 NAME: ___________________________

This exam includes 25 multiple-choice questions and three open-response questions that might be used as tie breakers. For questions 1 through 25 (the multiple-choice questions), mark your answer choice in the appropriate location on the sheet provided. After completing questions 1 through 25, answer each tie breaker question in sequential order (i.e. complete Question #1 first, then Question #2, and then Question #3 last). Be sure that your name is printed on each of the tie break questions. When time is called, you will be asked to turn in your multiple-choice question answer sheet and your written responses to the tie breaker questions.

1. Assuming the population standard deviation is known, which of the following specifications would result in the widest confidence interval for the population mean:
   A. 95% Confidence Level, $n = 100$
   B. 99% Confidence Level, $n = 100$
   C. 95% Confidence Level, $n = 500$
   D. 99% Confidence Level, $n = 500$

2. The statement that "A 95% confidence interval obtained from a simple random sample of 1000 people has a greater chance of containing the true population parameter than a 95% confidence interval obtained from a simple random sample of 500 people" is:
   A. Always True
   B. Never True
   C. Sometimes True
   D. Not Enough Information

3. The p-value in hypothesis testing represents which of the following:
   A. The probability of failing to reject the null hypothesis, given the observed results
   B. The probability that the null hypothesis is true, given the observed results
   C. The probability that the observed results are statistically significant, given that the null hypothesis is true
   D. The probability of observing results as extreme or more extreme than currently observed, given that the null hypothesis is true

4. A study of several track athletes, using dermatology clinic records, found that track athletes that did not wear any sunscreen during their daily workouts were more likely to have skin damage than those that wear a waterproof, SPF 35 sunscreen during their daily workouts.

   What type of study is described above?
   A. Survey
   B. Observational Study
   C. Experimental Study
   D. Single-Blind Experimental Study

5. The heights of 490 male high school students were recorded in inches. Summary statistics were then calculated for the dataset. However, after the summary statistics were calculated, it was realized that all of the original height measurements were in error by one inch. In order to correct this error, one inch was added to the heights of all 490 original values.

   Following this one inch correction, which of the following summary statistics would change when recalculated with data from the 'new' sample?

I.	The mean
II.	The range
III.	The variance
IV.	The coefficient of variation

   A. I only
   B. I and II
   C. I and IV
   D. I, II, III, and IV

6. According to statistics from the US Department of Agriculture, 64% of teenage boys are classified as 'calcium deficient', meaning they lack the recommended daily amount of calcium (1300 mg). In a random sample of 12 teenage boys, what is the expected value for the number that will be classified as 'calcium deficient'.
   A. 7.680
   B. 2.765
   C. 4.320
   D. 1.663

For Questions 7–10, refer to the table, which relates to the SimpliRED d-Dimer Test, which is one diagnostic tool used to assess Deep Venous Thrombosis (DVT).

	Deep Venous Thrombosis (DVT)	
SimpliRED d-Dimer	DVT	No DVT
Positive (+) Test	51	32
Negative (-) Test	8	30

7. What is the probability of one randomly selected individual presenting with DVT given the individual receives a positive SimpliRED d-Dimer test result? Round to 3 decimal places.
   A. 0.614
   B. 0.864
   C. 0.421
   D. 0.711

8. Assuming that each Simpli-RED d-Dimer test for DVT is independent of the other, what is the probability of getting two negative results given that you do actually have DVT?
   A. 0.016
   B. 0.004
   C. 0.044
   D. 0.018

9. Assume simple random sampling for the data summarized in the table before.
   Let p_D represent the proportion of individuals that present with DVT.
   What is the 95% confidence interval to estimate p_D, the population proportions of individuals presenting with DVT?
   A. $(0.510, 0.719)$
   B. $(0.399, 0.577)$
   C. $(0.477, 0.752)$
   D. $(0.371, 0.605)$

10. Refer to Question 9. Though a precise number is not known for the number of individuals that present with DVT, the US Centers for Disease Control and Prevention (CDC) estimate that the true population proportion of individuals presenting with DVT each year is approximately 0.002. Given this information, do the results from our sample contradict CDC expectations? Why might this be?
    A. Yes, the results contradict expectations. Our sample size might not be large enough.
    B. Yes, the results contradict expectations. Our assumption of random sampling might be violated.
    C. Yes, the results contradict expectations. Our confidence level might not be high enough.
    D. No, the results do not contradict expectations. Our confidence interval includes the sample proportion of individuals with DVT suggesting that our sample is reliable.

