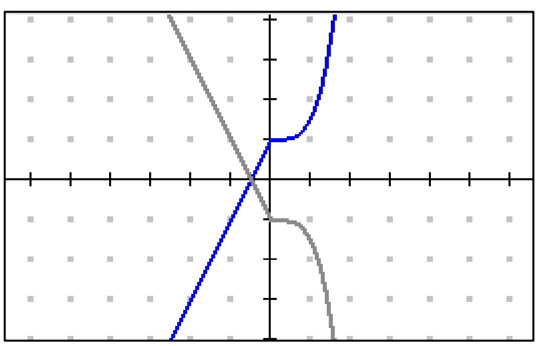
Arkansas Council of Teachers of Mathematics

Work on the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet, your tie-breaker pages, and your scratch work when you are finished. Figures are not necessarily drawn to scale.

1. Consider complex numbers $z_1 = 2 + 3i$ and $z_2 = 1 + 2i$. Which of the following is not true?
   A. $\frac{z_1}{z_2} = \frac{8}{5} - \frac{1}{5}i$
   B. $z_1 + z_2 = 3 + 5i$
   C. $z_1 \cdot z_2 = -4 + 7i$
   D. $|z_1 - z_2| = \sqrt{2}$
   E. All of the above are true.

2. For what values of b does the quadratic equation $x^2 + bx + 1 = 0$ have two distinct real solutions?
   A. $|b| < 2$
   B. $|b| \geq 2$
   C. $|b| \leq 2$
   D. $|b| > 2$
   E. None of the above

3. Which figure represents a function f and its inverse f^{-1}?

A.


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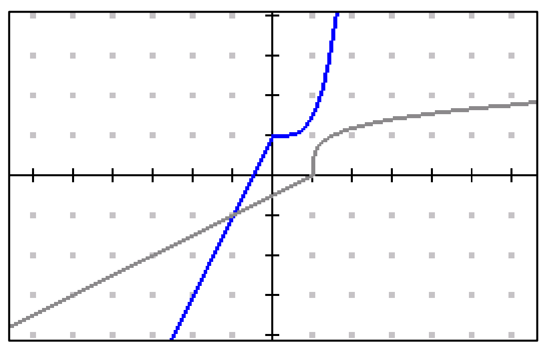

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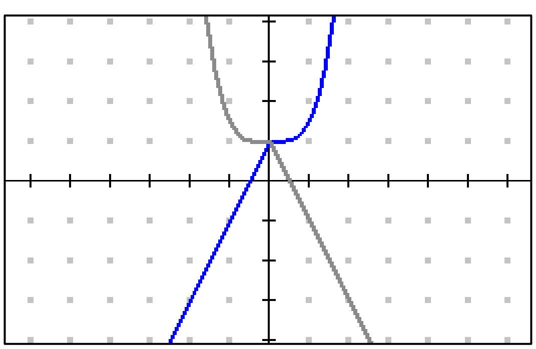

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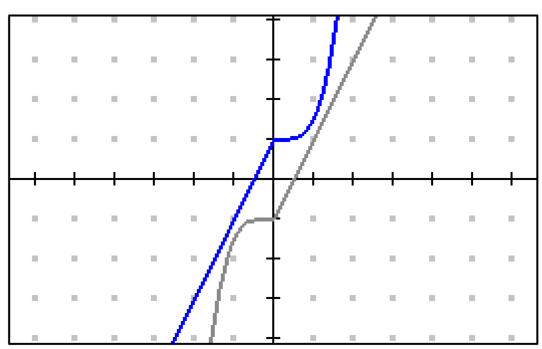


E. None of these graphs

4. Let $f(x) = \frac{3x+2}{x-4}$. Then the inverse function $f^{-1}(x)$ is given by
   - A. $f^{-1}(x) = \frac{x-4}{3x+2}$
   - B. $f^{-1}(x) = \frac{4x-2}{x+3}$
   - C. $f^{-1}(x) = \frac{4x+2}{x-3}$
   - D. $f^{-1}(x) = \frac{2x+3}{4x-1}$
   - E. The inverse function does not exist.

5. Let $f(x) = x^2 - 5$. If $h \neq 0$, the quotient $\frac{f(x+h)-f(x)}{h}$ equals
   - A. $2x + h$
   - B. h
   - C. $x + 2h$
   - D. $x^2 - h^2$
   - E. None of the above

6. Consider functions $f(x) = \frac{x+5}{x-3}$ and $g(x) = \frac{1}{x}$. Let $D_{f\circ g}$ represent the domain of the composition function $f \circ g$. Then
   - A. $D_{f\circ g} = (-\infty, \infty)$
   - B. $D_{f\circ g} = \left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, 0\right) \cup (0, \infty)$
   - C. $D_{f\circ g} = \left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$
   - D. $D_{f\circ g} = (-3,0) \cup (0, \infty)$
   - E. $D_{f\circ g}$ does not exist.

