Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

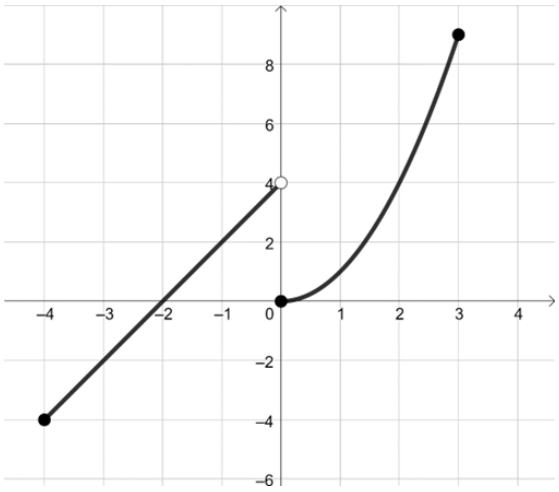
1. A third-degree polynomial named P has the following properties:
   - P has three real roots at $x = -3$, $x = 7$, and $x = 10$.
   - P has the following end behavior: As $x \to \infty, P \to \infty$ and as $x \to -\infty, P \to -\infty$.

   Which of the following could be true?
   a. The x-value of the local maximum is less than -3.
   b. The x-value of the local maximum is between -3 and 7.
   c. The x-value of the local maximum is between 7 and 10.
   d. The x-value of the local maximum is greater than 10.
   e. Not enough information is given to determine the x value.

Consider the figure shown to the right and each statement below for problems 2 and 3.

I. The constant rate of change is 2.

II. The average rate of change is 2.



2. Which of the statements are true on the interval $[-3, -1]$?
   a. I only
   b. II only
   c. Both I and II.
   d. Neither I nor II.
   e. Not enough information is given.

3. Which of the following statements are true on the interval $[0, 2]$?
   a. I only
   b. II only
   c. Both I and II.
   d. Neither I nor II.
   e. Not enough information is given.

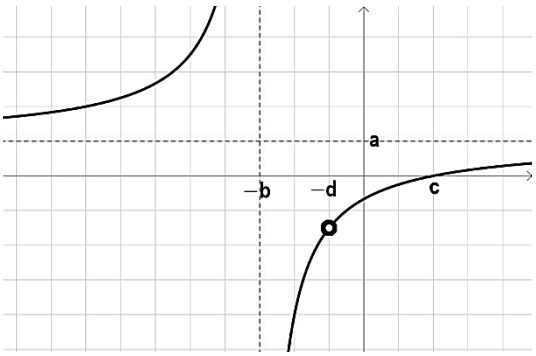
4. Which of the following has the lowest average rate of change as x increases from π to $\frac{3\pi}{2}$?

   a. $y = \sin(x)$
   b. $y = \sin(x) + 2$
   c. $y = \sin(2x)$
   d. $y = 2\sin(x)$
   e. $y = \sin^2(x)$

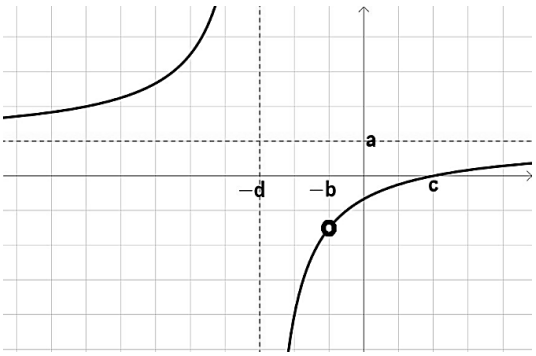
5. If a, b, c, and d are all positive integers, which of the following could be the graph of the function $r(x)$?

$$r(x) = \frac{a(x+b)(x-c)}{(x+b)(x+d)}$$

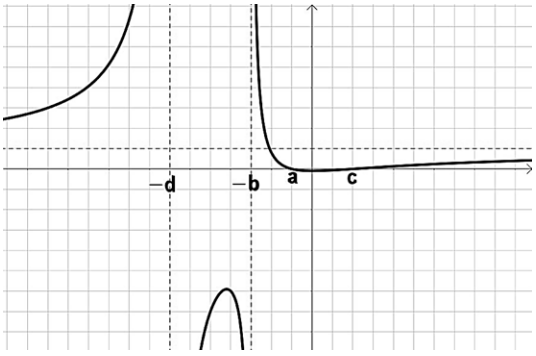
a.



