## 2023 State Trigonometry & Precalculus Competition

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tie breaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1. If n is an even number, then which of the following statements is true?
   A. The equation $x^n + 1 = 0$ has n nonreal roots.
   B. The equation $x^n + 1 = 0$ has $n + 1$ roots.
   C. The equation $x^n + 1 = 0$ has exactly one real root.
   D. The equation $x^n + 1 = 0$ has at least one real root.
   E. None of the above

2. Find the midpoint of the line segment whose end points are $\left(-7\sqrt{7}, -8\sqrt{6}\right)$ and $\left(-2\sqrt{7}, -5\sqrt{6}\right)$.
   A. $\left(\frac{-9\sqrt{7}}{2}, \frac{-13\sqrt{6}}{2}\right)$
   B. $\left(\frac{-5\sqrt{7}}{2}, \frac{-3\sqrt{6}}{2}\right)$
   C. $\left(\frac{5\sqrt{7}}{2}, \frac{3\sqrt{6}}{2}\right)$
   D. $\left(-9\sqrt{7}, -13\sqrt{6}\right)$
   E. Unable to determine

3. The graph of a function f is shown. Use the graph to find $f^{-1}(-5)$.
   A. -8
   B. -4
   C. -3
   D. 0
   E. 5
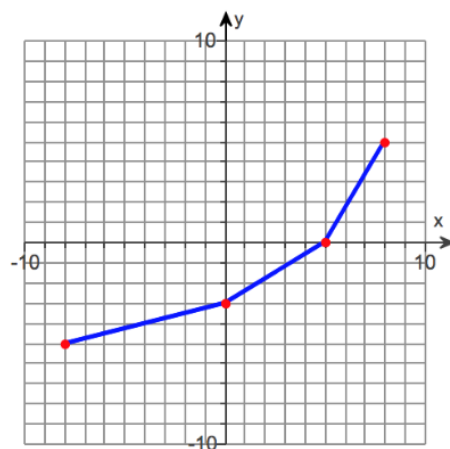
4. Assume that α is an angle in standard position whose terminal side contains the point $(1.7, 5.1)$ and $0 < \alpha < \frac{\pi}{2}$. Find the measure of α to the nearest tenth of a radian.

    A. 3.0

    B. -0.1

    C. 1.2

    D. 0.3

    E. None of the above

5. Find the solution set for the system of equations.
$$\begin{cases} x^2 + y^2 = 9 \\ 9x^2 + 25y^2 = 225 \end{cases}$$

    A. $\{(0,5), (0,-5)\}$

    B. $\{(3,0), (-3,0)\}$

    C. $\{(0,3), (0,-3)\}$

    D. $\{(5,0), (-5,0)\}$

    E. None of the above

6. Determine which of the following is true about the function $y = \cos\left(\frac{1}{2}x\right) + 3$.

    A. The period is 2π.

    B. The phase shift is 3

    C. The range is $[2, 4]$.

    D. All of the above are true.
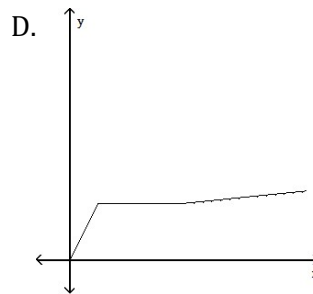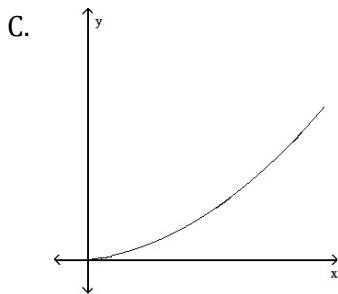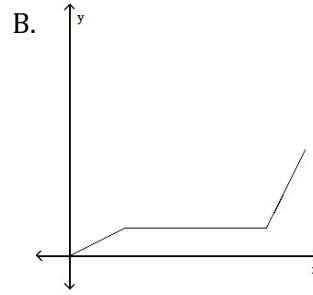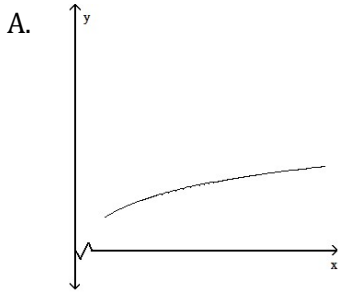
    E. None of the above are true.

7. Find the sum of the first 13 terms of the given sequence. The sequence is either arithmetic or geometric.
$$9, -3, 1, -\frac{1}{3}, \ldots$$

    A. 6.67

    B. 6.75

    C. 7.25

    D. 8.37

    E. 9.15

8. What is the exact value of s on the interval $\left[\frac{\pi}{2}, \pi\right]$ that satisfies $\sin(s) = \frac{\sqrt{2}}{2}$.

