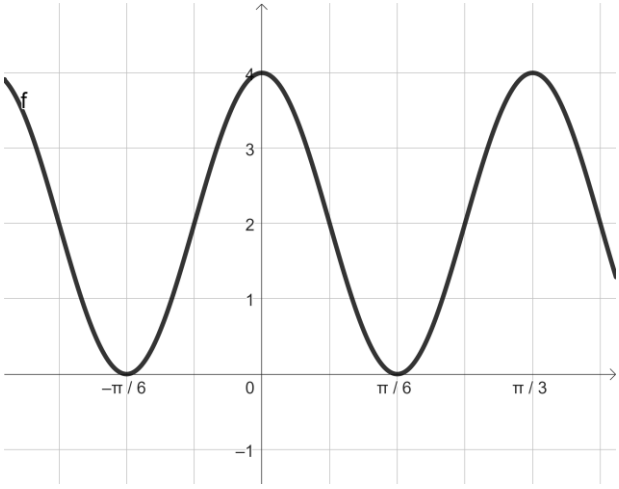
Work on the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

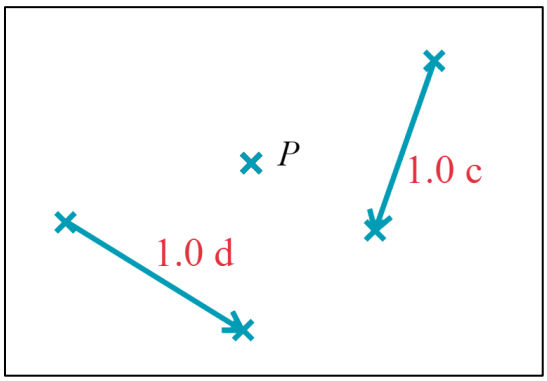
Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

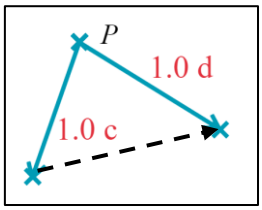
1. Let $p(x)$ be a quadratic function of the form $p(x) = x^2 + bx + c$, where b and c are integers. A rational function $r(x) = \frac{p(x)}{x+2}$ has a hole at the point $(-2, -5)$. Which of the following is the value of b?
   A. -6
   B. -5
   C. -3
   D. -1
   E. None of the above.

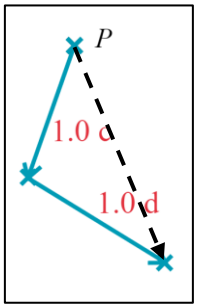
2. The graph of a periodic function is shown beside. Which of the following intervals could be used to restrict the domain to allow an inverse to be created?
   A. $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$
   B. $\left[0, \frac{\pi}{3}\right]$
   C. $\left[0, \frac{\pi}{6}\right]$
   D. $[0, 4]$
   E. $\left[-\frac{\pi}{12}, \frac{\pi}{12}\right]$

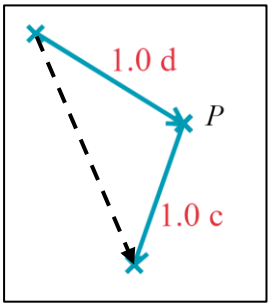


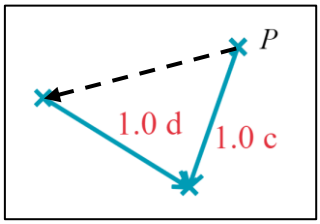
3. Which of the following correctly represents the resultant vector $\boldsymbol{c} + \boldsymbol{d}$ having initial point P given the vectors shown beside?



A. 

B. 

C. 

D. 