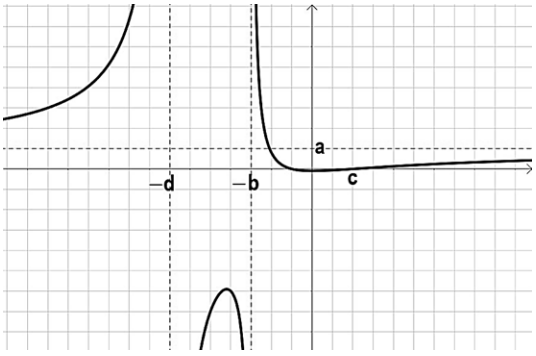
b.



c.



d.



   e. None of the above.

6. A circle has a radius of 8 ft. Find the length s of the arc intercepted by a central angle of 88°.

   a. 704.0 ft
   b. 110 ft
   c. 15.2 ft
   d. 12.3 ft
   e. None of the above.

7. Amber is the gliding champion of Mondstadt. She glides by jumping off a tall building or mountain cliff. The distance she can glide is a function (named g) of the height from which she jumps. Let d represent the distance Amber glides and let h represent the height from which she jumps. Which of the following is the correct function notation using the given variables?
   a. $d = h(g)$
   b. $h = g(d)$
   c. $g = h(d)$
   d. $d = g(h)$
   e. None of the above.

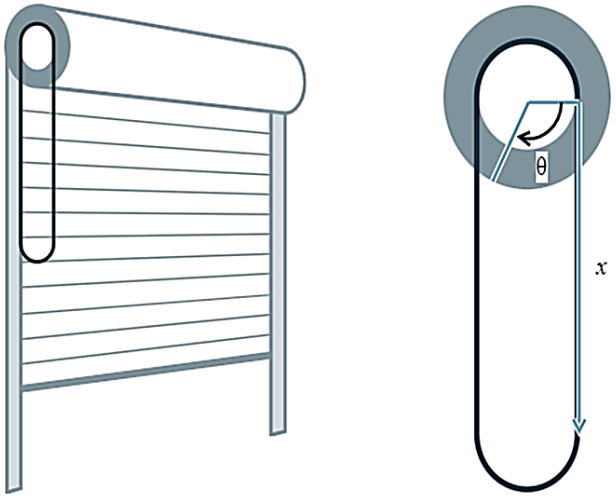
8. A rational function, R, has a vertical asymptote of $x = 3$ and a horizontal asymptote of $y = 1$. Which of the following statements could <u>not</u> correspond to these asymptotes?
   a. As x decreases without bound, $R(x)$ approaches 3.
   b. As x increases without bound, $R(x)$ approaches 1.
   c. As x approaches 3 from the right, $R(x)$ increases without bound.
   d. As x approaches 3 from the left, $R(x)$ increases without bound.
   e. All the limits above correspond to these asymptotes.

9. A spider rests on the blade of a rotating ceiling fan. A stopwatch is started, at which point the time is $t = 0$ seconds. When $t = 0$, the spider is at is maximum distance from the wall, 4.5 m away from the wall. The spider first reaches its minimum distance from the wall, 3.5 m away from the wall, when $t = 1$ second. When $t = 2$ seconds, the spider is back to its maximum distance from the wall. The spider continues to move in a circle, so that the spider's distance from the wall over time can be modeled by a sinusoidal function. Let $f(t)$ be the distance between the spider and the wall t seconds after starting the stopwatch. Which of the following is the equation $f(t)$?
   a. $f(t) = 0.5\cos(\pi t) + 4$
   b. $f(t) = 0.5\sin(2\pi t) + 4$
   c. $f(t) = 3.5\cos(\pi t) + 1$
   d. $f(t) = \sin(\pi t) + 4.5$
   e. None of the above.