7. Find the area of the triangle formed by x-axis and lines $y = \frac{3}{2}x$ and $y = -x + 5$. (Use inches as the unit.)
   - A. 15 $inch^2$
   - B. 12.5 $inch^2$
   - C. 10 $inch^2$
   - D. 7.5 $inch^2$
   - E. 5 $inch^2$

8. Data from a laboratory experiment shows that the number of bacteria in a Petri dish grows at an exponential rate of 5.4% per week. Approximately how many months will it take for an initial population of 460 bacteria to quadruple in number? (Round nearest whole number of months.)
   A. 6 months
   B. 5 months
   C. 16 months
   D. 23 months
   E. None of the above

9. Over a two-year period from January 1, 2014, to December 30, 2016, the weekly price of a share of a company's stock was approximated by the function $P = -0.0137x^2 + 1.8243x + 67.8$, where x is the number of weeks since January 1, 2014, and P is the price of a share of the company. According to this model, how many weeks after January 1, 2014, were needed for the price of the stock to reach its highest value? (Round to the nearest integer.)
   A. 60 weeks
   B. 67 weeks
   C. 70 weeks
   D. 74 weeks
   E. None of the above

10. Rewrite the expression $5\log_5(x) + \log_5(y) - 3\log_5(z)$ as a single logarithm.
    A. $\log_5\left(\frac{x^5y}{z^3}\right)$
    B. $\log_5(3xy - z)$
    C. $\log_5\left(\frac{5xy}{z^3}\right)$
    D. $\log_5(5x + y - 3z)$
    E. Cannot be done.

11. Use the method of completing square to find the radius and the coordinates of the center of the circle whose equation is given by $x^2 + y^2 - 4x + 6y - 36 = 0$.
    A. Center: $(-3, 2)$, Radius $= 7$
    B. Center: $(-2, 3)$, Radius $= 6$
    C. Center: $(2, 3)$, Radius $= 6$
    D. Center: $(2, -3)$, Radius $= 7$
    E. Center: $(-2, 3)$, Radius $= 7$

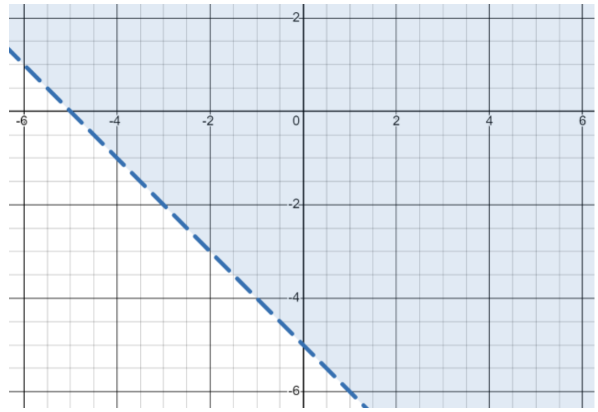
12. Suppose $P(x)$ is a polynomial of degree 6 with real coefficients and two x-intercepts. If $x_1 = 1 - 2i$ and $x_2 = 3 + 2i$ are two complex zeros of polynomial $P(x)$, then which other pairs below are also zeros of this polynomial?
    A. $x_3 = 1 - 2i$ & $x_4 = 3 + 2i$
    B. $x_3 = 1 + 2i$ & $x_4 = 3 - 2i$
    C. $x_3 = 2 - i$ & $x_4 = 2 + 3i$
    D. $x_3 = 2 + i$ & $x_4 = 3 - 2i$
    E. All of the above.

13. A farmer raises corn and soybeans on 350 acres of land. Because of the expected prices at harvest time, he thinks it would be wise to plant 100 more acres of corn than of soybeans. Which system of linear equations below should be used to model the number of acres that he must plant for each crop? (Let $x =$ acres of corn and let $y =$ acres of soybeans.)
    A. $\begin{cases} x + y = 100 \\ x + y = 350 \end{cases}$
    B. $\begin{cases} x - y = 350 \\ x + y = 100 \end{cases}$
    C. $\begin{cases} x + y = 350 \\ x - y = 100 \end{cases}$
    D. $\begin{cases} x - y = 100 \\ xy = 350 \end{cases}$
    E. Unable to determine.

