2024 Regional Algebra I Competition – Page 1 Arkansas Council of Teachers of Mathematics

Work on the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tiebreaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tiebreaker pages when you are finished. You may keep the pages with the multiple-choice questions. Figures are not necessarily drawn to scale.

- 1. Find a value for the constant *b* such that the points (2, 8), (1, 5), and (*b*, 11) lie on the same line.
 - a. 11
 - b. 13
 - c. 9
 - d. 3
 - e. No possible value of *b* exists.
- The water usage (in gallons), *W*, from a running sprinkler can be modeled by a linear function. After 10 minutes of operation, 25 gallons of water were used. After 30 minutes, a total of 65 gallons were used. Letting *t* represent time in minutes, find an equation to model the water usage, *W*, in terms of *t*.
 - a. W(t) = 5t + 2
 - b. $W(t) = \frac{1}{2}t + 30$
 - c. $W(t) = \frac{1}{2}t \frac{1}{5}$
 - d. W(t) = 2t + 5
 - e. None of the above.
- 3. In 1977, Jacoby purchased a new car for \$5456. Suppose the value of the care depreciates at a rate of 15% per year. Which of the following would model the value, *V*, of the car *t* years after purchase.
 - a. $V(t) = 5456(1.15)^t$
 - b. $V(t) = 5456(0.85)^t$
 - c. V(t) = 5456(0.15t)
 - d. $V(t) = 5456 (1.15)^t$
 - e. $V(t) = 5456(15)^{t-1977}$

4. Solve the following equation for *x*:

|4x + 1| + 3 = 2

- a. *x* = 1
- b. $x = 1, x = -\frac{3}{2}$
- c. x = 0
- d. $x = 0, x = \frac{1}{2}$
- e. No solution
- 5. Suppose the height, *h*, of a rocket is given by the equation $h(t) = 35t 5t^2$, where *h* is in meters and *t* is time in seconds. At what time(s) is the height of the rocket equal to 12 *meters*?
 - a. t = 0.36 seconds
 - b. t = 6.64 seconds
 - c. t = 7 seconds
 - d. Both a and c
 - e. Both a and b
- 6. A total of 390 people registered for the 49th annual Fun Run in the Park race. Race participants include both children and adults. The number of children who registered was twice the number of adults. How many adults registered to participate in the race?
 - a. 260
 - b. 150
 - c. 195
 - d. 200
 - e. 130
- 7. The Aerolite Furniture company produces luxury sofas. The unit cost (cost in dollars per sofa) depends on the number of sofas produced. If *x* sofas are produced, the unit cost is given by the function
 - $C(x) = 0.3x^2 120x + 19,683$. Find the minimum unit cost.
 - a. \$7,683
 - b. \$200
 - c. \$55,683
 - d. \$19,683
 - e. None of the above

<u>2024 Regional Algebra I Competition</u> – Page 3 Arkansas Council of Teachers of Mathematics

8. Choose the correct graph representing the solution to the system of linear inequalities:

$$\begin{cases} y \ge -x + 6\\ -2x + 3y < 1 \end{cases}$$



- e. None of these graphs
- 9. Solve for *x*:

$$10x + 3(x - 9) = -6 + 8x$$

a.
$$x = \frac{1}{3}$$

b. $x = \frac{21}{3}$
c. $x = \frac{3}{5}$
d. $x = \frac{21}{5}$

5

e. None of the above

Name_

2024 Regional Algebra I Competition – Page 4 Arkansas Council of Teachers of Mathematics

10. Let s = 2ab + 2bc + 2ac. Solve this equation for *b*.

a.
$$b = \frac{s-2ac}{2a+2c}$$

b.
$$b = \frac{2(s-ac)}{a+c}$$

c.
$$b = 2ac - \frac{s}{2a+2c}$$

d.
$$b = \frac{2a+2c}{ac-s}$$

e.
$$b = \frac{a+c}{2as+c}$$

- 11. Suppose the graph of the function y = 3|x| 2 is shifted up by 3 units and to the right by 4 units. Which of the following equations would describe the resulting graph?
 - a. y = 3|x 3| + 2
 - b. y = 3|x + 3| 2
 - c. y = 5|x| + 1
 - d. y = -5|x| + 3
 - e. y = 3|x 4| + 1

