Begin by removing the three tie breaker sheets at the end and writing your name on all three pages. Work on the multiple-choice questions first, choosing the single **best** (most detailed and complete correct) response from the choices available. Indicate your answer here and on your answer sheet. Make sure you attempt the tie-breaker questions at the end of the test starting with tie breaker 1, then 2, and then 3 if you have time. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

**Figures are not necessarily drawn to scale. All angles are in degrees.**

1. If the size of a rectangle's area is twice as large as the size of its perimeter, and the length of one of its sides is 4.5 cm, what is the length of the other side?
   A. 36 cm
   B. 3.6 cm
   C. 18 cm
   D. 16 cm
   E. None of these

2. A given circular cylindrical can is made up of a square piece of metal and two circular disks each with diameter n inches. What is the volume of this cylindrical can?
   A. $2\pi n^3 \ in^3$
   B. $\frac{\pi^2 n^3}{4} \ in^3$
   C. $\frac{\pi^3 n^3}{6} \ in^3$
   D. $2.467 n^3 \ in^3$
   E. None of the above

3. For the points $(0, 2)$, $(6, 6)$, and $(10, 0)$ which of the following statements are true?
   I.   The points form the vertices of a right triangle.
   II. The points for the vertices of an isosceles triangle.
   III. The largest angle is at the vertex located on $(6, 6)$.
   A. Only III.
   B. II and III.
   C. I and III.
   D. All are true.
   E. None are true.

Name _____

4. Two chords $\overline{AB}$ and $\overline{CD}$ of a circle intersect each other so that $AP = 4$, $BP = 9$, and $CP = DP$. How long is $\overline{DC}$?

A. 12 units

B. $\sqrt{12}$ units

C. 6 units

D. 18 units

E. None of these
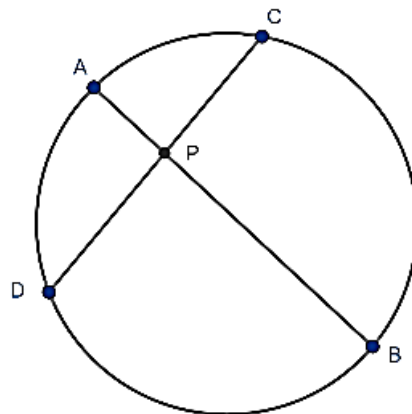
5. What is the size of an angle between two adjacent sides of a regular 10-sided polygon?

A. 180°

B. 100°

C. 36°

D. 144°

E. None of these

6. Let $\overrightarrow{OB}$ bisect $\angle AOC$. If $m\angle AOB = 2x + 10$ and if $m\angle BOC = 8x - 14$, what is $m\angle AOC$?

A. 22°

B. 25°

C. 36°

D. 40°

E. 44°

7. Three circles of radii 2, 4, and 6 are tangent to each other externally. Find the area of the triangle formed by connecting their centers.

A. 4

B. 6

C. 12

D. 24

E. Not enough information to determine.

8. In ΔABC, the point D is on $\overline{AB}$ and the point E is on $\overline{AC}$ such that $\overline{DE}$ is parallel to $\overline{BC}$.

   If $AD = 9$, $BD = 6$, and $DE = 4$, then $BC =$ _____ .

   A. $2\frac{2}{3}$

   B. $5\frac{1}{3}$

   C. $6\frac{2}{3}$

   D. $7\frac{1}{3}$
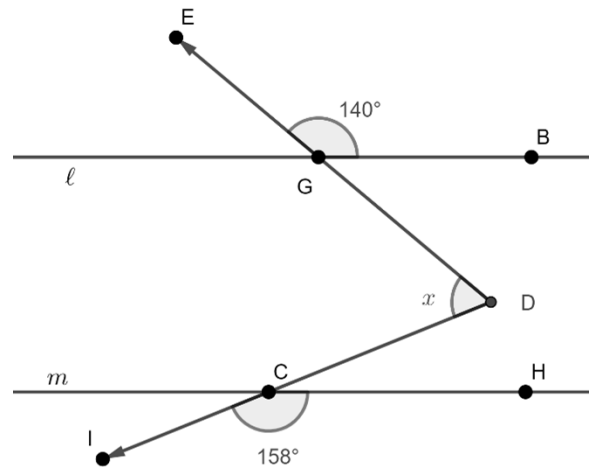
   E. $13\frac{1}{2}$

9. The point (a, b) is reflected over the y-axis to the point (c, d), which is reflected over the x-axis to the point (e, f). Compute $(ab - ef)$.