10. Suppose the vector $\boldsymbol{s}$ has a magnitude 41 and makes an angle of 303° with the positive x-axis (measured counterclockwise), when $\boldsymbol{s}$ is in standard position. Write the vector $\boldsymbol{s}$ in the form $\boldsymbol{s} = \langle a, b \rangle$ where a is the horizontal component of the vector $\boldsymbol{s}$ and b is the vertical component of the vector $\boldsymbol{s}$.
    a. $\boldsymbol{s} = \langle 41\sin(303°), 41\cos(303°) \rangle$
    b. $\boldsymbol{s} = \langle 41\sin(303°), -41\cos(303°) \rangle$
    c. $\boldsymbol{s} = \langle 41\cos(303°), 41\sin(303°) \rangle$
    d. $\boldsymbol{s} = \langle 41\cos(303°), -41\sin(303°) \rangle$
    e. None of the above

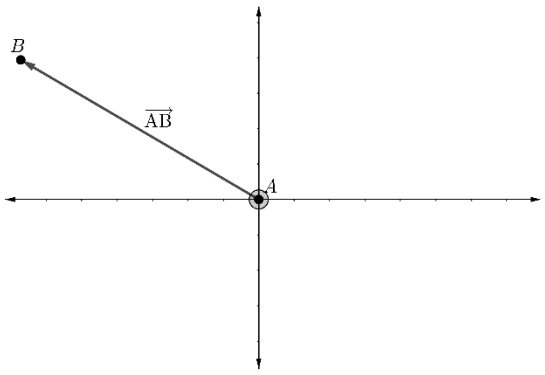
11. Which of the following is NOT the same as $\cos(\pi - x)$?
    a. $\cos(x - \pi)$
    b. $-\cos x$
    c. $\sin\left(x - \frac{\pi}{2}\right)$
    d. $-\cos(-x)$
    e. All the above are the same as $\cos(\pi - x)$.

12. A roll up door is operated by a chain on a round tension device (see beside figure). The wheel has a diameter of 54 cm, and the chain moves over it without sliding. Let x be the distance (in cm) the chain moves as the wheel rotates θ radians. Assume θ to be positive and consider only its magnitude. (So, its position on the circle is not relevant.) Assume also that the distance x is positive. Write a function, d, that gives θ in terms of x.

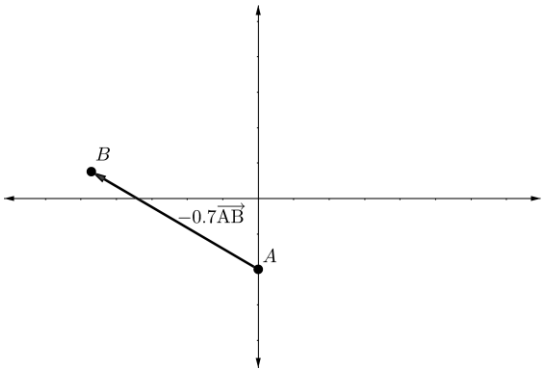


    a. $d(x) = 54x$
    b. $d(x) = \frac{x}{27}$
    c. $d(\theta) = 27\sin\theta$
    d. $d(x) = \frac{x}{54}$
    e. $d(\theta) = 27\theta$

