Work on the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tiebreaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tiebreaker pages when you are finished. You may keep the pages with the multiple-choice questions. Figures are not necessarily drawn to scale. You may assume that all variables are real values.

1. Define function $f(x) = -x^2 + bx - 75$. What is the coordinate for the vertex?
   A. $\left(-\frac{b}{2}, -\frac{3b^2}{4} - 75\right)$
   B. $\left(\frac{b}{4}, \frac{3b^2}{16} - 75\right)$
   C. $\left(\frac{b}{2}, \frac{b^2}{4} - 75\right)$
   D. $(-2b, -6b^2 - 75)$
   E. None of the above

2. Suppose the given expression represents the ratio of the power consumed by a circuit board to the total resistance in the circuit. Here, I is the current flowing through the circuit, V is the voltage across the circuit, and R_{load} is the resistance of the load. Simplify the expression to find a more manageable form for calculating the power to resistance ratio in the circuit.

$$\frac{2I^3\,V^2\,(R_{load})^{-4}}{4I^{-2}\,V^4\,(R_{load})^3}$$

   A. $\frac{I(R_{load})}{2V^2}$
   B. $I^2V^5(R_{load})^2$
   C. $\frac{V^2(R_{load})}{4I}$
   D. $\frac{I^5}{2V^2(R_{load})^7}$
   E. None of the above

3. Solve the inequality. Give your answer in interval notation.

$$\frac{x}{4} + 2 \geq \frac{6 - x}{4} - 3$$

   A. $[7, \infty)$
   B. $[-7, \infty)$
   C. $(7, \infty)$
   D. $(-\infty, 7]$
   E. None of the above

4. If $a^2 + b^2 = 16$ and $a^2 - b^2 = 5$, then find $a^4 - b^4$.
   A. 25
   B. 80
   C. 61
   D. 121
   E. None of the above

5. Which of the following sets does not represent a function?
   A. $\{(2,5), (1,6), (3,2), (4,5), (5,6)\}$
   B. $\{(x, y) \mid y = 3x + 5\}$
   C. $\{(x, y) \mid x^2 + y^2 = 25, x \geq 0\}$
   D. $\{(x, y) \mid y = e^x + 3\}$
   E. None of the above

6. Suppose a hiker is climbing a mountain trail which gains 6 feet in elevation for every 100 feet of hiking. If the hiker maintains a steady pace of 2 miles per hour for 30 minutes, how much elevation does the hiker gain in that time?
   A. 1,240.4 ft
   B. 496.1 ft
   C. 542.3 ft
   D. 316.8 ft
   E. None of the above

7. Find the product of the solutions to the equation $3x^2 - 26x - 9 = 0$.
   A. 5
   B. -4
   C. 7
   D. -3
   E. None of the above

8. Find the x-intercept of the line that passes through the points $(2, 12)$ and $(5, 21)$.
   A. $\frac{1}{3}$
   B. 4
   C. -2
   D. $\frac{1}{2}$
   E. None of the above

9. Simplify the expression:

$$\frac{a^{2b}-9}{a^{2b}-2a^{b}-15}$$

A. $\frac{9}{2a^{b}+15}$
B. $\frac{a^{b}-3}{a^{b}-5}$
C. $\frac{a^{b}+3}{a^{b}-5}$
D. $\frac{a^{b}-9}{15}$
E. None of the above

10. Sally is Betsy's aunt. The sum of their ages is 66. In 9 years, Sally will be three times as old as Betsy. Find their current ages.
A. Betsy 12, Sally 54
B. Betsy 11, Sally 53
C. Betsy 19, Sally 47
D. Betsy 17, Sally 49
E. None of the above

11. Suppose that $g(x)$ and $h(x)$ are linear functions such that $g(-1)=2$, $g(2)=17$ and the graph of $h(x)$ is perpendicular to the graph of $g(x)$. What is the slope of $h(x)$?
A. 5
B. $\frac{1}{5}$
C. $\frac{2}{15}$
D. $-\frac{1}{5}$
E. None of the above

12. Suppose $-a+4b+c=-8$. What is the value of b in the following system of equations:

$$5a+2b+3c=9$$
$$6a+2b+2c=5$$

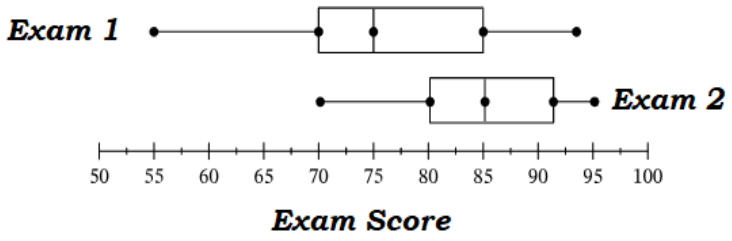
A. $b=-3$
B. $b=-2$
C. $b=0$
D. $b=1$
E. None of the above