   A. 0

   B. 2

   C. $2ab$

   D. $2cd$

   E. $-2cd$

10. Assume $\ell \parallel m$ in the figure shown. Note that $m\angle EGB = 140°$ and $m\angle HCI = 158°$. Find the value of x in degrees.

   (Note that $\angle GDC$ is angle x.)

   A. 18°

   B. 22°

   C. 40°

   D. 62°

   E. Not enough information to determine.

11. An isosceles triangle with base 10 cm has an area of 60 cm^2. Find its perimeter.

   A. 23 cm

   B. 26 cm

   C. 31 cm

   D. 34 cm

   E. 36 cm

12. If the diagonals of a quadrilateral are congruent, then the quadrilateral must be a

    A. Parallelogram

    B. Rhombus

    C. Rectangle

    D. All the above.

    E. None of the above.

13. Two of the exterior angles of a pentagon have measures 75° and 105°. The measures of the remaining three

    exterior angles have the ratio $3:4:5$. The measure of the smallest angle, is

    A. 15

    B. 30

    C. 40

    D. 45

    E. 75

14. A round table can be made square by dropping the four leaves down. If a side
    of the square table measures 36 inches, approximately how much larger is
    the area of the table when the leaves are up? Round to the nearest ten.
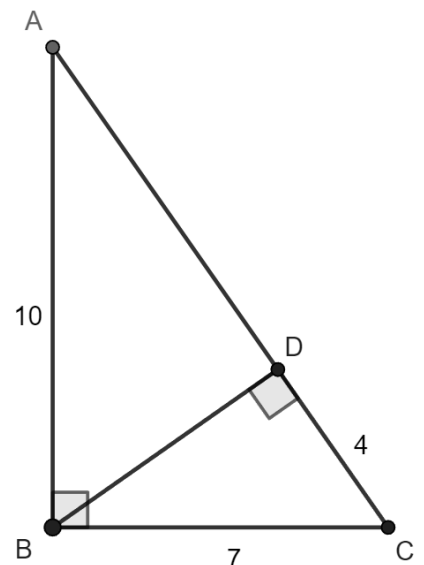
    A. 740 in^2

    B. 850 in^2

    C. 1000 in^2

    D. 1250 in^2

    E. 1300 in^2
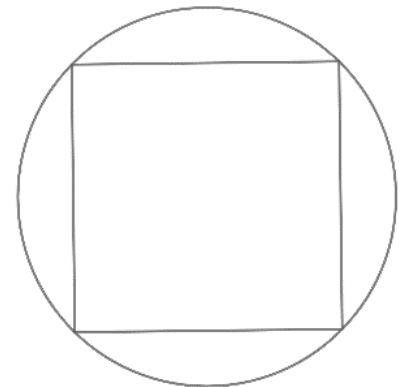
15. Find the area of ΔABD in the figure beside.

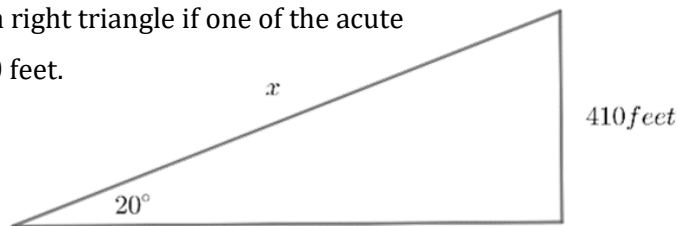   Round to the nearest half square unit.

    A. About 6.0 $units^2$

    B. About 11.5 $units^2$

    C. About 23.5 $units^2$

    D. About 35.0 $units^2$

    E. None of the above.

16. To the nearest foot, find the length of the hypotenuse of a right triangle if one of the acute

    angles measures 20° and the opposite side measures 410 feet.