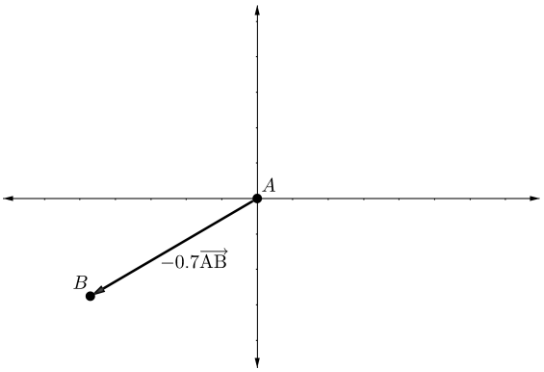
13. The vector $\overrightarrow{AB}$ is shown in the figure to the right. (Not necessarily drawn to scale.) Which of the following represents $-0.7\overrightarrow{AB}$?



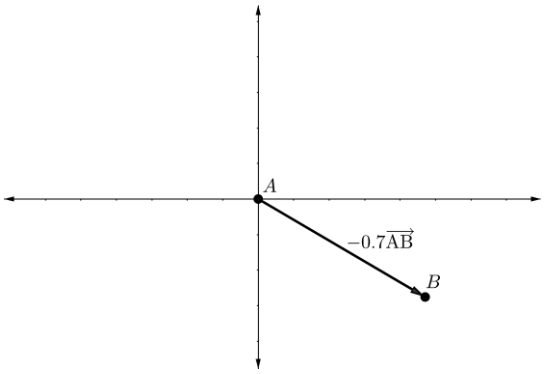
a.



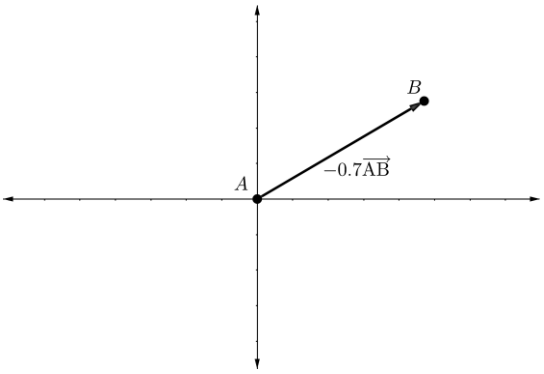
b.



c.



d.



e. None of the above.

14. A cell phone tower is located 4 miles east and 5 miles north of the city center. It has a coverage radius of 2.25 miles from its location. The locations of several houses with respect to the city center are provided in the answer choices. Which of the houses is NOT within the cell tower's coverage radius?
    a. House A is 2 miles east and 4 miles north of the city center.
    b. House B is 3 miles east and 7 miles north of the city center.
    c. House C is 6 miles east and 5 miles north of the city center.
    d. House D is 3 miles east and 3 miles north of the city center.
    e. All the houses above are in the cell tower's coverage radius.

15. Superman flies at a constant airspeed of 500 mph headed due west. The jet stream is 100 mph in the south-easterly direction. Find Superman's actual speed relative to the ground.
    a. 429.3 mph
    b. 70.7 mph
    c. 400 mph
    d. 435.1 mph
    e. None of the above.

16. Multiply the following matrices. What expression represents the second row and first column of the product?

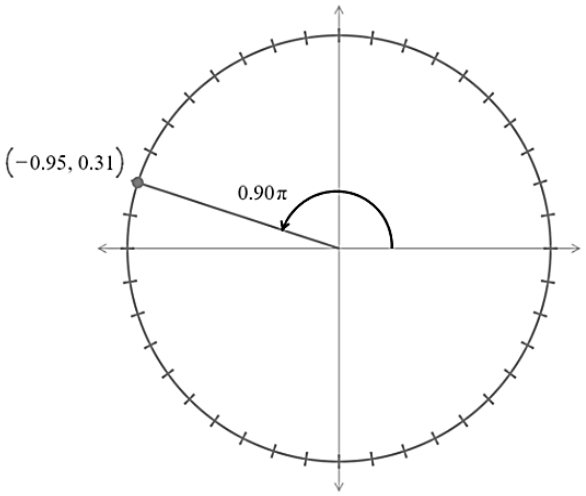
$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \cdot \begin{bmatrix} 7 & 2 & -1 \\ 0 & 3 & 5 \end{bmatrix}$$

    a. $2b$
    b. 0
    c. $3e + 5f$
    d. $2b + 3e$
    e. The matrices cannot be multiplied.

