Arkansas Council of Teachers of Mathematics

2023 State Algebra I Competition

Name_

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tiebreaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tiebreaker pages when you are finished. You may keep the pages with the multiple-choice questions. You may assume all variables are real values.

- 1. Find the solution to the following system of equations: $\begin{cases} x + y 3x = -6 \\ -7y + 7z = 7 \\ 3z = 9 \end{cases}$
 - A. x = 1; y = 2; z = -3
 - B. x = -1; y = -2; z = 3
 - C. x = 1; y = -2; z = 3
 - D. x = 1 -; y = 2; z = -3
 - E. None of the above
- 2. The fuel for a chain saw is a mix of oil and gasoline. The ratio of ounces of oil to gallons of gasoline is 9:14. There are 112 gallons of gasoline. How many ounces of oil are there?
 - A. 174.2
 - B. 1.13
 - C. 78
 - D. 72
 - E. None of the above
- 3. A quadratic function *f* is given by $f(x) = ax^2 + bx + c$ where *a* is not 0. Select the statement that *must* be true about the graph of *f*.
 - A. The graph has an x-intercept at (c, 0).
 - B. The graph has two *x*-intercepts.
 - C. If b = 0, then the vertex is on the *y*-axis.
 - D. All of the above.
 - E. None of the above.
- 4. Tickets to the zoo cost \$12 for adults and \$8 for children. The school has a budget of \$240 for the field trip. Which of the following statements is **NOT TRUE**.
 - A. If no adult chaperones were needed, 30 children could go to the zoo.
 - B. If four more adults go to the zoo, that means there will be room for six fewer children.
 - C. If two more children go to the zoo, that means there will be room for three fewer adults.
 - D. If 16 adults go to the zoo, then 6 children can go.
 - E. None of the above.

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5. Which of the following equations have two distinct real solutions?

I. $x^2 = 16$ II. $4x^2 = 0$ III. $x^2 = -16$ IV. 3x + 2 = 14V. $x^2 - 1 = 24$ VI. (x + 8)(x - 8) = 0

- A. I, II, III, V, and VI
- B. I and II only
- C. I, V, and VI
- D. All of the above
- E. None of the above
- 6. Which of the following expressions could be equivalent to $x^2 + bx 36$ where *b* is negative?
 - A. (x+3)(x-12)
 - B. (x-2)(x+18)
 - C. (x 13)(x 3)
 - D. (x+4)(x+9)
 - E. None of the above
- 7. The graph on the right shows the equation $y = x^2 + 4x + 5$. Which statement is true about the real solutions to the equation $x^2 + 4x + 5 = 0$?
 - A. The equation has no real solution.
 - B. The equation has one real solution: 5.
 - C. The equation has two real solutions: -2 and 0.
 - D. The equation has infinitely many solutions.
 - E. None of the above.
- 8. What is the solution set to the inequality?

$$\frac{x+2}{2} \ge -7 - \frac{x}{2}$$

- A. $x \leq -8$
- B. $x \ge -8$
- C. $x \ge -\frac{9}{2}$
- D. $x \ge 8$
- E. None of the above.



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9. At the beginning of the year, the number of books owned by a library was 10,000. Since then, it has grown by 1% each month.

Which expressions represent the number of books, in thousands, owned by the library 5 years later if it continues to grow at that rate?

- A. $10 \cdot (1 + 0.01^{12})^{60}$
- B. $10 \cdot (1.01)^5$
- C. $10 \cdot (1.01)^{60}$
- D. $10 \cdot (0.01)^{60}$
- E. None of the above
- 10. Function *w* gives the weight of a cat, in kilograms, when it is *m* months old. Which statement represents the meaning of the equation w(7) = 4 in this situation?
 - A. The cat weighs 7 kilograms when it is 4 months old.
 - B. The cat weighs 4 kilograms when it is 7 months old.
 - C. The weight of the cat has been 7 kilograms for 4 months.
 - D. The cat weighs 4 kilograms when it is 7 years old.
 - E. None of the above
- 11. To raise funds for a trip, members of a high school math club are holding a game night in the gym. They sell tickets at \$5 per person. The gym holds a maximum of 250 people. The amount of money raised is a function of the number of tickets sold.

Which statement accurately describes the domain of the function?

- A. All numbers less than 250
- B. All integers
- C. All positive integers
- D. Non-negative integers less than or equal to 250
- E. None of the above

12. Which equation is equivalent to $\frac{(5x+6)}{2} = 3 - (4x + 12)?$

- A. 5x + 6 = 6 (4x + 12)
- B. $\frac{5}{2}x + 6 = 3 (4x + 12)$
- C. 5x + 6 = -8x 18
- D. All of the above
- E. None of the above

13. A new computer loses $\frac{1}{3}$ of its value every year. Which graph could represent the relationship between the year and the computer's value?