    A. 1199 feet

    B. 449 feet

    C. 436 feet

    D. 140 feet

    E. 385 feet

17. Consider the conditional statement: "If two lines are parallel, then the lines do not intersect."

    Which of the following conditional statements is <u>logically equivalent</u> to the given statement?

    A. If two lines do not intersect, then the lines are parallel.

    B. If two lines intersect, then the lines are not parallel.

    C. If two lines are not parallel, then the lines intersect.

    D. If two lines are not parallel, then the lines do not intersect.

    E. If two lines do not intersect, then the lines are not parallel.

18. In a right triangle ΔABC, $\tan A = \frac{12}{5}$. The sum $\sin A + \cos A$ is equal to what?

    A. 1

    B. $\frac{17}{13}$

    C. $\frac{60}{169}$

    D. $\frac{60}{17}$

    E. None of these

19. A 25-foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of

    the building. If the top of the ladder slips 4 feet down the wall, how much further will the foot of the ladder

    slide away from the wall?

    A. 4 feet

    B. 5 feet

    C. 8 feet

    D. 9 feet

    E. 15 feet

20. What is the equation of the circle with center $(-3, 4)$ and containing the point $(-1, 6)$.

   A. $(x - 3)^2 + (y + 4)^2 = 8$

   B. $(x + 3)^2 + (y - 4)^2 = \sqrt{8}$

   C. $(x + 3)^2 + (y - 4)^2 = 4$

   D. $(x + 3)^2 + (y - 4)^2 = 8$

   E. $(x + 1)^2 + (y - 6)^2 = \sqrt{8}$

21. Twice the measure of the supplement of an angle is added to three times the measure of the complement of the same angle. The sum is the measure of an interior angle of a regular nine-sided polygon. What is the measure of the supplement of the angle?

   A. $82°$

   B. $86°$

   C. $90°$

   D. $94°$

   E. None of these

22. A 16-inch diameter pizza is cut into 12 pieces. What is the distance around one slice?

   A. $16 + \frac{1}{12}\pi$

   B. $16 + 8\pi$

   C. $16 + \frac{1}{6}\pi$

   D. $16 + \frac{4}{3}\pi$

   E. $32 + \frac{1}{12}\pi$

23. Which of these can be used to prove congruence of two triangles?

	I. Side-Angle-Side	II. Side-Side-Side	III. Angle-Angle-Angle
	IV. Side-Side-Angle	V. Angle-Angle-Side	VI. Angle-Side-Angle

   A. All of them
   B. All but III
   C. I, II and VI only
   D. All but III and IV
   E. None of these

24. The sum of the number of vertices, edges, and faces of an octagonal pyramid is
   A. 26
   B. 34
   C. 50
   D. 65
   E. None of these

25. The segment $\overline{CD}$ is an altitude of right triangle ΔACB. Find the exact length of $\overline{CD}$
   in the figure if $\overline{DB} = 5cm$ and $\overline{AD} = 16\ cm$.
   A. 80
   B. $4\sqrt{5}$
   C. $8\sqrt{5}$
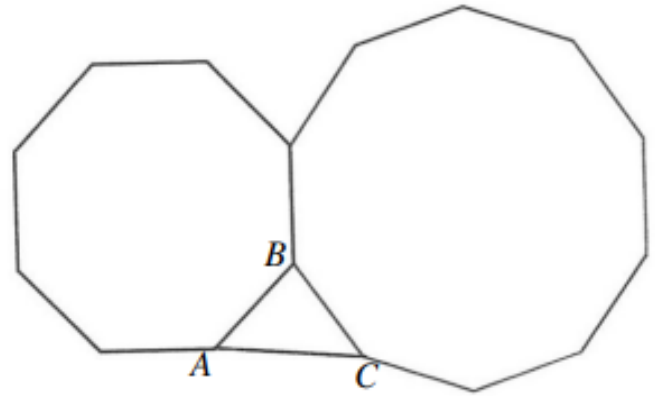   D. $\frac{16}{5}$
   E. $21\sqrt{11}$
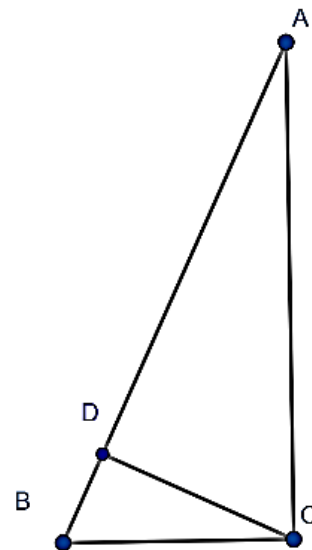
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Tie Breaker 1