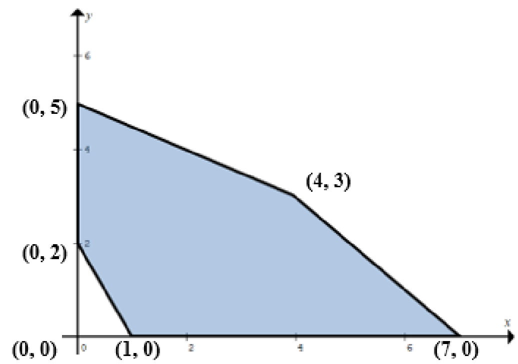
14. Continue the previous problem. How many acres of corn and soybeans should the farmer plant?
    A. 125 acres of corn and 225 acres of soybeans
    B. 225 acres of corn and 125 acres of soybeans
    C. 350 acres of corn and 0 acres of soybeans
    D. 175 acres of corn and 175 acres of soybeans
    E. Either 350 acres of corn or 350 acres of soybeans

15. The diameter of a car tire is 20 inches. How many times will this tire rotate if the car travelled a distance of 2 miles? (Round to the nearest integer.)
    A. 1008
    B. 2017
    C. 985
    D. 4034
    E. It cannot be determined.

16. Which inequality describes the region on the right?
    - A. $y \leq -5 - x$
    - B. $y \geq -5 - x$
    - C. $y > -5 - x$
    - D. $y < -5 - x$
    - E. None of the above.



17. Find the minimum and the maximum of objective function $z = 3x + 5y + 10$ over the feasible region shown on the right.
    - A. Minimum= 13, Maximum= 31
    - B. Minimum= 10, Maximum= 45
    - C. Minimum= 20, Maximum= 35
    - D. Minimum= 13, Maximum= 37
    - E. None of the above



18. A community theater is set up with 15 rows of seats. The number of seats in a row increases by two with each successive row. The first row has 10 seats. What is the total number of seats?
    - A. 38 seats
    - B. 150 seats
    - C. 360 seats
    - D. 400 seats
    - E. None of the above.

19. Because of friction and air resistance, each swing of a pendulum is a little shorter than the previous one. The lengths of the swings form a geometric sequence. Suppose the first swing of a pendulum has a length of 100cm and the return swing is 99cm. On which swing will the arc first have a length that is less than 50cm?
    - A. 13 swings
    - B. 51 swings
    - C. 70 swings
    - D. 92 swings
    - E. Not enough information given.

20. Solve the equation $16^{x-2} - 4^{x-2} = 2$.
    A. $x = 2$
    B. $x = \frac{1}{2}$ & $\frac{5}{2}$
    C. $x = \frac{5}{2}$
    D. $x = -\frac{3}{2}$
    E. The equation cannot be solved.

21. For what value of a does the rational function $f(x) = \frac{2x-7}{ax^2-3x}$ have a vertical asymptote of $x = 2$?
    A. $a = 0$
    B. $a = \frac{7}{2}$
    C. $a = \frac{7}{3}$
    D. $a = \frac{3}{2}$
    E. $a = \frac{3}{7}$

22. Consider matrices $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Then the solution of the matrix equation $AX = B$ is
    A. $X = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$
    B. $X = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$
    C. $X = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$
    D. $X = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$
    E. $X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

23. Solve $\log_2(x) + \log_2(x-2) = 3$.
    A. $x = 4$
    B. $x = -2$ & 4
    C. $x = 3$ & -1
    D. $x = 3$
    E. The equation cannot be solved.

24. A manufacturer of snowblades makes a regular model and a deluxe model. The regular model requires 3 hours of fabricating time and 1 hour of finishing time. The deluxe model requires 4 hours of fabricating time and 2 hours of finishing time. The maximum labor-hours available per week in the fabricating and finishing departments are 60 and 24, respectively. If x is the number of regular model blades and y is the number of deluxe model blades produced, which set of inequalities represents this situation?

A. $\begin{cases} 3x + 4y \leq 24 \\ x + 2y \leq 60 \\ x \geq 0, y \geq 0 \end{cases}$

B. $\begin{cases} 3x + 4y \leq 60 \\ x + 2y \leq 24 \\ x \geq 0, y \geq 0 \end{cases}$

C. $\begin{cases} 3x + 2y \leq 60 \\ x + 4y \leq 24 \\ x \geq 0, y \geq 0 \end{cases}$

D. $\begin{cases} 3x + 2y \leq 24 \\ x + 4y \leq 60 \\ x \geq 0, y \geq 0 \end{cases}$

E. None of the above

25. Continue the previous problem. If the profit function (the objective function) from the sale of x regular snowboards and y deluxe ones is given by $P = 20x + 65y$, how many of each type of snowboards should be produced to give the maximum profit?

A. 20 regular and no deluxe
B. 12 regular and 6 deluxe
C. No regular and 12 deluxe
D. 9 regular and 9 deluxe
E. No maximum profit is possible.