E. None of the above.

4. Give the radian measure of the angle that subtends $\frac{9}{16}$ of the circumference of the unit circle.

A. $\frac{9}{16}$

B. $\frac{9}{16}\pi$

C. $\frac{9}{8}$

D. $\frac{9}{8}\pi$

E. Not enough information.

5. Let θ be an angle in quadrant II such that $\sin\theta = a$, where $a > 0$. Write an expression that gives the exact value of $\sec\theta$.

   A. $\frac{1}{1-a}$

   B. $-\frac{1}{\sqrt{1-a^2}}$

   C. $-\frac{1}{1-a}$

   D. $\frac{1}{\sqrt{1-a^2}}$

   E. None of the above.

6. The following matrix gives the number of vehicles sold in January at a car dealership. In February, the new car sales decreased by 2, but the new truck sales increased by 25%. The used car sales increased by 50% while the used truck sales increased by 9. Write the vehicle sales matrix for February.

$$\begin{matrix} & \text{Cars} & \text{Trucks} \\ \text{New} \\ \text{Used} \end{matrix} \begin{bmatrix} 15 & 12 \\ 22 & 14 \end{bmatrix}$$

   A. $\begin{bmatrix} 13 & 15 \\ 33 & 24 \end{bmatrix}$

   B. $\begin{bmatrix} 13 & 15 \\ 32 & 23 \end{bmatrix}$

   C. $\begin{bmatrix} 17 & 15 \\ 33 & 23 \end{bmatrix}$

   D. $\begin{bmatrix} 13 & 16 \\ 33 & 23 \end{bmatrix}$

   E. None of the above

7. A and B are 3×3 matrices, and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Which of the following is NOT true for all possible matrices of this size?

   A. $AB = BA$

   B. $AI = IA$

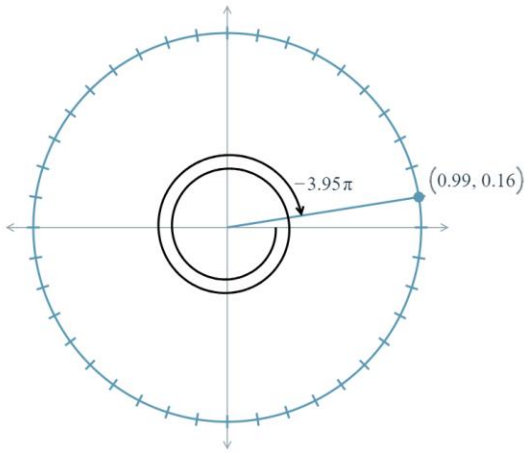
   C. $A + B = B + A$

   D. $IB = B$

   E. $A - B = A + (-B)$

8. Which of the following gives the direction angle of the vector $\boldsymbol{v} = -3\boldsymbol{i} - 5\boldsymbol{j}$? That is, which formula would find the angle between 0° and 360° that $\boldsymbol{v}$ makes with the positive x-axis (measured counterclockwise), when $\boldsymbol{v}$ is in standard position.
   A. $\tan\left(\frac{5}{3}\right)$
   B. $\arctan\left(\frac{5}{3}\right)$
   C. $\arctan\left(-\frac{5}{3}\right)$
   D. $\arctan\left(-\frac{5}{3}\right) + 180°$
   E. None of the above.

9. The figure on the right gives the ordered pair on the unit circle that corresponds to the angle -3.95π. Use the coordinates shown, which are rounded to the hundredths place, to find $\sec(-2.95\pi)$.



   A. $\sec(-2.95\pi) = 0.99$
   B. $\sec(-2.95\pi) = 0.16$
   C. $\sec(-2.95\pi) = -\frac{1}{0.99}$
   D. $\sec(-2.95\pi) = -\frac{1}{0.16}$
   E. None of the above.

10. Which of the following is the same as $\cos^2(\theta) - 1$?
   A. $\sin^2(\theta)$
   B. $\sec^2(\theta)$
   C. $-\sin^2(\theta)$
   D. $-\sec^2(\theta)$
   E. None of the above.

