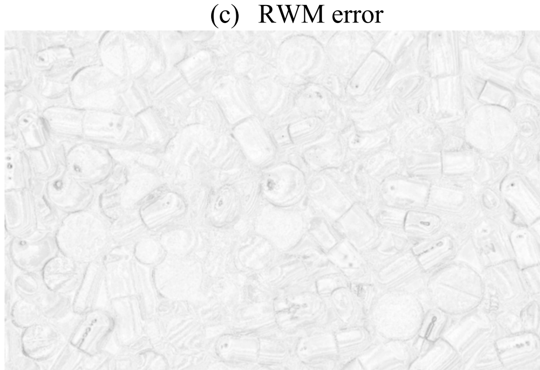
(b) MPOP error



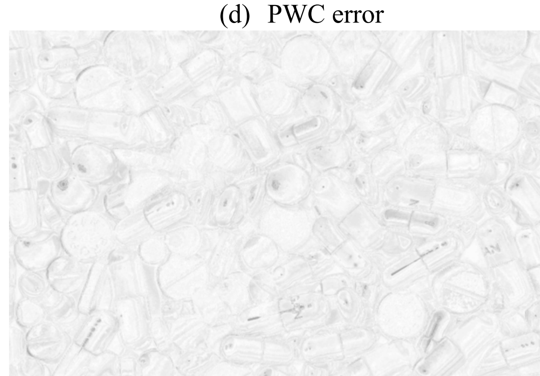
(c) RWM error



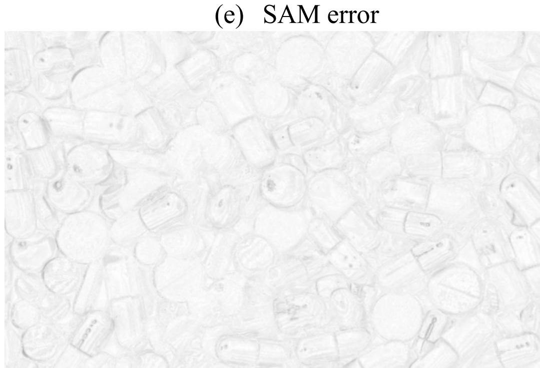
(d) PWC error



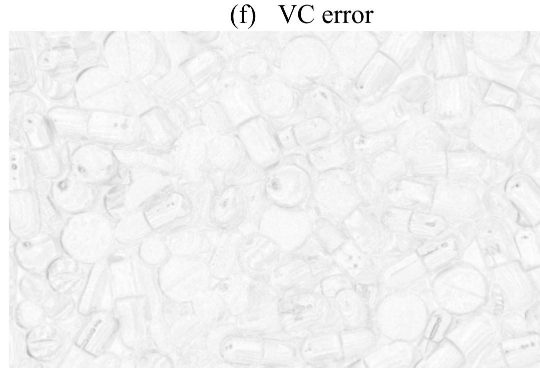
(e) SAM error



(f) VC error



(g) BKM error



(h) MKM_q error

CHAPTER 5: CONCLUSIONS AND FUTURE WORK

In this thesis, a variation of MacQueen's online k-means algorithm was introduced as a means to quantize color images. A series of experiments were performed in order to determine the best parameter combination for the proposed algorithm. Included within the tested algorithms, was both a pseudo-random and quasi-random implementation of MacQueen's k-means clustering algorithm. Both utilized maximin as the initialization method due to its deterministic nature and superior results to that of random selection. The pseudo-random implementation utilized the MT19937 generator and the quasi-random implementation used the Sobol' sequence. The results of the experiment demonstrated that MacQueen's algorithm was comparable to Lloyd's algorithm in effectiveness and was several times faster.

In the experiments, MacQueen's algorithm was compared against a number of CQ algorithms: popularity, median-cut, modified popularity, octree, variance-based method, greedy orthogonal bipartitioning, center-cut, radius-weighted mean-cut, pairwise clustering, split and merge, Cheng and Yang, variance-cut, variance-cut with Lloyd iterations, self-organizing map, and modified maximin. Along with these were two custom implementations of Lloyd's algorithm. While Lloyd's algorithm performed best in nearly all scenarios, typically the resulting difference between it and MacQueen's algorithm was negligible. The primary difference between the two was speed. MacQueen's algorithm consistently performed many times faster than Lloyd's due to its online nature.

In this study, only maximin was used as an initialization method. Many other initialization methods have been developed (Celebi *et al*., 2013), and through further experimentation, perhaps faster and better results could be achieved. The quasi-random variant of MacQueen's algorithm was of particular interest during this study due to its deterministic nature, and the only presentation method tested for this was the Sobol' sequence. It would be interesting to compare other sampling methods (Ros and Guillaume, 2020), such as coreset sampling (Valenzuela *et al.*, 2018) to perhaps achieve better results.

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