Work on the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tiebreaker questions at the end starting with Tie Breaker \#1, then \#2, and finally \#3. Turn in your answer sheet and the tiebreaker pages when you are finished. You may keep the pages with the multiple-choice questions. Figures are not necessarily drawn to scale.

1. Find a value for the constant $b$ such that the points $(2,8),(1,5)$, and $(b, 11)$ lie on the same line.
a. 11
b. 13
c. 9
d. 3
e. No possible value of $b$ exists.
2. The water usage (in gallons), $W$, from a running sprinkler can be modeled by a linear function. After 10 minutes of operation, 25 gallons of water were used. After 30 minutes, a total of 65 gallons were used. Letting $t$ represent time in minutes, find an equation to model the water usage, $W$, in terms of $t$.
a. $\quad W(t)=5 t+2$
b. $\quad W(t)=\frac{1}{2} t+30$
c. $W(t)=\frac{1}{2} t-\frac{1}{5}$
d. $W(t)=2 t+5$
e. None of the above.
3. In 1977, Jacoby purchased a new car for $\$ 5456$. Suppose the value of the care depreciates at a rate of $15 \%$ per year. Which of the following would model the value, $V$, of the car $t$ years after purchase.
a. $\quad V(t)=5456(1.15)^{t}$
b. $V(t)=5456(0.85)^{t}$
c. $\quad V(t)=5456(0.15 t)$
d. $V(t)=5456-(1.15)^{t}$
e. $\quad V(t)=5456(15)^{t-1977}$
4. Solve the following equation for $x$ :

$$
|4 x+1|+3=2
$$

a. $x=1$
b. $x=1, x=-\frac{3}{2}$
c. $x=0$
d. $x=0, x=\frac{1}{2}$
e. No solution
5. Suppose the height, $h$, of a rocket is given by the equation $h(t)=35 t-5 t^{2}$, where $h$ is in meters and $t$ is time in seconds. At what time(s) is the height of the rocket equal to 12 meters?
a. $t=0.36$ seconds
b. $t=6.64$ seconds
c. $t=7$ seconds
d. Both a and c
e. Both a and b
6. A total of 390 people registered for the $49^{\text {th }}$ annual Fun Run in the Park race. Race participants include both children and adults. The number of children who registered was twice the number of adults. How many adults registered to participate in the race?
a. 260
b. 150
c. 195
d. 200
e. 130
7. The Aerolite Furniture company produces luxury sofas. The unit cost (cost in dollars per sofa) depends on the number of sofas produced. If $x$ sofas are produced, the unit cost is given by the function $C(x)=0.3 x^{2}-120 x+19,683$. Find the minimum unit cost.
a. $\$ 7,683$
b. $\$ 200$
c. $\$ 55,683$
d. $\$ 19,683$
e. None of the above
$\qquad$ Arkansas Council of Teachers of Mathematics
8. Choose the correct graph representing the solution to the system of linear inequalities:

$$
\left\{\begin{array}{c}
y \geq-x+6 \\
-2 x+3 y<1
\end{array}\right.
$$


c. NORMAL FLOAT AUTO REAL RADIAN MP INEQUALITY MENU: [2nd] [CALC]

b. NORMAL FLOAT AUTO REAL RADIAN MP INEQUALITY MENU: [2nd] [CALC]

d. NORMML FLOAT AUTO REAL RADIAN MP INEQUALITY MENU: [2nd] [CALC]

e. None of these graphs
9. Solve for $x$ :

$$
10 x+3(x-9)=-6+8 x
$$

a. $x=\frac{5}{3}$
b. $x=\frac{21}{3}$
c. $x=\frac{3}{5}$
d. $x=\frac{21}{5}$
e. None of the above
$\qquad$ Arkansas Council of Teachers of Mathematics
10. Let $s=2 a b+2 b c+2 a c$. Solve this equation for $b$.
a. $b=\frac{s-2 a c}{2 a+2 c}$
b. $\quad b=\frac{2(s-a c)}{a+c}$
c. $b=2 a c-\frac{s}{2 a+2 c}$
d. $b=\frac{2 a+2 c}{a c-s}$
e. $b=\frac{a+c}{2 a s+c}$
11. Suppose the graph of the function $y=3|x|-2$ is shifted up by 3 units and to the right by 4 units. Which of the following equations would describe the resulting graph?
a. $\quad y=3|x-3|+2$
b. $y=3|x+3|-2$
c. $y=5|x|+1$
d. $y=-5|x|+3$
e. $y=3|x-4|+1$
12. Suppose the average score on a physics test was 80 . Which of the following cannot be true?
a. Half of the students scored 60 and the other half scored 100.
b. The range of scores for the class was 80.
c. No student scored higher than 80.
d. All students scored exactly 80 .
e. All options above might be true.
13. Which of the following equations could define the function $f(x)$, shown in the graph beside.
a. $f(x)=-x^{2}+4 x-3$
b. $f(x)=x^{2}-4 x+3$
c. $f(x)=-(x-2)^{2}-1$
d. $f(x)=(x+2)^{2}-1$
e. None of the above