11. An equilateral triangle has been bisected to create a right triangle. If the hypotenuse of this triangle is 1 inch, what is the length of the longer leg?
    A. $\frac{\sqrt{2}}{2}$
    B. $\frac{\sqrt{3}}{2}$
    C. $\frac{1}{2}$
    D. 1
    E. Not enough information is provided to determine this length.

12. Simplify the following expression:
$$\cos(x+y)\cos y + \sin(x+y)\sin y$$
    A. $\cos x$
    B. $\cos(x+2y)$
    C. 1
    D. $\sin x$
    E. None of the above are equivalent to the given expression.

13. Tighnari is fighting the Hydro Hypostasis with his bow and arrow. The Hypostasis floats in the air above a point that is 25 meters away from Tighnari if measured horizontally along the ground. He notices that the distance his arrow needs to travel to hit the Hypostasis, d, varies with the angle of elevation between him and the Hypostasis, θ. Write a function T that calculated the distance Tighnari's arrow must travel to hit the Hypostasis in terms of the angle of elevation between him and the Hypostasis.
    A. $T(\theta) = 25\cos(\theta)$
    B. $T(d) = 25\sec(d)$
    C. $d(T) = \frac{\cos(T)}{25}$
    D. $T(\theta) = 25\csc(\theta)$
    E. None of the above.

14. A hitchhiker is standing at an intersection of roads X and Y. Road X forms a straight road to the east that passes first through City A and then through City B. Road Y is also straight, but angles N50°E and leads to City C, which is 75 miles from the hitchhiker's current location. If City A and City B are each 60 miles from City C, how far is the hitchhiker from City A?
    A. 73 miles
    B. 103 miles
    C. 28 miles
    D. 22 miles
    E. None of the above.

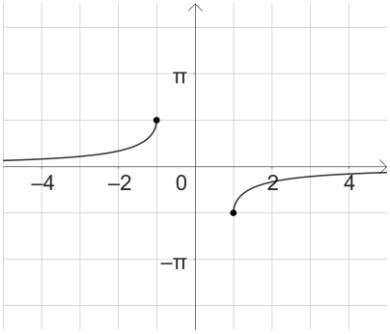
15. Which property is demonstrated by the following equation:

$$\cos(-\theta) = \cos(\theta)$$

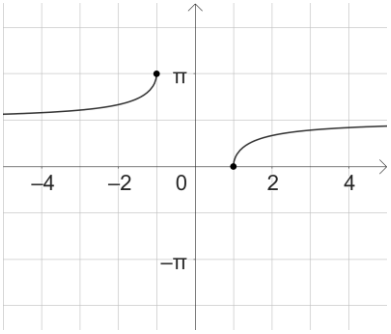
    A. Cosine is an even function.
    B. Cosine is an odd function.
    C. Cosine is a periodic function.
    D. Cosine is a reciprocal function.
    E. The equation is false, therefore there is no property demonstrated.

16. Which of the following represents the graph of $y = \text{arcsec}(x)$?

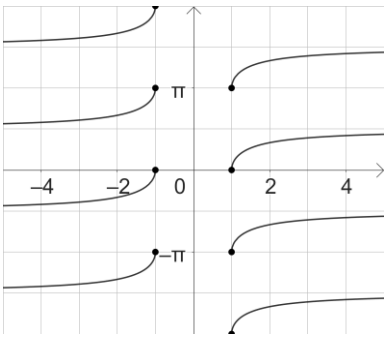
A.



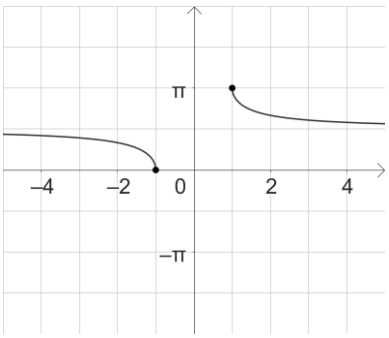
B.



C.