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Tie Breaker #1

Name: ___________________

School: __________________

Find all solutions to this equation. Show all work.

$$27^7 + 27^7 + 27^7 = 3^{3x+7}$$

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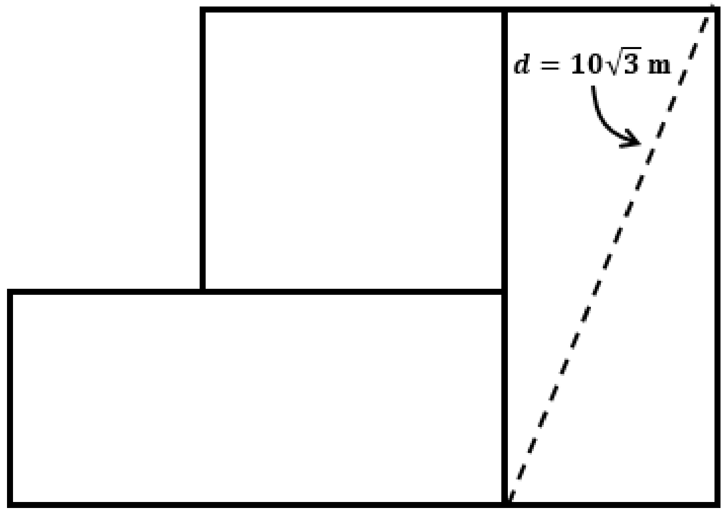
Tie Breaker #2

Name: ________________

School: ________________

You want to build three adjacent storage rooms. The total area is partitioned into two identical rectangular regions and a square region, as shown in the figure on the right.



The diagonal of the rectangular region(s) is $10\sqrt{3}$ meters.

A. Use this information to find the total area of this figure.

B. How much will be the total cost if it costs $3.45 per square meter for certain material to cover the entire floor of the three storage rooms.

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Tie Breaker #3

Name: ____________________

School: ___________________

A small theater has a seating capacity of 2000. When the ticket price is $20, the attendance is 1500. For each $1 decrease in price, attendance is increased by 100.

A. Find a linear function representing attendance N in terms of ticket price x.

B. Write the revenue R as a function of ticket price x.

C. At what ticket price will the revenue be maximum?

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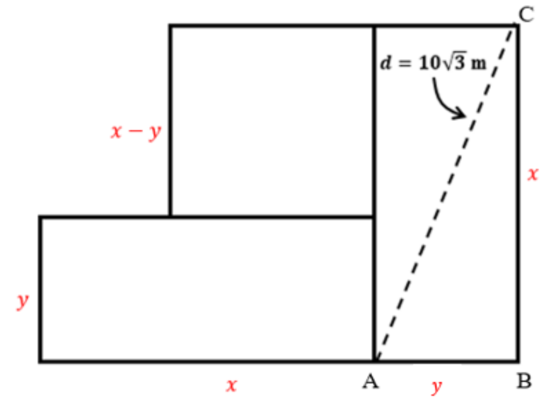
ANSWER KEY

1) E	11) D	21) D
2) D	12) B	22) E
3) B	13) C	23) A
4) C	14) B	24) B
5) A	15) B	25) C
6) B	16) C	
7) D	17) D	
8) A	18) C	
9) B	19) C	
10) A	20) C	

TB1: There are multiple ways to solve this problem. This is one option.

$27^7 + 27^7 + 27^7 = 3^{3x+7}$	
$3(27)^7 = 3^{3x+7}$	**Combine similar terms.**
$(27)^7 = 3^{3x+6}$	**Divide both sides by 3.**
$(3^3)^7 = 3^{3x+6}$	**Replace 27 by 3^3.**
$3^{21} = 3^{3x+6}$	**Simplify Exponents.**
$21 = 3x + 6$	**If the bases are equal, the exponents must also be equal. Alternately: Take a base-3 logarithm of both sides.**
$3x = 15$	
$x = 5$	

TB2: Multiple options are available. This is one possibility.



Part A) Total area

$$A = xy + xy + (x - y)^2 = 2xy + x^2 - 2xy + y^2 = x^2 + y^2$$

Now, for the right triangle ABC, we have

$$x^2 + y^2 = \left(10\sqrt{3}\right)^2$$

$$x^2 + y^2 = 300$$

So, total area= 300 m^2

Part B) Total cost $= 300 \times 3.45 = \$1,035.$

TB3:

A. Find a linear function representing attendance N in terms of ticket price x.

a. The graph of the linear function representing attendance N as a function of x passes through point $(20, 1500)$, and its slope is $m = -100$. So, we have

$$N - 1500 = -100(x - 20)$$

$$N = -100x + 3500$$

b. Write the revenue R as a function of ticket price x.

We know that $Revenue = (attendance)(ticket\ price)$, so we have

$$R = N \cdot x = (-100x + 3500)x$$

$$R = -100x^2 + 3500x$$

c. At what ticket price will the revenue be maximum?

Calculate the vertex of R.

$$x = \frac{3500}{-2(-100)} = \$17.50$$

Maximum Revenue =$30,625