14. Solve the following equation for $x$ :

$$
\sqrt{2 x+5}-3=2
$$

a. $x=5$
b. $x=15$
c. $x=10$
d. $x=2,-2$
e. None of the above.
15. Tim bought a new suitcase for an upcoming vacation. The new suitcase is $x$ inches wide with a length of $2 x+2$ inches and a height of $3 x-1$ inches. Which of the following would represent the volume of Tim's suitcase?
a. $\quad V=x^{2}(2 x+2)$
b. $\quad V=6 x^{2}+4 x-2$
c. $\quad V=2 x^{3}+2 x-2$
d. $V=6 x^{3}+4 x^{2}-2 x$
e. None of the above
16. The Highflyer Trampoline Park charges a fee of $\$ 35$ for 60 minutes of jumping time. The price for 90 minutes of jumping time is $\$ 50$. In that interval, what is the average rate of change in the cost of jumping time per minute?
a. $\$ 1.75$ per minute
b. $\$ 2.00$ per minute
c. $\$ 1.35$ per minute
d. $\$ 0.50$ per minute
e. None of the above.
17. Which of the following polynomials have zeros of 4 and -2 ?
a. $f(x)=-x^{2}+2 x+8$
b. $f(x)=3 x^{2}-6 x-24$
c. $f(x)=x^{2}-2 x-8$
d. $f(x)=(x-1)^{2}-9$
e. All of the above.
18. Solve the given system of equations.

$$
\left\{\begin{array}{c}
2 y-x=-6 \\
y^{2}-x=9
\end{array}\right.
$$

a. $(0,-3),(16,5)$
b. $(12,3),(-4 .-5)$
c. $(3,0),(5,-16)$
d. $(-3,0),(-4,-5)$
e. None of the above.
19. Suppose a grocery delivery company charges a monthly fee of $\$ 20$ plus an additional $\$ 5$ for each grocery delivery. Let $x$ represent the number of grocery deliveries. Which of the following equations could be used to calculate a customer's monthly bill, $B$ ?
a. $B(x)=20 x+5$
b. $B(x)=5 x+20$
c. $B(x)=20(x+5)$
d. $B(x)=5(x-20)$
e. None of the above.
20. Let $f(x)=2 x^{2}+4 x-6$. Which of the following statements is true about the graph of $f(x)$ ?
a. The graph has a $y$-intercept of $(0,-6)$.
b. The $x$-intercepts are $(-3,0)$ and $(1,0)$.
c. The coordinates of the vertex are $(-1,-4)$.
d. Both $a$ and $b$ are true.
e. Both b and c are true.
21. Solve the following equation for $t$ :

$$
3 t^{2}-4 t=-7
$$

a. $t=-1, t=\frac{7}{3}$
b. $\quad t=-1$
c. $t=-\frac{7}{3}$
d. $t=-\frac{7}{3}, t=\frac{7}{3}$
e. No Solution
$\qquad$ Arkansas Council of Teachers of Mathematics
22. Suppose a positive correlation coefficient is found between two quantities. Which of the following must be true?
a. Since the correlation coefficient is positive, one of the quantities is causing the other to increase.
b. A positive correlation coefficient means that the variables will be perfectly aligned in a straight line on a scatter plot.
c. A positive correlation coefficient ensures that the relationship will remain stable over time.
d. A positive correlation coefficient indicates that the line of best fit will have a positive slope.
e. None of the above.
23. Which of the following best represents a function with a range of all real numbers greater than or equal to -3 .
a. NORMAL FLOAT GUTO REAL RADIfiN MP $\quad \square$

b. NORMAL FLoAt futo Refl Radian mp $\quad \square$

c. NORMAL FLOAT AUTO REAL RADIAN MP Ø
d.

e. None of these
$\qquad$
24. Which of the following could be the exponential function whose graph contains the following points.

| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 6 | 9 | 13.5 | 20.25 |

a. $y=6(0.5 x)^{2}$
b. $y=9(1.5)^{x}$
c. $y=7.5^{x}$
d. $y=6(1.5)^{x}$
e. None of the above.
25. Which of the following represents a function?