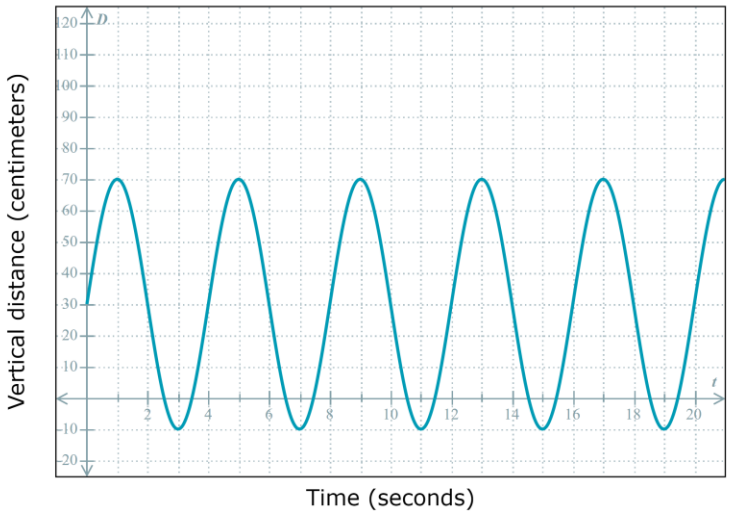


D.



    E. None of the above.

17. Bob pedals his water trike across the lake. The vertical distance between a sticker on the side of the wheel and the surface of the water is recorded. The vertical distance, D (in cm), as a function of time, t (in seconds), is shown in the graph beside. Assume the sinusoidal trend continues.



Relative to the surface of the water, what is the vertical distance of the sticker (in centimeters) at $t = 35$ seconds?

A. 70
B. 30
C. 10
D. -10
E. None of the above.

18. The pilot of an aircraft wishes to head directly east but is faced with a wind speed of 40 mph from the northwest. If the pilot maintains an airspeed of 250 mph, what is the actual speed of the aircraft (rounded to the nearest tenth)?

A. 223.5 mph
B. 250.0 mph
C. 279.7 mph
D. 281.1 mph
E. None of the above.

19. A pendulum is a weight attached to a fixed rod. Suppose that during an experiment, a pendulum moves back and forth in a periodic manner. At the beginning of the experiment, when the time is $t = 0$ seconds, the pendulum is at its maximum distance from the wall, 4 meters away from the wall. The pendulum first reaches its minimum distance from the wall, 1 meter from the wall, when $t = 2$ seconds. When $t = 4$ seconds, the pendulum is back to its maximum distance from the wall. The pendulum continues to move back and forth so that the distance between the pendulum and the wall over time can be modeled by a sinusoidal function. Let $f(t)$ be the distance between the wall and the pendulum t seconds after the beginning of the experiment. Select an equation for $f(t)$.

    A. $f(t) = 3\cos\left(\frac{\pi}{2}t\right) + 2.5$
    B. $f(t) = 1.5\cos(4t)$
    C. $f(t) = 3\sin\left(\frac{\pi}{2}t\right) + 2.5$
    D. $f(t) = 1.5\sin(4t)$
    E. None of the above.

20. Determine the standard form of an equation of the parabola with focus $(1, 2)$ and directrix $x = 7$.

    A. $(y-4)^2 = -12(x-2)$
    B. $(y-2)^2 = -12(x-4)$
    C. $(x-2)^2 = 12(x-4)$
    D. $(x-4)^2 = 12(x-2)$
    E. None of the above.

21. Which of the following is the equation of an asymptote of this hyperbola.

$$-\frac{(x-2)^2}{9} + (y+3)^2 = 1$$

    A. $y = 3x - 9$
    B. $y = \frac{1}{3}x - \frac{11}{3}$
    C. $y = -3x + 3$
    D. $y = \frac{1}{3}x - \frac{7}{3}$
    E. None of the above

22. Complete the square to write the equation of the circle in standard form. What is the center and radius of the circle?

$$x^2 + y^2 + 10x - 6y + 25 = 0$$

A. Center: $(-5, 3)$; Radius: 3
B. Center: $(5, -3)$; Radius: 5
C. Center: $(-5, 3)$; Radius: 5
D. Center: $(5, -3)$; Radius: 3
E. None of the above.