17. What is the radian measure of an angle that subtends $\frac{1}{5}$ of the circumference of a circle?
    a. $\frac{2\pi}{5}$
    b. $\frac{\pi}{5}$
    c. $\frac{\pi}{10}$
    d. 10π
    e. None of the above.

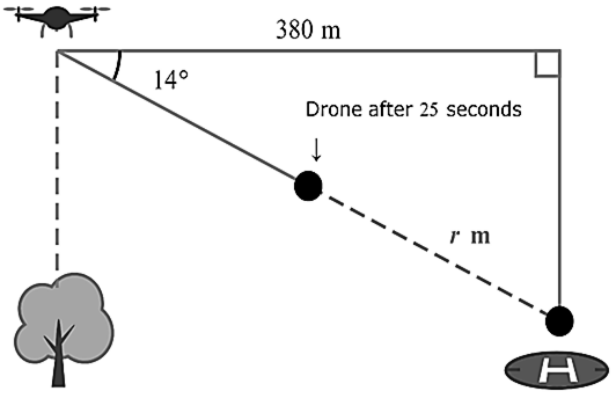
18. Let θ be an angle such that $\tan\theta = -\frac{7}{15}$ and $\cos\theta < 0$. Find the exact value of $\sin\theta$.
    a. $\frac{15\sqrt{274}}{274}$
    b. $\frac{7\sqrt{274}}{274}$
    c. $-\frac{15\sqrt{274}}{274}$
    d. $-\frac{7\sqrt{274}}{274}$
    e. None of the above.

19. Using the figure to the right, gives the coordinates of the point on the unit circle that terminates at an angle of 0.90π. Use the coordinates shown, which are rounded to the hundredths place, to find $\tan(0.90\pi)$.



    a. $\tan(0.90\pi) = \frac{\sin(0.31)}{\cos(-0.95)}$
    b. $\tan(0.90\pi) = -\frac{0.95}{0.31}$
    c. $\tan(0.90\pi) = \frac{\sin(-0.95)}{\cos(0.31)}$
    d. $\tan(0.90\pi) = -\frac{0.31}{0.95}$
    e. None of the above.

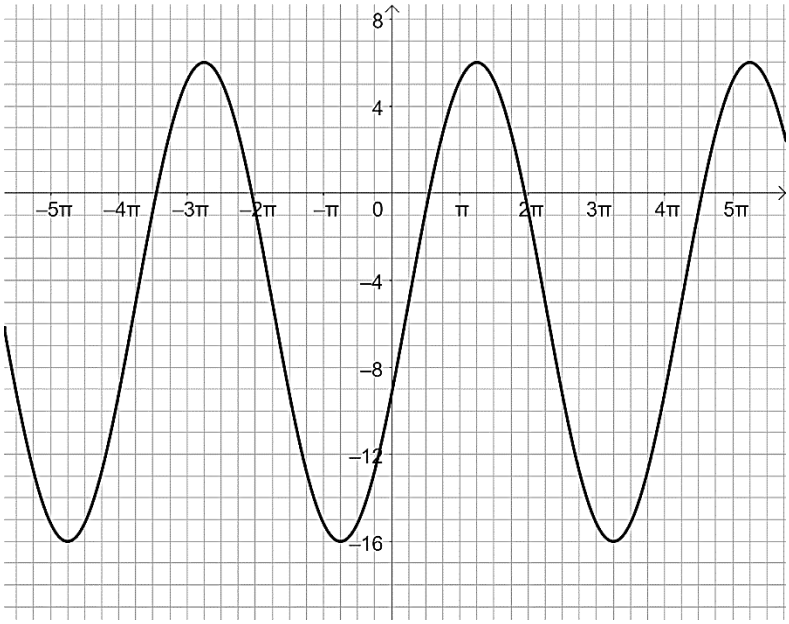
20. A drone flying at a constant speed of 11 m/s is currently located directly above a tree. The drone begins to descend towards a landing pad, as shown in the figure beside. (Not necessarily drawn to scale.) The tree and the landing pad are 380m apart, and the drone's angle of descent is 14° from the horizontal. Find the remaining distance, r, that will be between the drone and the landing pad after 25 seconds.