13. Write an equation for the function, $g(x)$, whose graph has the same shape as the graph of $f(x) = x^3$ but has been shifted 6 units to the left, 4 units downward, and reflected across the x-axis.
    A. $g(x) = -(x+6)^3 - 4$
    B. $g(x) = -(x-4)^3 + 6$
    C. $g(x) = -x^3 - 2$
    D. $g(x) = -(x+6)^3 + 4$
    E. None of the above

14. Sarah paid \$30,500 for a new car. Due to depreciation, each year, the value of the car is only worth $8/9^{\text{ths}}$ of its value for the previous year. Find a model, $C(t)$, for the value of the car after t years.
    A. $C(t) = -30{,}500\left(\frac{1}{9}\right)^t$
    B. $C(t) = 30{,}500 - \left(\frac{8}{9}\right)t$
    C. $C(t) = 30{,}500\left(\frac{8}{9}\right)^t$
    D. $C(t) = \left(\frac{1}{9}\right)(30{,}500)^t$
    E. None of the above

15. Which expression is equivalent to $(z^2 + 2z - 3)(4z + 5)$?
    A. $(2z+1)(2z^2+4z-1)$
    B. $4z^3 + 13z^2 - 2z - 15$
    C. $z(4z^2 + 13z - 5)$
    D. $4z^3 - 10z + 15$
    E. None of the above

16. The box plots beside represent student exams scores for Exam 1 and Exam 2 in Mrs. Corley's Calculus class.



Based on this figure, which of the following is <u>not</u> true.
    A. The median exam score for Exam 1 was lower than for Exam 2.
    B. The data for Exam 2 has a higher interquartile range than Exam 1.
    C. More students earned a passing grade ($\geq 70\%$) on Exam 2 than on Exam 1.
    D. At least 25% of the students in the class earned an A on Exam 2.
    E. All of the above are true.

17. If $a^2 - 3a = 0$ and $2b^2 - b = 0$, and assume $a \neq 0$ & $b \neq 0$. Then which of the following is equal to $\frac{a}{b}$?

A. 6
B. $\frac{1}{6}$
C. 0
D. $\frac{3}{2}$
E. $\frac{2}{3}$

18. Let $f(x) = \frac{\sqrt{x+5}}{|x-3|}$. Determine the domain of $f(x)$ in interval notation.

A. $(-\infty, 3) \cup (3, \infty)$
B. $(-\infty, -5) \cup (-5, 3)$
C. $[-5, 3) \cup (3, \infty)$
D. $[-5, \infty)$
E. None of the above

19. Solve the following equation for x.

$$0.4(6 - x) = \frac{1}{3}(8x + 5) - 0.6$$

A. 10/23
B. 23/10
C. 10/17
D. 17/10
E. None of the above

20. For the equation $6x^2 = 4x + 2$, which of the following are true:

I. The sum of the solutions is negative.
II. The product of the solutions is negative.
III. Exactly one of the solutions is a fraction.

A. I and II only
B. II and III only
C. I and III only
D. All of the above
E. None of the above

21. The following table gives the populations for two small cities in various years.

Year	Clarencetown Population	Kirkleton Population
1980	37,300	62,000
1990	40,600	63,500
2000	58,800	82,600
2010	73,300	87,900
2020	85,400	93,700

Suppose we use linear regression to create models for each population. Which of the following is true?

A. The population of Kirkleton has a faster growth rate.
B. The populations of the two towns will never be equal.
C. By the year 2032, the population of Clarencetown will be greater than the population of Kirkleton.
D. In the year 2050, the population of Clarencetown is expected to be 1.5 times the population of Kirkleton.
E. None of the above.

22. Solve the given inequality. Express your answer in interval notation.

$$-2 \geq 4 - |x + 3|$$

A. $[-9, 3]$
B. $(-\infty, -9] \cup [3, \infty)$
C. $[-3, 9]$
D. $(-\infty, -3] \cup [3, \infty)$
E. None of the above

23. Consider the two lines $3x + 4y = 6$ and $-3x - 4y = -12$. Which of the following statements is true?

A. The two lines intersect at the at the point $\left(1, \frac{3}{4}\right)$.
B. The two lines are perpendicular.
C. The two lines have the same x-intercept.
D. The two lines have the same y-intercept.
E. None of the above.

24. The average high temperature over a 14-day period in Balentine, KS was found to be 73°. However, when the high temperature for one day was removed, the average temperature was found to be 75°. What temperature was removed from the data set?
    A. 71°
    B. 32°
    C. 47°
    D. 56°
    E. None of the above.

25. Suppose $f(x)$ is a 12^{th} degree polynomial function and $h(x)$ is a 15^{th} degree polynomial function with three repeated roots. Which of the following is true?
    A. $f(x)$ will cross the y-axis more.
    B. $h(x)$ will cross the y-axis more.
    C. Neither function will cross the y-axis.
    D. $f(x)$ will cross the y-axis at least one time, but $h(x)$ will not cross the y-axis.
    E. None of the above.