23. Suppose an individual is paid \$0.01 on day 1 and every day thereafter, the payment is doubled. What is the total amount earned in 30 days?

A. \$5,368,709.11
B. \$5,368,709.12
C. \$10,737,418.24
D. \$10,737,418.23
E. None of the above.

24. Suppose f, g, h, and j are functions such that:

- $f(r)$ represents the circumference (in cm) of a circle whose radius is r cm.
- $g(C)$ represents the radius (in cm) of a circle whose circumference is C cm.
- $h(r)$ represents the area (in cm^2) of a circle whose radius is r cm.
- $j(A)$ represents the radius (in cm) of a circle whose area is A cm^2.

Use function notation to represent the area of a circle whose circumference is 108 cm.

A. $h\big(g(108)\big)$
B. $g\big(h(108)\big)$
C. $f\big(g(108)\big)$
D. $g\big(j(108)\big)$
E. None of the above.

25. Which of the following describes the end behavior of the function $f(x) = ax^p$ if $a < 0$ and p is even?

A. As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$.
B. As $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$.
C. As $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$.
D. As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.
E. Not enough information is given to know the end behavior.

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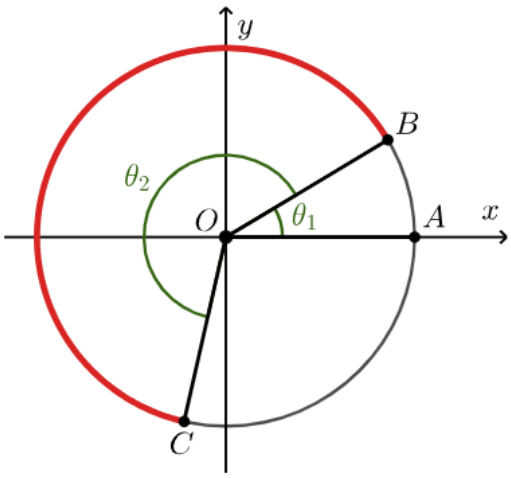
Tie Breaker #1

Name: _____

School: _____

Show all your work.

Consider the figure shown to the right. Suppose that $O = (0, 0)$, $A = (3.4, 0)$, and $B = (2.81, 1.92)$.
The arc between B and C is 13.056 units long.
What are the x and y coordinates of point C?



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Tie Breaker #2

Name: _____

School: _____

Prove or disprove this trig identity. Show all your work.

$$\cos(x - y)\sin x - \sin(x - y)\cos x = \sin y$$

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Tie Breaker #3

Name: _____

School: _____

Show all your work.

Expand using the binomial theorem:

$$(3x^2 - 5y)^5$$

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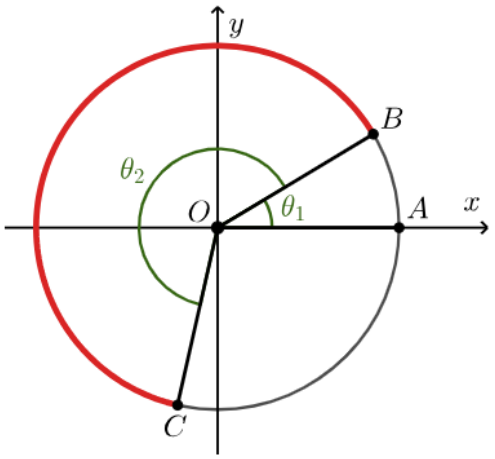
Multiple Choice Answer Key