    a. 275.0 m
    b. 116.6 m
    c. 1295.8 m
    d. 105.0 m
    e. None of the above.

21. Samantha works a total of 40 hours each week at two part-time jobs. She works at a restaurant, where she earns $13 per hour, and as a freelance blog editor, where she earns $15 per hour. Write a function f that inputs the number of hours Samantha works at the restaurant in a week (x) and outputs the number of hours she can work as a freelance blog editor that week.
    a. $f(x) = 15(40 - x)$
    b. $f(x) = 13x$
    c. $f(x) = 40 - x$
    d. $f(x) = 13(40 - x)$
    e. None of the above.

Use this graph of a trigonometric function to answer questions 22 through 24.



22. What is the amplitude of this function?
    a. 11
    b. 6
    c. 5
    d. 9
    e. None of the above.

23. What is the equation of the midline of the function shown above?
    a. $y = 0$
    b. $y = -4$
    c. $x = 0$
    d. $x = -4$
    e. None of the above.

24. What is the period of the function above?
    a. $\frac{\pi}{4}$
    b. π
    c. 2π
    d. 4π
    e. None of the above.

25. Sam has 320 meters of fencing. He will use it to form three sides of a rectangular garden. The fourth side will be along a house and will not need fencing. What is the maximum area that the garden can have?
    a. 6400 m^2
    b. 12,000 m^2
    c. 12,800 m^2
    d. 25,600 m^2
    e. None of the above.

Tie Breaker #1

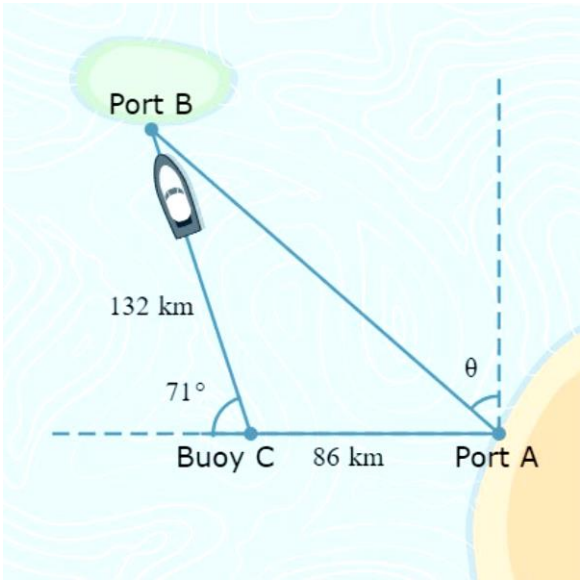
Name: _____

School: _____

Show all your work.

A ship leaves Port A and travels 86 km due west to Buoy C. It then adjusts its course 71° northward. It travels 132km in that direction until it reaches Port B. What angle θ with respect to due north could the ship have used to travel directly from Port A to Port B?

See the figure beside, not necessarily drawn to scale.



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Tie Breaker #2

Name: _____

School: _____

Prove or disprove this trig identity. Show all work.

$$\cot x\,(1-\cos(2x)) = \sin(2x)$$

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Tie Breaker #3

Name: _____

School: _____

Solve the following system of equations. Give exact solutions, not decimal approximations.

