$\qquad$ Arkansas Council of Teachers of Mathematics
Begin by removing the three tie breaker sheets at the end and writing your name on all three pages. Work on the multiple-choice questions first, choosing the single best (most detailed and completely correct) response from the choices available. Indicate your answer here and on your answer sheet. Make sure you attempt the tie-breaker questions at the end of the test starting with tie breaker 1 , then 2 , and then 3 if you have time. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

## Figures are not necessarily drawn to scale. All angles are in degrees.

1. Which of the following are NOT the sides of a triangle?
A. $3,3,1$
B. $5,6,12$
C. $3,4,5$
D. $9,9,9$
E. $7,8,13$
2. The area of an isosceles right triangle is what fraction of the square of the length of the hypotenuse?
A. One-half
B. One- third
C. One-fourth
D. One-fifth
E. None of the above
3. If each of two isosceles triangles have an angle that measures $120^{\circ}$, then the two isosceles triangles:
A. must be congruent
B. must be similar
C. Could be similar
D. Could not be congruent
E. None of the above
4. If $\overline{A D}$ bisects $\angle A$, and $A B=6 f t, A C=9 f t$, and $B D=2 f t$, then $D C$ is.
A. 5 ft
B. $\sqrt{17} \mathrm{ft}$
C. 3 ft
D. 4.5 ft
E. None of the above

5. Given $\triangle A B C$ with $X, Y, Z$ midpoints of the respective sides with $A B=10 \mathrm{~cm}, B C=14 \mathrm{~cm}$, and $A C=18 \mathrm{~cm}$, what is the perimeter of $\triangle X Y Z$ ?
A. 42 cm
B. 21 cm
C. 14 cm

D. 10.5 cm
E. None of the above
6. Given circle $Q$ as shown, where $m \overparen{X Y}: m \overparen{Y Z}: m \overparen{Z X}=5: 6: 7$, what is $m \overparen{X Y}$ in degrees?
A. $100^{\circ}$
B. $120^{\circ}$
C. $50^{\circ}$
D. $60^{\circ}$
E. None of the above

7. Suppose $m \overparen{D G}=110^{\circ}$ and $m \overparen{B F}=40^{\circ}$, what is the measure of $\angle A$ in the diagram beside?
A. $75^{\circ}$
B. $45^{\circ}$
C. $70^{\circ}$
D. $35^{\circ}$
E. None of the above

8. What is the measure of each central angle in a regular octagon?
A. $60^{\circ}$
B. $30^{\circ}$
C. $45^{\circ}$
D. $90^{\circ}$
E. None of the above
9. What is the area of a slice of pizza cut from a pizza with diameter 20 inches if the arc of the slice measures 30 degrees?
A. $\frac{100}{3} \pi \mathrm{in}^{2}$
B. $\frac{5}{6} \pi i n^{2}$
C. $\frac{1}{18} \pi i n^{2}$
D. $\frac{25}{3} \pi i n^{2}$
E. None of the above
10. If quadrilateral $A B C D$ is a rhombus and $A$ is the point $(-1,3)$, while $C$ has coordinates $(1,-1)$, what is the slope of diagonal $\overline{B D}$ ?
A. -2
B. $-1 / 2$
C. 2
D. $1 / 2$
E. None of the above
11. Calculate the perimeter of the rectangle shown beside. Round your answer to the nearest whole number.
A. 141 m
B. 169 m
C. 155 m
D. 77 m

E. None of the above.
12. In the figure beside, lines $l$ and $m$ are parallel and cut by transversal $t$. Which of these expressions must add to $360^{\circ}$ ?
A. $A+B+Y+X$
B. $W+X+A+B$
C. $A+B+C+W$
D. $Z+X+D+C$
E. $C+D+X+Y$

13. Which of the following must be true for $\triangle A B C$ where $\overline{A B}$ is congruent to $\overline{B C}$ and $\overline{A C}$ is 1.5 times the length of $\overline{A B}$ ?
A. $m \angle B C A>m \angle B A C$
B. $m \angle B C A>m \angle C B A$
C. $m \angle A B C>m \angle B C A$
D. $m \angle A B C<m \angle B C A$
E. $A B$ is congruent to $A C$
14. Given $\overline{A E} \perp \overline{E D}, \overline{D B} \perp \overline{A C}$, and $m \angle C=30^{\circ}$, find the measure of angle 3 .
A. $120^{\circ}$
B. $80^{\circ}$
C. $150^{\circ}$
D. $145^{\circ}$
E. None of the above