11. A 99% t-based confidence interval for the mean mercury content for King Mackerel fish (ppm) is calculated using a simple random sample of observed mercury content for 50 King Mackerel fish. Given that the 99% confidence interval is $0.622 < \mu < 0.796$, what is the sample mean mercury content for King Mackerel fish (ppm)?
    A. 0.709
    B. 0.087
    C. Not Enough Information; we would need to know the variation in mercury content (ppm) for the sample of King Mackerel fish.
    D. Not Enough Information; we would need to know the variation in mercury content (ppm) for the population of King Mackerel fish.

12. Refer to the confidence interval in Question 11. What percentage of mercury content (ppm) values in the sample fall between 0.622 and 0.796 ppm?
    A. 99%
    B. 50%
    C. 100%
    D. Cannot Be Determined

13. A quiz consists of 9 True/False questions. Assume that the questions are independent. In addition, assume that (T) and (F) are equally likely outcomes when guessing on any one of the questions. What is the probability of guessing on each of the 9 quiz questions and getting at least one of the True/False questions wrong?
    A. 0.998
    B. 0.018
    C. 0.020
    D. 0.980

For Questions 14–16, refer to the experimental setting and data provided below.

An experiment was conducted to see whether sensory deprivation for an extended period of time affects alpha-wave frequencies in the brain. To determine this, ten inmates at a prison were randomly split into two groups of five. One group was placed in solitary confinement ('Confined') while the other group was not in solitary confinement ('Not Confined'). After seven days, alpha-wave frequencies were measured for the inmates.

Alpha-wave frequencies are listed below for both groups of inmates. In addition, a variable labeled 'Differences' is provided, which is the alpha-wave frequency of the non-confined row observation minus the alpha-wave frequency of the confined row observation.

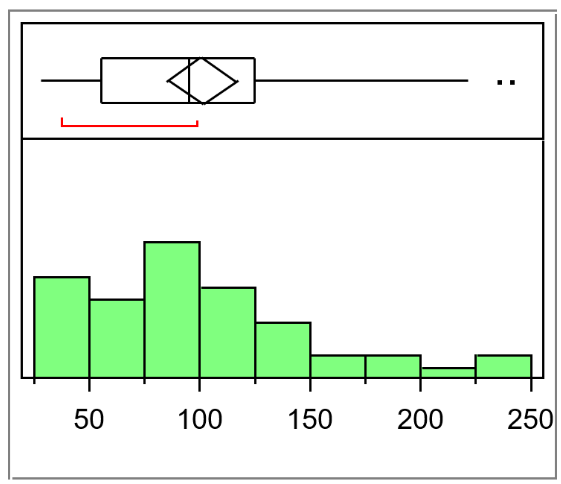
Not Confined	10.7	10.7	10.9	10.3	9.6
Confined	9.5	10.5	10.3	9.2	9.3
Differences	1.2	0.2	0.6	1.1	0.3

14. Which of the following statistical procedures would be most appropriate to test the claim that average alpha-wave frequencies are the same whether an inmate is in solitary confinement ('Confined') or not ('Not Confined')?
    Assume that any necessary normality requirements hold.
    A. Two-tailed two-sample paired/dependent t-test of means
    B. Two-tailed two-sample independent t-test of means
    C. Two-tailed two-sample paired/dependent z-test of means
    D. One-tailed two-sample z-test of proportions.

15. Referring to the setting and data provided above, what is the test statistic for testing the claim that average alpha-wave frequencies are the same whether an inmate is in solitary confinement ('Confined') or not ('Not Confined')?
    - A. 3.342
    - B. 3.736
    - C. 1.922
    - D. 2.149

16. Refer to Question 15. Using a 0.05 significance level, which of the following is the most appropriate conclusion for the hypothesis test given the results?
    - A. Reject the null hypothesis; there is sufficient evidence to suggest that average alpha-wave frequencies differ based upon confinement.
    - B. Fail to reject the null hypothesis; there is not sufficient evidence to suggest that average alpha-wave frequencies differ based upon confinement.
    - C. Fail to reject the null hypothesis; there is sufficient evidence to suggest that average alpha-wave frequencies differ based upon confinement.
    - D. Accept the null hypothesis; there is not sufficient evidence to suggest that average alpha-wave frequencies differ based upon confinement.