Make sure to give ALL solutions.

$$\begin{cases} 4x^2 + y^2 = 54 \\ x^2 + y^2 = 21 \end{cases}$$

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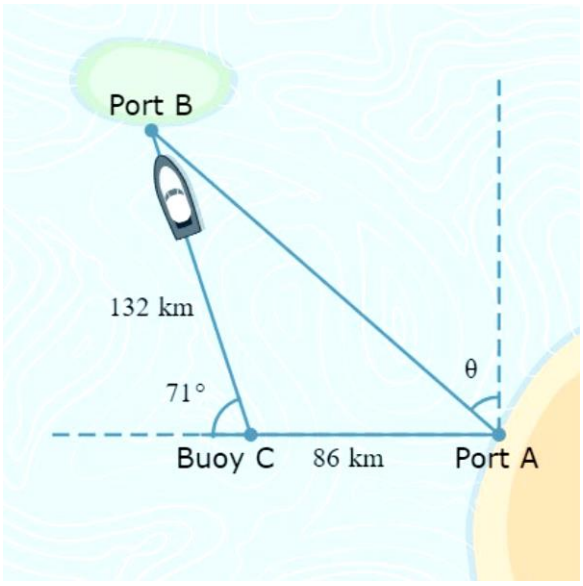
Multiple Choice Answers

1. B	11. E	21. C
2. C	12. B	22. A
3. B	13. C	23. E
4. D	14. E	24. D
5. B	15. D	25. C
6. D	16. E	
7. D	17. A	
8. A	18. B	
9. A	19. D	
10. C	20. B	

TB1:

A ship leaves Port A and travels 86 km due west to Buoy C. It then adjusts its course 71° northward. It travels 132 km in that direction until it reaches Port B. What angle θ with respect to due north could the ship have used to travel directly from Port A to Port B?

See the figure beside, not necessarily drawn to scale.



This is one way to work this:

$\angle ACB = 180° - 71° = 109°$

$c = \sqrt{a^2 + b^2 - 2ab\cos C}$

$c = \sqrt{132^2 + 86^2 - 2 \cdot 132 \cdot 86\cos 109°} \approx 179.4762$

$\angle CAB = \arccos\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) = \arccos\left(\frac{132^2 - 86^2 - 179.4762^2}{-2 \cdot 86 \cdot 179.4762}\right) \approx 44.0594°$

$\theta = 90° - \angle CAB = 90° - 44.0594° = 45.9406° \approx 45.9°$

TB2:

Prove or disprove this trig identity.

$$\cot x\,(1-\cos 2x)=\sin 2x$$

Statement	Rule
$\cot x\,(1-\cos 2x)$	
$\cot x\left(1-\left(1-2\sin^2 x\right)\right)$	Double angle identity
$\cot x\left(2\sin^2 x\right)$	Algebra
$\frac{\cos x}{\sin x}\left(2\sin^2 x\right)$	Ratio identity
$\cos x\,(2\sin x)$	Algebra
$\sin 2x$	Double angle identity

TB3:

Solve the following system of equations. Give exact solutions, not decimal approximations.

Make sure to give ALL solutions.

$$\begin{cases} 4x^2+y^2=54 \\ x^2+y^2=21 \end{cases}$$

Multiply the bottom equation by -1 on both sides of the equal sign and add the two equations together:

$$3x^2=33$$

$$x^2=11$$

$$x=\pm\sqrt{11}$$

Multiply the bottom equation by -4 on both sides of the equal sign and add the two equations together:

$$-3y^2=-30$$

$$y^2=10$$

$$y=\pm\sqrt{10}$$

There are four solutions:

$$\left(\sqrt{11},\sqrt{10}\right),\left(\sqrt{11},-\sqrt{10}\right),\left(-\sqrt{11},\sqrt{10}\right),\left(-\sqrt{11},-\sqrt{10}\right)$$