15. The diagonals of a kite are
A. Are perpendicular bisectors of each other.
B. Are bisectors of each other.
C. Are perpendicular.
D. Are angle bisectors of the vertex angles.
E. Divide the kite into four congruent triangles.
16. What is the center of the circle defined by the equation $x^{2}+y^{2}-10 x+8 y+29=0$ ?
A. $(-5,4)$
B. $(5,-4)$
C. $(25,16)$
D. $(10,-8)$
E. None of the above
17. Assume that line $l$ is perpendicular to line $k$ and the angles are as shown in the figure beside. What is the measure of angle $x$ ?
A. $20^{\circ}$
B. $25^{\circ}$
C. $45^{\circ}$
D. $65^{\circ}$
E. None of the above

18. A large window composed of a rectangle and a semicircle is shown below with a total area of $16,000 \mathrm{~cm}^{2}$. If the window has a width of 80 cm , how tall is the window?
A. 200 cm
B. $200-10 \pi \mathrm{~cm}$
C. $240-10 \pi \mathrm{~cm}$
D. $\sqrt{120}+\frac{40}{\pi} \mathrm{~cm}$

E. $\sqrt{240}-\frac{40}{\pi} \mathrm{~cm}$
19. Congruent parts of two triangles are indicated beside on each triangle. Considering the congruent pairs marked, name the additional pair of parts that must be congruent in order to prove $\Delta W V Y$ and $\Delta Z V X$ are congruent using Angle-SideAngle (ASA).
A. $\angle W V Y$ and $\angle Z V X$
B. $\overline{W X}$ and $\overline{Z Y}$

C. $\angle W X V$ and $\angle Z X V$
D. $\angle X V Y$ and $\angle Y V X$
E. None of the above
20. For which of these polygons does a rotation of $120^{\circ}$ about the center of the polygon map the polygon onto itself?
A. Square
B. Regular hexagon
C. Regular octagon
D. Regular decagon
E. None of these
21. Lily designs a tent in the shape of a square pyramid. Each of the sides of the square base measures $x$ feet, and the tent's height is $h$ feet. If Lily were to increase the length of each side of the base by 1 foot, how much more interior space would the tent have?
A. $\frac{h\left(x^{2}+2 x+1\right)}{3} f t^{3}$
B. $\frac{h(2 x+1)}{3} f t^{3}$
C. $\frac{x^{2} h+3}{3} f t^{3}$
D. $1 \mathrm{ft}^{3}$
E. None of the above
22. Two lines with equations $3 x-2 y+7=0$ and $2 x+3 y-6=0$ are
A. Lines with the same slope
B. Perpendicular lines
C. The same line
D. Have undefined slopes
E. None of these
23. Rory while walking away from a 10 - ft lamppost casts a shadow 6 ft long. If Rory is at a distance 10 ft from the lamppost at the moment, what is Rory's height?
A. 3 ft 3 in .
B. 3 ft 7 in .
C. 3 ft 9 in .
D. 6 ft 4 in .
E. 6 ft 7 in .
24. The vertices of a polygon are $(2,3),(8,1),(6,-5)$, and $(0,-3)$. Which of the following describes the polygon most specifically?
A. Parallelogram
B. Trapezoid
C. Rectangle
D. Square
E. None of the above
25. A cylindrical carrot stick is sliced with a knife. Which of the following shapes is not a possible crosssection?
A. Circle
B. Rectangle
C. Ellipse
D. Triangle
E. None of the above

## Tie Breaker 1

Name: $\qquad$

School: $\qquad$

Find the length $d$ of a diagonal of a cube with sides of length $x$ in the figure on the right.


## Tie Breaker 2

## Name:

$\qquad$

School: $\qquad$

Find the area of $\triangle A O B$ with altitude $\overline{O D}$ inside the circle in the figure beside if the measure of $\angle A O B=60^{\circ}$ and $A O=20 \mathrm{~cm}$ to the nearest tenth of a square centimeter. Explain your reasoning.