12. Suppose the average score on a physics test was 80. Which of the following cannot be true?

- a. Half of the students scored 60 and the other half scored 100.
- b. The range of scores for the class was 80.
- c. No student scored higher than 80.
- d. All students scored exactly 80.
- e. All options above might be true.
- 13. Which of the following equations could define the function f(x), shown in the graph beside.
 - a. $f(x) = -x^2 + 4x 3$
 - b. $f(x) = x^2 4x + 3$
 - c. $f(x) = -(x-2)^2 1$
 - d. $f(x) = (x+2)^2 1$
 - e. None of the above



14. Solve the following equation for *x*:

 $\sqrt{2x+5} - 3 = 2$

- a. *x* = 5
- b. x = 15
- c. x = 10
- d. x = 2, -2
- e. None of the above.
- 15. Tim bought a new suitcase for an upcoming vacation. The new suitcase is *x* inches wide with a length of 2x + 2 inches and a height of 3x 1 inches. Which of the following would represent the volume of Tim's suitcase?
 - a. $V = x^2(2x + 2)$
 - b. $V = 6x^2 + 4x 2$

c.
$$V = 2x^3 + 2x - 2$$

- d. $V = 6x^3 + 4x^2 2x$
- e. None of the above
- 16. The Highflyer Trampoline Park charges a fee of \$35 for 60 minutes of jumping time. The price for 90 minutes of jumping time is \$50. In that interval, what is the average rate of change in the cost of jumping time per minute?
 - a. \$1.75 per minute
 - b. \$2.00 per minute
 - c. \$1.35 per minute
 - d. \$0.50 per minute
 - e. None of the above.
- 17. Which of the following polynomials have zeros of 4 and -2?
 - a. $f(x) = -x^2 + 2x + 8$
 - b. $f(x) = 3x^2 6x 24$
 - c. $f(x) = x^2 2x 8$
 - d. $f(x) = (x-1)^2 9$
 - e. All of the above.

18. Solve the given system of equations.

$$\begin{cases} 2y - x = -6\\ y^2 - x = 9 \end{cases}$$

- a. (0, -3), (16, 5)
- b. (12, 3), (-4. -5)
- c. (3, 0), (5, −16)
- d. (−3, 0), (−4, −5)
- e. None of the above.
- 19. Suppose a grocery delivery company charges a monthly fee of \$20 plus an additional \$5 for each grocery delivery. Let *x* represent the number of grocery deliveries. Which of the following equations could be used to calculate a customer's monthly bill, *B*?
 - a. B(x) = 20x + 5
 - b. B(x) = 5x + 20
 - c. B(x) = 20(x+5)
 - d. B(x) = 5(x 20)
 - e. None of the above.

20. Let $f(x) = 2x^2 + 4x - 6$. Which of the following statements is true about the graph of f(x)?

- a. The graph has a *y*-intercept of (0, -6).
- b. The *x*-intercepts are (-3, 0) and (1, 0).
- c. The coordinates of the vertex are (-1, -4).
- d. Both a and b are true.
- e. Both b and c are true.

21. Solve the following equation for *t*:

$$3t^2 - 4t = -7$$

- a. $t = -1, t = \frac{7}{3}$ b. t = -1c. $t = -\frac{7}{3}$ d. $t = -\frac{7}{3}, t = \frac{7}{3}$
- e. No Solution

- 22. Suppose a positive correlation coefficient is found between two quantities. Which of the following must be true?
 - a. Since the correlation coefficient is positive, one of the quantities is causing the other to increase.
 - b. A positive correlation coefficient means that the variables will be perfectly aligned in a straight line on a scatter plot.
 - c. A positive correlation coefficient ensures that the relationship will remain stable over time.
 - d. A positive correlation coefficient indicates that the line of best fit will have a positive slope.
 - e. None of the above.
- 23. Which of the following best represents a function with a range of all real numbers greater than or equal to -3.



e. None of these

24. Which of the following could be the exponential function whose graph contains the following points.

x	0	1	2	3
у	6	9	13.5	20.25

a. $y = 6(0.5x)^2$

b.
$$y = 9(1.5)^x$$

c.
$$y = 7.5^x$$

d.
$$y = 6(1.5)^{\chi}$$

e. None of the above.

25. Which of the following represents a function?

Relation #1







- a. Relation #1only
- b. Relation #3 only
- c. Relations #1 and 2 only
- d. Relations #2 and 3 only
- e. All of the above

2024 Regional Algebra I Competition – Page 9 Arkansas Council of Teachers of Mathematics

Tie Breaker #1

Name: _____

School: _____

James purchased a new house and wishes to enclose a rectangular area in his backyard so that his house makes up one side of the fenced in area and fencing materials are used for the remaining three sides.

a. If James has 1200m of fencing available, find the dimensions of the rectangular enclosure with the maximum area.

b. What is the maximum area that James can enclose?