   A. $s = \frac{\pi}{4}$

   B. $s = \frac{5\pi}{6}$

   C. $s = \frac{3\pi}{4}$

   D. $s = \frac{2\pi}{3}$

   E. None of the above

9. Assume towns A, B, and C form a triangle. Town A is 19 miles from town B. Town B is 13 miles from town C. Town A is 12 miles from town C. What is the measure of $\angle ACB$?

   A. 98.8°

   B. 81.2°

   C. 56.4°

   D. 33.6°

   E. Unable to determine

10. Find the equation of the parabola with focus $(-5, 2)$ and directrix $y = -3$.

   A. $y = \frac{1}{10}(x - 5)^2 + \frac{1}{2}$

   B. $y = \frac{1}{3}(x + 5)^2 + \frac{1}{2}$

   C. $y = \frac{1}{3}(x - 5)^2 + 2$

   D. $y = \frac{1}{10}(x + 5)^2 - \frac{1}{2}$

   E. None of the above

11. Let the x-axis represent time and the y-axis represent distance. Which graph corresponds to the scenario of "A runner starts jogging, pauses due to cramping, then walks the rest of the way" ?

A.

B.

C.

D.

E.   None of the above.

12. What is the coefficient of x^4 in the expansion of $(2x + 1)^8$?

    A.   26,680

    B.   16

    C.   256

    D.   1120

    E.   1680

13. Given $f(x) = 4x + 8$ and $g(x) = \sqrt{x}$, find the domain of the composite function $f \circ g$.

    A. $[0, \infty)$

    B. $(-\infty, \infty)$

    C. $(-\infty, -2]$ or $[0, \infty)$

    D. $[-2, \infty)$

    E.   None of the above

14. Are the directed line segments $\overrightarrow{AB}$ and $\overrightarrow{CD}$ equivalent? Justify your answer.

$$A = (-4, 6), B = (2, 1), C = (0, 0), \text{ and } D = (6, -5)$$

    A. No, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are not equivalent because they have different initial and terminal points.

    B. Yes, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equivalent because they both represent the vector $\langle 6, -5 \rangle$.

    C. No, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are not equivalent because they point in opposite directions.

    D. Yes, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equivalent because they have the same length.

    E. No, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are not equivalent because they are not in the same quadrant.

15. Decide whether or not the equation has a circle as its graph. If it does not, describe the graph.

$$x^2 + y^2 + 2x - 8y + 18 = 0$$

    A. The graph of the equation is a point

    B. The graph of the equation is a line.

    C. The graph of the equation is a circle.

    D. The graph of the equation is a parabola.

    E. The graph of the equation is nonexistent.

16. Indicate whether the statement is true always, sometimes, or never:

"The product of a pair of complex conjugates (with $b \neq 0$) is the difference of the squares of the real and imaginary parts."

    A. Always

    B. Sometimes

    C. Never

17. What is the domain of $\frac{x^2}{4} - \frac{y^2}{25} = 1$?

    A. Domain: $(-\infty, -2]$

    B. Domain: $(-\infty, -2]$ or $[2, \infty)$

    C. Domain: $(-\infty, \infty)$

    D. Domain: $[2, \infty)$

    E. None of the above

18. Find the magnitude $\|v\|$ for the vector $v = -3i - 4j$.

    A. 3
    B. 5
    C. 7
    D. 12
    E. None of the above.

19. Find the equation of a hyperbola satisfying the given conditions:

    Vertices at $(0, 16)$ and $(0, -16)$; asymptotes $y = \pm 4x$

    A. $\dfrac{x^2}{256} - \dfrac{y^2}{16} = 1$

    B. $\dfrac{x^2}{16} - \dfrac{y^2}{1} = 1$

    C. $\dfrac{y^2}{4} - \dfrac{x^2}{1} = 1$

    D. $\dfrac{y^2}{256} - \dfrac{x^2}{16} = 1$

    E. None of the above.

20. A ship is heading due north for 100 miles to reach a port. Then, it turns (counter-clockwise) 120° and travels for an unknown number of miles to a city. If it makes an additional counter-clockwise turn of 140° before heading back to its starting point, how far apart are the port and city?

    A. 100 miles
    B. 94.8 miles
    C. 153.2 miles
    D. 138.8 miles
    E. None of the above

21. Find all real numbers that satisfy the equation $\sin(x) = -\dfrac{1}{2}$.

    A. The solution set is $\left\{ x \middle| x = \dfrac{7\pi}{6} + 2\pi k \text{ or } x = \dfrac{11\pi}{6} + 2\pi k \right\}$, where k is any integer

    B. The solution set is $\left\{ x \middle| x = \dfrac{7\pi}{6} + \pi k \text{ or } x = \dfrac{11\pi}{6} + \pi k \right\}$, where k is any integer

    C. The solution set is $\left\{ x \middle| x = \dfrac{\pi}{6} + 2\pi k \text{ or } x = \dfrac{5\pi}{6} + 2\pi k \right\}$, where k is any integer

    D. The solution set is $\left\{ x \middle| x = \dfrac{\pi}{6} + \pi k \text{ or } x = \dfrac{5\pi}{6} + \pi k \right\}$, where k is any integer

    E. None of the above

22. If a new car is valued at $28,800 and 8 years later it is valued at $8000, then what is the average rate of change of its value during those 8 years?