## Tie Breaker 3

Name: $\qquad$

School: $\qquad$

A rainbow pattern is designed from semi-circles as shown beside. What is the area of the shaded region as a function of $x$ ?


| 1. B | 11. C | 21. B |
| :--- | :--- | :--- |
| 2. C | 12. A | 22. B |
| 3. B | 13. C | 23. C |
| 4. C | 14. A | 24. D |
| 5. B | 15. C |  |
| 6. A | 16. B |  |
| 7. D | 17. C |  |
| 8. C | 18. C |  |
| 9. D | 19. A |  |
| 10. D | 20. B |  |

## Tie Breaker \#1 Solution

Answer: The length of $d$ is given by $d=\sqrt{3} x$.

Let c be the hypotenuse of the square base when the base is divided into two right triangles. Using the Pythagorean Theorem, we have $x^{2}+x^{2}=c^{2}$. Then the length of the diagonal of the base is $c=\sqrt{2 x^{2}}$, or $c=\sqrt{2} x$.
Notice that the diagonal of the base and the vertical side of the cube form a right triangle. Now that we have the length of the diagonal, we can use this length and the vertical length of the cube to create the equation
$x^{2}+c^{2}=d^{2}$. Using substitution, we have $x^{2}+(\sqrt{2} x)^{2}=d^{2}$. Simplifying we have

$$
\begin{gathered}
x^{2}+2 x^{2}=d^{2} \\
3 x^{2}=d^{2}
\end{gathered}
$$

Thus, $d=\sqrt{3} x$.

Answer: The area is $100 \sqrt{3} \mathrm{~cm}^{2}$.

First, find the area of the triangle. Sinc e $\overline{O B}$ and $\overline{O A}$ are radii of the circle, $\triangle A O B$ is isosceles and $\angle A \cong \angle B$. Thus, all angles in $\triangle A O B$ are 60 degrees and the triangle is equiangular, and therefore, equilateral. This makes $A B=20 \mathrm{~cm}$. Also, $\triangle B O D$ is a $30^{\circ}-60^{\circ}-90^{\circ}$ triangle since the altitude $\overline{O D}$ is perpendicular to the base. Using the $30^{\circ}-60^{\circ}-90^{\circ}$ triangle relationship, $\mathrm{DB}=10 \mathrm{~cm}, \mathrm{OD}=10 \sqrt{3} \mathrm{~cm}$ because the hypotenuse is 20 cm .
Then the area of $\triangle A O B=\frac{1}{2}(A B)(O B)=\frac{1}{2}(20)(10 \sqrt{3})=100 \sqrt{3} \mathrm{~cm}^{2}$

Tie Breaker \#3 Solution

Answer: The area is $\frac{21 \mathrm{x}^{2} \pi}{2}$.

If we consider the area of the largest semicircle with nothing removed (if we had a solid, filled-in semicircle), we would have $r=6 x$ and so the area of the largest semicircle would be $\frac{\pi(6 x)^{2}}{2}$.

Next, if we look at the second largest semicircle possible (had we filled in the semicircle starting with the second longest shaded area) we would have $r=4 x$. Then the area of the second largest semicircle would be $\frac{\pi(4 x)^{2}}{2}$. The third smallest semicircle would have a radius of $r=2 x$ and area of $\frac{\pi(2 x)^{2}}{2}$. Then because we have not subtracted the blank (or missing) portions of the diagram. We can think of those blank or missing areas as missing semicircles. The area of the first missing semicircle is $\frac{\pi(5 x)^{2}}{2}$ because it will have a radius of $r=5 x$. The next largest missing semicircle will have area $\frac{\pi(3 x)^{2}}{2}$, and the smallest missing semicircle with halve area $\frac{\pi x^{2}}{2}$.

Now we add the area of the shaded semicircles and subtract the areas of the missing semicircles to find the total area of the shaded region.
This gives us $\frac{\pi\left(36 x^{2}-25 x^{2}+16 x^{2}-9 x^{2}+4 x^{2}-x^{2}\right)}{2}=\frac{\left(21 x^{2}\right) \pi}{2}$