17. The boxplot and histogram to the right both represent the storm index for each year from 1950 to 2007. A storm index of 100 represents a typical year whereas a storm index above 100 represents a year with stronger storms than typically observed. Determine the relationship between the mean and the median.
    - A. Mean = Median
    - B. Mean ≈ Median
    - C. Mean < Median
    - D. Mean > Median



18. Refer to the boxplot and histogram provided in Question 17.
    Suppose that from 2008 to 2012 the recorded storm index values were equal to 100, 101, 99, 103, and 98. What happens to the standard deviation for storm index after adding these five values to the 1950-2007 data?
    - A. The standard deviation increases.
    - B. The standard deviation decreases.
    - C. The standard deviation remains the same.
    - D. There is not enough information to determine the effect of these values.

19. The skewness of a distribution can be measured by Pearson's index of skewness. This index can be calculated as follows:

$$I = \frac{3 \cdot (\bar{x} - \tilde{x})}{s}$$

If $I \leq -1.0$ or if $I \geq 1.0$, the data is considered significantly skewed.

Based upon the boxplot and histogram provided in Question 17, would you expect Pearson's index of skewness to be a positive or negative value? Note: $\tilde{x}$ refers to the sample median

A. Positive
B. Negative
C. Not Enough Information; The sample standard deviation must be known
D. Not Enough Information; The population standard deviation must be known

20. Refer to the discrete probability distribution provided in the table below. Notice that X can only take on one discrete value from 0 to 4, $x \in \{0, 1, 2, 3, 4\}$.

$X = x$	0	1	2	3	4
$P(X = x)$	0.040	?	0.450	0.110	0.230

Find the probability that x is equal to 0 and 1. Round to 3 decimal places.

A. 0.000
B. 0.210
C. 0.170
D. 1.000

21. The statement "If there is not sufficient evidence to reject a null hypothesis at the 1% significance level, then there is also not sufficient evidence to reject it at the 5% significance level" is:

A. Always True
B. Never True
C. Sometimes True; the p-value for the statistical test needs provided for a conclusion
D. Not Enough Information; this would depend on the type of statistical test used

22. Provide the probabilistic definition for statistical power.

A. P(Reject H0 | H0 True)
B. P(Reject H0 | H0 False)
C. P(Fail to Reject H0 | H0 True)
D. P(Fail to Reject H0 | H0 False)

23. The GRE General Test is a standardized exam that is often required for and used to make decisions about admission into graduate school programs. The GRE test is comprised of three components: Verbal Reasoning, Quantitative Reasoning, and Analytical Writing. Verbal Reasoning and Quantitative Reasoning scores are reported on a scale from 130 to 170. Analytical Writing scores are on a scale from 0.0 to 6.0. Using performance statistics provided from ETS data, which of the following exam component scores is better relative to other scores on the same exam component?
    - A Verbal Reasoning score of 167; the mean Verbal Reasoning score is 150.22 with a standard deviation of 8.45
    - A Quantitative Reasoning score of 170; the mean Quantitative Reasoning score is 152.47 with a standard deviation of 8.93
    - An Analytical Writing score of 5.5; the mean Analytical Writing score is 3.50 with a standard deviation of 0.87

    A. The Verbal Reasoning score is relatively better
    B. The Quantitative Reasoning score is relatively better
    C. The Analytical Writing score is relatively better
    D. All of the exam scores are relatively equivalent

24. Assume that independent simple random samples are drawn from normally distributed populations with equal standard deviations. Five groups of respondents, each associated with different religious affiliations, were asked what the ideal number of children was for a family. A one-way ANOVA was conducted to determine if religious affiliation had an effect on the reported ideal number of children. The output is provided below:

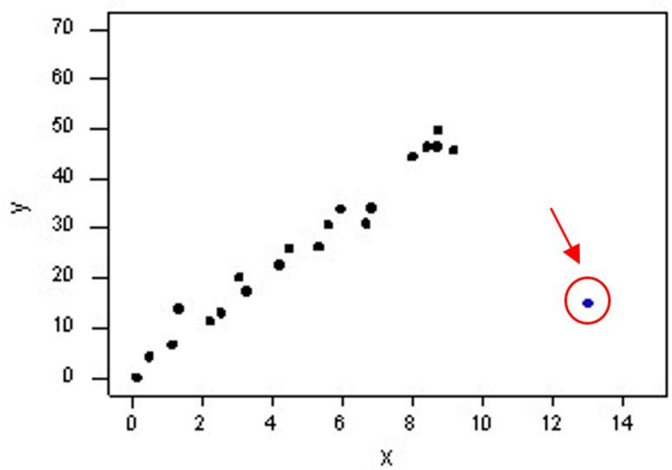
Ideal number of kids in a family

Source	DF	SS	MS	F	P
Religion	4	10.19	2.55	3.59	0.006
Error	1291	920.06	0.71		
Total	1295	930.25			