<u>2024 Regional Algebra I Competition</u> – Page 10 Arkansas Council of Teachers of Mathematics

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2024 Regional Algebra I Competition – Page 11 Arkansas Council of Teachers of Mathematics

Tie Breaker #2

Name: _____

School: _____

Which of the following could be the graph of the function ax - by = -c? Assume that *a*, *b*, & *c* are positive real numbers.

Please circle your choice.



Using complete sentences, explain your reasoning.

Name _____

<u>2024 Regional Algebra I Competition</u> – Page 12 Arkansas Council of Teachers of Mathematics

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2024 Regional Algebra I Competition – Page 13 Arkansas Council of Teachers of Mathematics

Name _____

Tie Breaker #3

Name: _____

School: _____

The table shown beside lists the number of students enrolled in a contemporary literature course at Midtown Community College for various years.

a. Let *x* represent the number of years after 2007, find the line of best fit for this data. (Round all values to the nearest tenth as necessary.)

YEAR	ENROLLMENT
2007	27
2008	42
2009	48
2010	64
2011	83
2012	104
2013	129
2014	147
2015	169
2016	189

b. Use your model to predict course enrollment in 2022.(Round your answer to the nearest whole number.)

c. Based on your model, in what year would enrollment be expected to reach 400 students. (Round your answer to the nearest year.)

<u>2024 Regional Algebra I Competition</u> – Page 14 Arkansas Council of Teachers of Mathematics

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<u>2024 Regional Algebra I Competition</u> – Page 15 Arkansas Council of Teachers of Mathematics

Answer Key

1. D	11. E	21. E	
2. D	12. E	22. D	
3. B	13. D	23. A	
4. E	14. C	24. D	
5. E	15. D	25. E	
6. E	16. D		
7. A	17. E		
8. A	18. A		
9. D	19. B		
10. A	20. D		

TB1: House 2y = -x + 1200 $y = -\frac{1}{2}x + 600$ $y = -\frac{1}{2}x + 600$ y Objective Function: A = xy $A(x) = x(-\frac{1}{2}x + 600) = -\frac{1}{2}x^2 + 600x$

Since the leading coefficient is negative, the graph of A(x) will be a parabola opening downward with the maximum occurring at the vertex.

$$x = -\frac{b}{2a}$$
$$x = -\frac{600}{2\left(-\frac{1}{2}\right)} = 600$$
$$y = -\frac{1}{2}(600) + 600 = 300$$

The dimensions leading to the maximum area are: $600m \times 300m$

The maximum area is $A = xy = (600)(300) = 180,000m^2$

<u>TB2:</u>

We note that constants *a*, *b*, & *c* are all positive real numbers.

We can solve the function ax - by = -c into slope-intercept form.

$$ax - by = -c$$
$$-by = -ax - c$$
$$y = \frac{a}{b}x + \frac{c}{b}$$

2.65"The line represented by the equation $y = \frac{a}{b}x + \frac{c}{b}$ will have a positive slope and a positive y-intercept.

Hence, graph C is the only possible graph.

<u>TB3:</u>

We enter the data into our calculator. Since x represents the years after 2007, each of the years is determined by $L_1 = year - 2007$. Then we calculate a linear regression.

a. Our line of best fit is y = 18.53x + 16.8.

Note: 2022 is 15 years after 2007. Apply this value to our line of best fit. The result is 294.8 (or about this value due to rounding.)

According to this model, the enrollment in 2022 would be 295 students.

b. We can find the expected year that enrollment reaches 400 by solving our best fit line set equal to 400.

y = 18.53x + 16.8 = 400 $x = \frac{400 - 16.8}{18.53} = 20.7 \approx 21$

According to this model, the enrollment ~21 years after 2007, or year 2028, would be expected to reach 400.

L1	L2	Lз	Lu	Ls	:
0	27				
1	42				
2	48				
3	64				
4	83				
5	104				
6	129				
7	147				
8	169				
9	189				

NORMAL FLOAT AUTO REAL RADIAN MP

LinRe9 9=ax+b a=18.53333333 b=16.8 r²=0.9863508948 r=0.9931519998

2022-2007	15
Y1(15)	294.8
18.53*15+16.8	294.75
Y1(21)	406
Y1(20.7)	400.44

Name ____