    A. −2,600 dollars per year
    B. 20,800 years per dollar
    C. −20,800 dollars per year
    D. 2,600 dollars per year
    E. None of the above
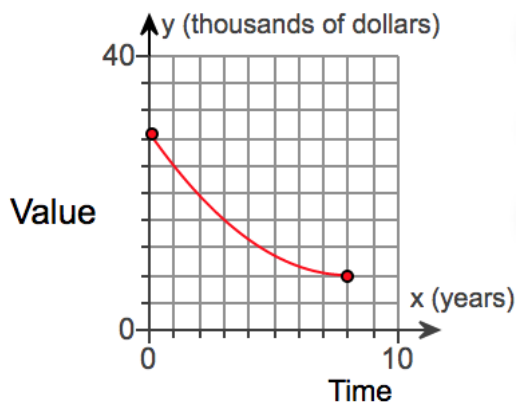
23. Consider the cost benefit equation $\ln(1 - P) = -0.0029 - 0.0058T$, which models the approximate relationship between a tax of T dollars per ton of carbon and the corresponding percent reduction P (in decimal form) of emissions of carbon dioxide. What would be the percent reduction of emissions in carbon dioxide when the tax is $60 per ton of carbon?

    A. 0.704
    B. 0.554
    C. −0.295
    D. −0.351
    E. 0.296

24. Two forces of 425 newtons and 267 newtons act at a point. The resultant force is 507 newtons. Find the angle between the forces.

    A. 52.7°
    B. 163.7°
    C. 88.7°
    D. 91.3°
    E. None of the above

25. Find the inverse of $f(x) = \frac{6}{7x+5}$.

    A. $f^{-1}(x) = \frac{5}{7} - \frac{6}{7x}$

    B. $f^{-1}(x) = \frac{6}{7y} - \frac{5}{7}$

    C. $f^{-1}(x) = \frac{6}{7x} - \frac{5}{7}$

    D. $f^{-1}(x) = \frac{7x+5}{6}$

    E. None of the above

Name _____

School _____

TIEBREAKER #1

SHOW YOUR WORK.

If the radius of a circle is doubled, how is the length of the arc intercepted by a fixed central angle changed?

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Name _____

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TIEBREAKER #2

SHOW YOUR WORK

A one compartment vertical file is to be constructed by bending the long side of an 8 in. by 10 in. sheet of plastic along two lines to form a U shape. How tall should the file be to maximize the volume that it can hold? What is the maximum volume?
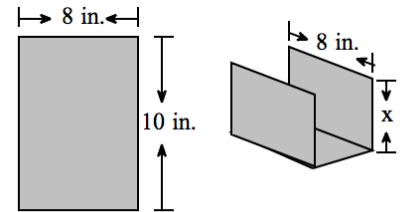
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TIEBREAKER #3

SHOW YOUR WORK.

Prove the following trig identity.

$$\frac{1 - \sin^2(x)}{\sin(x) - \csc(x)} = -\sin(x)$$

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KEY Multiple-Choice Answers

1) A	11) D	21) A
2) A	12) D	22) A
3) A	13) A	23) E
4) C	14) B	24) C
5) C	15) E	25) C
6) C	16) C	
7) B	17) B	
8) C	18) B	
9) A	19) D	
10) D	20) C	

Tie-Breaker Solutions

1) Given that arc length is $s = r\theta$, so if r is doubled then s (the length of the arc) is doubled also.

2) Volume equation is $V = 8x(10 - 2x) = 80x - 16x^2$. Find the vertex $x = \frac{-80}{-2 \cdot 32} = 2.5$. The file should be 2.5 inches high. Maximum volume is $V(2.5) = 100\ in^3$.

3) One possible proof:

$$\frac{1 - sin^2x}{sinx - cscx} = -sinx$$

$$\frac{cos^2x}{sinx - cscx} = -sinx$$

$$\frac{cos^2x}{sinx - \dfrac{1}{sinx}} = -sinx$$

$$\frac{cos^2x}{\dfrac{sin^2 x - 1}{sinx}} = -sinx$$

$$\frac{sinx \cdot cos^2x}{sin^2x - 1} = -sinx$$

$$\frac{sinx \cdot cos^2x}{-cos^2x} = -sinx$$

$$-sinx = -sinx$$