Using the results of the one-way ANOVA, can you conclude at the 1% significance level that every religious affiliation has a different population mean for the ideal number of children? Please select the best answer of those provided below.

A. Yes. Reject the null hypothesis. There is sufficient evidence to suggest that every religious affiliation has a different population mean ideal number of children
B. No. Fail to reject the null hypothesis. There is not sufficient evidence to suggest that every religious affiliation has a different population mean ideal number of children
C. No. Accept the null hypothesis. There is sufficient evidence to suggest that every religious affiliation has the same population mean ideal number of children
D. No. Reject the null hypothesis. There is sufficient evidence only to suggest that at least one religious affiliation has a different population mean ideal number of children

25. The scatterplot to the right reveals that one data point lies very far from the rest of the data. What is the effect of including this data point when fitting a simple linear regression model?
    A. Including the point would increase the intercept term and increase the slope term
    B. Including the point would decrease the intercept term and increase the slope term
    C. Including the point would increase the intercept term and decrease the slope term
    D. Including the point would decrease the intercept term and decrease the slope term



NAME: ____________________________

School: _________________________

Tie Breaker 1

The prevalence[1] of HIV infection in the US population is estimated to be 35 per 10,000. Certain types of diagnostic tests for HIV are estimated to be 99% accurate. This means that given a person has HIV, the test indicates a positive result 99% of the time (sensitivity). Also, given a person does not have HIV, the test indicates a negative result 99% of the time (specificity).

If a test result is negative, what is the probability that the person does not have HIV?
Note: This is known as the negative predictive value of the diagnostic test.

Round only your final answer to *5* decimal places. Support your answer.

[1]Prevalence is distinct from incidence. Prevalence is a measurement of all individuals affected by the disease within a particular period of time, whereas incidence is a measurement of the number of new individuals who contract a disease during a particular period of time.

NAME: ____________________________

This page is blank.

NAME: ____________________________

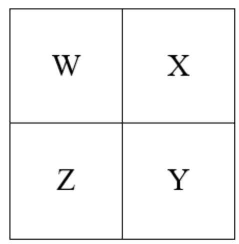
School: ___________________________

Tie Breaker 2

Let S denote the unit square. Suppose that 52 random points are chosen independently such that they are equally likely to be located anywhere on S.

Furthermore, let w, x, y, and z denote the observed number (count) of these points in each quadrant of S.

S



Answer the following questions. Support your answers.

A. Is X a discrete or continuous random variable?

B. What is the sum of w, x, y, and z?

C. State the probability distribution of the random variable X.

D. What is the expected value for the random variable X?

This page is blank.

NAME: ____________________________

School: ___________________________

<u>Tie Breaker 3</u>

Assume a researcher wishes to compare five groups. If no multiple comparison procedure is applied to each of the resulting pairwise tests, what is the probability of the researcher making at least one Type I Error? Assume each pairwise test is conducted at the $\alpha = 0.01$ significance level.

Round to 3 decimal places. Support your answer.

This page is blank.

Multiple Choice Key

1. B
2. B
3. D
4. B
5. C
6. A
7. A
8. D
9. B
10. B
11. A
12. D
13. A
14. B
15. C
16. B
17. D
18. B
19. A
20. A
21. C
22. B
23. C
24. D
25. C

-Tie Breaker -
Question 1

ABBREVIATED Question: The prevalence[1] of HIV infection in the US population is estimated to be 35 per 10,000 ... both the sensitivity and the specificity of the diagnostic test to be 99%. If a test result is negative, what is the probability that the person does not have HIV?