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Tie Breaker #1

Name: _____

School: _____

A company has decided to upgrade its conference rooms by purchasing new projectors, whiteboards, and additional chairs. The total number of items purchased is 42. The cost of each projector is $118, the cost of each whiteboard is $131, and the cost of each chair is $84. The total amount of money spent on the new equipment was $4351. The company purchased two times as many chairs as whiteboards.

Determine the number of each item that was purchased.

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Tie Breaker #2

Name: _____

School: _____

The management team of the Floaty Recreational Rentals company has determined that the number of canoes rented each day can be modeled by the function $n(x) = 1440 - 6x$, where n is the number of canoes rented, x is the daily rental price, and $60 \leq x \leq 240$.

How much should the management team charge per daily rental to maximize their revenue?

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Tie Breaker #3

Name: _____

School: _____

Susan invested money in a business 8 years ago. Since that time, her investment has grown at an average rate of 1.3% compounded quarterly. Her investment is now worth $250,000.

a. What was the amount of Susan's initial investment?

b. How long will it take for Susan's current investment worth to double?

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Answer Key

1. C	11. D	21. E
2. D	12. A	22. B
3. B	13. A	23. E
4. B	14. C	24. C
5. C	15. B	25. E
6. D	16. B	
7. D	17. A	
8. C	18. C	
9. B	19. A	
10. A	20. B	

Tiebreaker Answer Key: Tiebreaker #1

A company has decided to upgrade its conference rooms by purchasing new projectors, whiteboards, and additional chairs. The total number of items purchased is 42. The cost of each projector is $118, the cost of each whiteboard is $131, and the cost of each chair is $84. The total amount of money spent on the new equipment was $4351. The company purchased two times as many chairs as whiteboards. Determine the number of each item that was purchased.

Let x =the number of projectors, y =the number of whiteboards, and z =the number of chairs

$$x + y + z = 42$$
$$118x + 131y + 84z = 4351$$
$$2y - z = 0$$

We can solve the third equation for z to get:

$$2y - z = 0 \Rightarrow 2y = z$$

We can then substitute this expression for z in the first two equations to get:

$$x + y + 2y = 42 \Rightarrow x + 3y = 42$$
$$118x + 131y + 84(2y) = 4351 \Rightarrow 118x + 299y = 4351$$

This gives a system of two equations and two unknowns. We can solve the first equation for x to get $x = -3y + 42$. Substituting in the second equation gives us:

$$118(-3y + 42) + 299y = 4351$$
$$-354y + 4956 + 299y = 4351$$
$$-55y = -605$$
$$y = 11$$

Then, $y = 11 \Rightarrow z = 2(11) = 22$ and $x = -3(11) + 42 = 9$.
Hence, the company purchased 9 projectors, 11 whiteboards, and 22 chairs.

Alternatively, the system of equations could be solved using Gauss-Jordan elimination, either by hand or by using the graphing calculator:

$$\begin{bmatrix} 1 & 1 & 1 & 42 \\ 118 & 131 & 84 & 4351 \\ 0 & 2 & -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 22 \end{bmatrix}$$

Tiebreaker #2

The management team of the Floaty Recreational Rentals company has determined that the number of canoes rented each day can be modeled by the function $n(x) = 1440 - 6x$, where n is the number of canoes rented, x is the daily rental price, and $60 \leq x \leq 240$. How much should the management team charge per daily rental to maximize their revenue?

$$\begin{aligned} R(x) &= x \cdot n(x) \\ &= x(1440 - 6x) \\ &= -6x^2 + 1440 \end{aligned}$$

The graph of $R(x)$ is a parabola opening downward. The maximum value will occur at the x-coordinate of the vertex.

$$x = -\frac{1440}{2(-6)} = \frac{1440}{12} = 120$$

Hence the management team should charge $120 per daily canoe rental to maximize revenue.

Tiebreaker #3

Susan invested money in a business 8 years ago. Since that time, her investment has grown at an average rate of 1.3% compounded quarterly. Her investment is now worth $250,000.

a. What was the amount of Susan's initial investment?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$250{,}000 = P\left(1 + \frac{.013}{4}\right)^{4(8)}$$

$$25000 = P(1.00325)^{32}$$

$$\frac{250{,}000}{(1.00325)^{32}} = P$$

$$P = \$225{,}344.32$$

b. How long will it take for Susan's current investment worth to double?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$50000 = 250000\left(1 + \frac{.013}{4}\right)^{4t}$$

$$2 = (1.00325)^{4t}$$

$$\ln(2) = ln(1.00325)^{4t}$$

$$\ln(2) = 4t \cdot \ln(1.00325)$$

$$\frac{\ln(2)}{4 \cdot \ln(1.00325)} = t$$

$$t \approx 53.41\ years$$