Relation \#1

| $x$ | 1 | 4 | 7 | 5 | 8 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 2 | 3 | 6 | 2 | 4 | 1 |

Relation \#2


Relation \#3
Domain Range

a. Relation \#1only
b. Relation \#3 only
c. Relations \#1 and 2 only
d. Relations \#2 and 3 only
e. All of the above

## Tie Breaker \#1

Name: $\qquad$

School: $\qquad$

James purchased a new house and wishes to enclose a rectangular area in his backyard so that his house makes up one side of the fenced in area and fencing materials are used for the remaining three sides.
a. If James has 1200 m of fencing available, find the dimensions of the rectangular enclosure with the maximum area.
b. What is the maximum area that James can enclose?

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$\qquad$

## Tie Breaker \#2

Name: $\qquad$

School: $\qquad$

Which of the following could be the graph of the function $a x-b y=-c$ ?
Assume that $a, b, \& c$ are positive real numbers.

Please circle your choice.

Graph B


Graph D


Using complete sentences, explain your reasoning.

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$\qquad$

## Tie Breaker \#3

Name: $\qquad$

School: $\qquad$

The table shown beside lists the number of students enrolled in a contemporary literature course at Midtown Community College for various years.
a. Let $x$ represent the number of years after 2007, find the line of best fit for this data. (Round all values to the nearest tenth as necessary.)
b. Use your model to predict course enrollment in 2022.
(Round your answer to the nearest whole number.)

| YEAR | ENROLLMENT |
| :---: | :---: |
| 2007 | 27 |
| 2008 | 42 |
| 2009 | 48 |
| 2010 | 64 |
| 2011 | 83 |
| 2012 | 104 |
| 2013 | 129 |
| 2014 | 147 |
| 2015 | 169 |
| 2016 | 189 |

c. Based on your model, in what year would enrollment be expected to reach 400 students.
(Round your answer to the nearest year.)

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$\qquad$

Answer Key


## TB1:

Constraint: $x+2 y=1200$

$$
\begin{aligned}
& 2 y=-x+1200 \\
& y=-\frac{1}{2} x+600
\end{aligned}
$$



Since the leading coefficient is negative, the graph of $A(x)$ will be a parabola opening downward with the maximum occurring at the vertex.

$$
\begin{gathered}
x=-\frac{b}{2 a} \\
x=-\frac{600}{2\left(-\frac{1}{2}\right)}=600 \\
y=-\frac{1}{2}(600)+600=300
\end{gathered}
$$

The dimensions leading to the maximum area are: $\mathbf{6 0 0 m} \times 300 \mathrm{~m}$

The maximum area is $A=x y=(600)(300)=180,000 m^{2}$

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Arkansas Council of Teachers of Mathematics

## TB2:

We note that constants $\boldsymbol{a}, \boldsymbol{b}, \& \boldsymbol{c}$ are all positive real numbers.
We can solve the function $a x-b y=-c$ into slope-intercept form.

$$
\begin{gathered}
a x-b y=-c \\
-b y=-a x-c \\
y=\frac{a}{b} x+\frac{c}{b}
\end{gathered}
$$

2.65"The line represented by the equation $y=\frac{a}{b} x+\frac{c}{b}$ will have a positive slope and a positive $y$-intercept.

Hence, graph C is the only possible graph.

## TB:

We enter the data into our calculator. Since $x$ represents the years after 2007, each of the years is determined by $L_{1}=$ year -2007 . Then we calculate a linear regression.
a. Our line of best fit is $y=18.53 x+16.8$.

Note: 2022 is 15 years after 2007. Apply this value to our line of best fit. The result is 294.8 (or about this value due to rounding.)

According to this model, the enrollment in 2022 would be 295 students.
b. We can find the expected year that enrollment reaches 400 by solving our best fit line set equal to 400 .

$$
\begin{aligned}
& y=18.53 x+16.8=400 \\
& x=\frac{400-16.8}{18.53}=20.7 \approx 21
\end{aligned}
$$

According to this model, the enrollment $\sim 21$ years after 2007, or year 2028, would be expected to reach 400.

$$
0
$$

He, graph is the only possible graph.

$$
\text { y- } 0
$$

$$
\infty
$$

$\qquad$
$\square$
$\square$
$\square$
E


NORMAL FLOAT AUTO REAL RADIAN MP
LinReg(ax+b) Li, L2, Y1

NORMAL FLOAT AUTO REAL RADIAN MP
Linked
$y=a x+b$
$a=18.53333333$
$b=16.8$
$r^{2}=0.9863508948$
$r=0.9931519998$