Solution: $P(\text{no } HIV|\text{-}) \approx 0.99996$

**** Steps***

$P(HIV) = 35/10000 = 0.0035$ -> $P(\text{no } HIV) = 1 - P(HIV) = 1 - 0.0035 = 0.9965$

Sensitivity: $P(+|HIV) = 0.99$ -> $P(-|HIV) = 1 - P(+|HIV) = 1 - 0.99 = 0.01$

Specificity: $P(-|\text{no HIV}) = 0.99$

**** The 'Tough' Step Before Getting to the Solution***

$$P(\text{-}) = P((\text{-} \cap HIV) \cup (\text{-} \cap \text{no } HIV)) = P(\text{-} \cap HIV) + P(\text{-} \cap \text{no } HIV)$$
$$= P(\text{-}|HIV) * P(HIV) + P(\text{-}|\text{no } HIV) * P(\text{no } HIV)$$
$$= 0.01 * 0.0035 + 0.99 * 0.9965 = 0.98657$$

**** The Final Step in the Solution***

$$P(no\ HIV|-) = \frac{P(-\ \cap no\ HIV)}{P(-)} = \frac{P(-|no\ HIV) * P(no\ HIV)}{P(-)} = \frac{0.99 * 0.9965}{0.98657} = 0.99996$$

Rubric:

0 pts – No Correct Probabilities or Work Provided

1 pts – Probabilities Known from the Question are Provided but the 'Tough' Step is not Completed

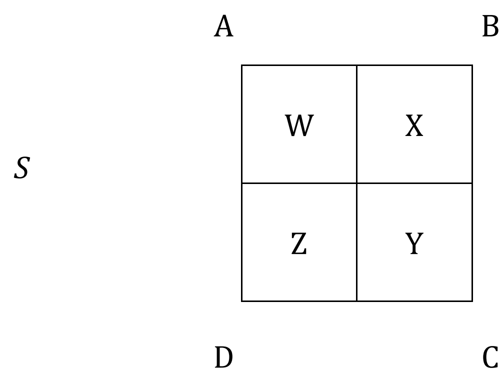
2 pts – The 'Tough' Step is Completed but the Final Solution is Incorrect 3 pts – The Final Solution is Correct

[1] Prevalence is distinct from incidence. Prevalence is a measurement of *all* individuals affected by the disease within a particular period of time, whereas incidence is a measurement of the number of *new* individuals who contract a disease during a particular period of time.

-Tie Breaker - Question 2

Let S denote the unit square, with vertices labeled A, B, C, and D. Suppose that 52 random points are chosen independently according to the uniform distribution on S. Thus, the 52 random points are equally likely to be located anywhere on S.

Furthermore, let W, X, Y, and Z denote the number (count) of these points that are closest to A, B, C, and D respectively.



Answer the following questions. You must provide reasoning for your answers.

Solution Provided in Red:

a. Is X a discrete or continuous RV?

Discrete

b. What is the sum of W, X, Y, and Z?

52

c. What is the distribution of X?

X ~ BIN (n = 52, p = ¼)

d. What is the expected value for X?

E(X) = n*p = 52 * (1/4) = 13

Rubric: 0 pts to 4 pts Possible

1 point for each completely correct answer and 0 points for each incorrect answer on parts a – d

-Tie Breaker -
Question 3

Assume a researcher wishes to compare five groups.

If no multiple comparison procedure is applied to each of the resulting pairwise tests, which are each conducted at the $\alpha = 0.01$ significance level, what is the probability of the researcher making at least one Type I Error?

You must provide reasoning for your answer.

Solution: P(At Least One Type I Error) ≈ 0.096

**** Steps***

1. Notice that with four groups there are $\binom{5}{2} = 10$ pairwise comparisons needed

2. The probability of making a Type I Error on one pairwise comparison is: $\alpha = 0.01$

3. So, the probability of NOT making a Type I Error on one pairwise comparison is:

$$1 - \alpha = 1 - 0.01 = 0.99$$

4. The probability of making NO Type I Errors on all six pairwise comparisons is:

$$(1 - \alpha)^{10} = (1 - 0.01)^{10} = (0.99)^{10} \approx 0.904382$$

5. So, the probability of making at least one Type I Error on all six pairwise comparisons is:

P(At Least One Type I Error) = 1 − P(No Type I Errors) ≈ 1 − 0.904382 ≈ 0.096

Rubric: 0 pts to 5 pts Possible

1 point for each completely correct step and 0 points for each incorrect step on the problem steps labeled #1-5 above.

If the correct final solution is provided but a step (#1-5) is skipped, 5 points should be awarded as long as sufficient work was shown or sufficient reasoning was provided for the answer.