1. D	11. B	21. B
2. C	12. A	22. A
3. B	13. E	23. D
4. D	14. D	24. A
5. B	15. A	25. C
6. E	16. B	
7. A	17. D	
8. E	18. C	
9. C	19. E	
10. C	20. B	

Tie Breaker #1 Solution

Consider Figure 5 shown to the right. Suppose that $O = (0, 0)$, $A = (3.4, 0)$, and $B = (2.81, 1.92)$. The arc between B and C is 13.056 units long. What are the x and y coordinates of point C? Do not round any intermediate computations, but round your final coordinates to the nearest thousandth.



$$\cos(\theta_1) = \frac{2.81}{3.4} \rightarrow \theta_1 = \arccos\left(\frac{2.81}{3.4}\right)$$

$$\theta_2 = \frac{13.056}{3.4} = 3.84$$

The x-value of C is…

$$3.4\cos(\theta_1 + \theta_2) =$$

$$= 3.4\cos\left(arccos\left(\tfrac{2.81}{3.4}\right) + 3.84\right)$$

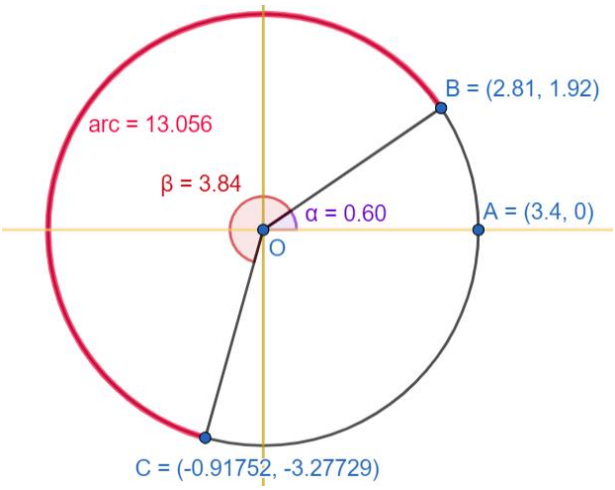
$$\approx -0.921$$

The y-value of C is…

$$3.4\sin(\theta_1 + \theta_2) =$$

$$= 3.4\sin\left(arccos\left(\tfrac{2.81}{3.4}\right) + 3.84\right)$$

$$\approx -3.273$$



Tie Breaker #2 Solution

Prove or disprove this trig identity.

$$\cos x \cos(x+y) + \sin x \sin(x+y) = \cos y$$

Statement	Rule
$\cos x \cos(x+y) + \sin x \sin(x+y)$	
$\cos\big(x-(x+y)\big)$	Difference Identity for Cosine
$\cos(-y)$	Algebra
$\cos(y)$	Even Identity for Cosine

OR

Statement	Rule
$\cos x \cos(x+y) + \sin x \sin(x+y)$	
$\cos x\,(\cos x \cos y - \sin x \sin y) + \sin x\,(\sin x \cos y + \cos x \sin y)$	Difference Identity for Cosine Sum Identity for Sine
$\cos^2 x \cos y - \cos x \sin x \sin y + \sin^2 x \cos y + \cos x \sin x \sin y$	Algebra
$\cos y\,(\cos^2 x + \sin^2 x)$	Algebra
$\cos y$	Pythagorean Identity

Tie Breaker #3

Expand using the binomial theorem: $(3x^2 - 5y)^5$

The entries of the sixth row of Pascal's triangle are: 1, 5, 10, 10, 5, 1

$(3x^2 - 5y)^5 =$

$$= 1(3x^2)^5 + 5(3x^2)^4(-5y) + 10(3x^2)^3(-5y)^2 + 10(3x^2)^2(-5y)^3 + 5(3x^2)(-5y)^4 + 1(-5y)^5$$

$$= 243x^{10} - 2025x^8y + 6750x^6y^2 - 11250x^4y^3 + 9375x^2y^4 - 3125y^5$$