Name: _____

School: _____

The adjacent figure is a regular octagon adjacent to a regular decagon. The side lengths of the octagon and decagon are equal. What is the measure of $\angle ABC$?

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Name _____

Tie Breaker 2

Name: _____

School: _____

A circle with center $(4, -2)$ passes through $(7, 2)$. What is the equation of the line tangent to the circle at $(7, 2)$?

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Tie Breaker 3

Name: _____

School: _____

Find the length of the perimeter of a right triangle whose area is 30 cm^2 and whose hypotenuse is 13 cm.

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ACTM Regional Geometry Competition 2024 Answer Key

1	A		11	E		21	A
2	B		12	C		22	D
3	D		13	D		23	D
4	A		14	A		24	B
5	D		15	C		25	B
6	C		16	A			
7	D		17	B			
8	C		18	B			
9	A		19	C			
10	D		20	D			

TB1:

The following figure is a regular octagon adjacent to a regular decagon. The side lengths of the octagon and decagon are equal. What is the measure of $\angle ABC$?

TB1 Solution: 81°

> Since each angle in a regular octagon measures 135° and each angle in a regular decagon measures 144°, the remaining angle measure must be 360° − 135° − 144° = 81°. Therefore, $m\angle ABC = 81°$.
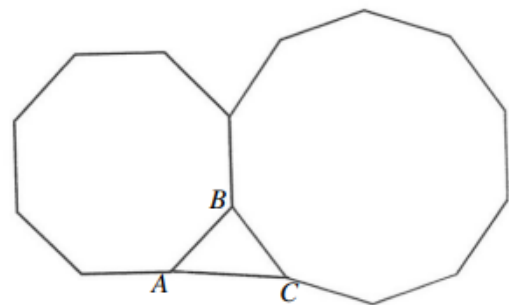
TB2:

A circle with center $(4, -2)$ passes through $(7, 2)$.

What is the equation of the line tangent to the circle at $(7, 2)$?

TB2 Solution: $3x + 4y = 29$

We first find the slope of the radius segment formed by points $(4, -2)$ and $(7, 2)$.

The slope is $\frac{2-(-2)}{7-4} = \frac{4}{3}$.

The slope of the tangent will be perpendicular to the slope of the radius.

Therefore, the slope of the tangent line is $-\frac{3}{4}$.

The equation of the tangent line through point (7,2) in point-slope form is then $y - 2 = -\frac{3}{4}(x - 7)$.

After multiplying both sides by 4 and writing in standard form we have $3x + 4y = 29$.

TB3:

Find the length of the perimeter of a right triangle whose area is $30 \ cm^2$ and whose hypotenuse is $13 \ cm$.

TB3 Solution: 30 cm

There are several approaches to this problem. This is one option.

Because the triangle is a right triangle and the hypotenuse is 13, start with the Pythagorean Theorem.

Let the two legs of the right triangle be a and b. We know $a^2 + b^2 = 13^2 = 169$. (1)

We also know the area of this triangle is $\frac{1}{2}ab = 30$ which is equivalent to $2ab = 120$. (2)

Adding equations (1) and (2) together yields $a^2 + 2ab + b^2 = 169 + 120 = 289$.

Subtracting equation (2) from (1) yields $a^2 - 2ab + b^2 = 169 - 120 = 49$.

Factoring the left side of each equation above yields $(a + b)^2 = 289$ and $(a - b)^2 = 49$.

After taking the square roots of both sides of each equation, we have $a + b = 17$ and $a - b = 7$.

Then adding these two equations together yields $2a = 24$ so $a = 12$. By substitution, $b = 5$.

The sum of a, b, and c is then $13 + 12 + 5 = 30$.