- E. None of the above.
- 14. A ball bounces several times after it is dropped. The graph shows the height of the ball over time. Height is measured in meters and time is measured in seconds.

Select which statement is true about the graph and the situation it represents.

- A. The vertical intercept shows the time when the ball hits the ground.
- B. The function reaches its maximum value after the first bounce.
- C. The function has no minimum value.
- D. All of the above statements are true.
- E. None of the above statements are true.



- 15. Function *p* represents the number of people in the library on a Monday as a function of hours since the library opened. Here is the graph of function *p*. Based on the graph and the situation, which domain and range make sense for function *p*?
 - A. Domain: all numbers from 0 to 85. Range: all numbers from 0 to 14.
 - B. Domain: whole numbers only from 0 to 14. Range: whole numbers only from 0 to 85.
 - C. Domain: all numbers from 0 to 14. Range: whole numbers only from 0 to 85.
 - D. Domain: whole numbers only from 0 to 14. Range: all numbers from 0 to 85.
 - E. None of the above.



16. A student on the track team runs 45 minutes each day as a part of her training. She begins her workout by running at a constant rate of 8 miles per hour for *a* minutes, then slows to a constant rate of 7.5 miles per hour for *b* minutes.

Which equation describes the relationship between the distance she runs in miles, *D*, and her running speed, in miles per hour?

A.
$$a + b = 45$$

B.
$$8a + 7.5b = D$$

- C. $8\left(\frac{a}{60}\right) + 7.5\left(\frac{b}{60}\right) = D$
- D. 8(45 b) + 7.5b = D
- E. None of the above
- 17. Which of the following expressions are equivalent to 2^{10} ?
 - A. $2^9 + 2$
 - $\frac{2^{11}}{2}$ B.

 - C. 10²
 - D. 10 · 2
 - E. None of the above

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- 18. Twenty-one young men got a quote for the cost of a car insurance policy. The policy cost and ages of the men are shown in the scatter plot. Which of the following statements is true?
 - A. Every 19-year-old in this group has a lower cost of car insurance than the 18-year-olds.
 - B. The value of the correlation coefficient is very close to -1, so there is no association between age and car insurance policy cost.
 - C. The line of best fit indicates that, in general, the cost of a car insurance policy for a 21-year-old should be, on average, about \$316 less than the cost of a car insurance policy for a 20-year-old.
 - D. The *y*-intercept from the line of best fit indicates that, if the linear trend holds, a car insurance policy would be \$0 for a person who is about 8,987 years old.
 - E. None of the above.



- 19. Which function could represent the height in meters of an object thrown upwards from a height of 25 meters above the ground *t* seconds after being launched?
 - A. $f(t) = -5t^2$

B.
$$f(t) = -5t^2 + 25$$

- C. $f(t) = -5t^2 + 25t + 50$
- D. $f(t) = -5t^2 + 50t + 25$
- E. None of the above

20. Choose the graph that shows the solution to this system: $\begin{cases} y > 3x + 2 \\ -4x + 3y \le 12 \end{cases}$



E. None of the above.

21. Here is a pattern of squares. *S* represents the number of small squares in the pattern as a function of *n*, the step number.



Which expression could define *S*?

- A. 3*n*
- B. *n* + 3
- C. $n^2 + 2$
- D. $n^2 + 3$
- E. None of the above.

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22. An orange is shot up into the air with a catapult. The function *h* given by $h(t) = 15 + 60t - 16t^2$ models the orange's height, in feet, *t* seconds after it was launched.

Which of the below statements about the situation are true?

- A. The domain of function h only contains values greater than or equal to 0.
- B. The orange is 15 feet above the ground when it is launched.
- C. The value t = 10 does not belong to the domain of *h*.
- D. All of the above.
- E. None of the above.

23. Which of the following equations doesn't have a real solution?

- A. $x^2 = 64$
- B. (x)(-x) = 4
- C. $x^2 = 15$
- D. All of the above.
- E. None of the above.
- 24. A square garden with side length 10 meters has a square pond in one corner, as shown in the diagram.

Select the expression that represents the area of the pond, in square meters.

- A. $100 20p + p^2$
- B. $10^2 p^2$
- C. 100 20p
- D. p(10 p)
- E. None of the above.
- 25. Here are two equations that relate two quantities, p and Q:

$$Q = 7p + 1,999$$

 $p = \frac{Q - 1,999}{7}$

Select all statements that are true about p and Q.

- A. Q = 7p + 1,999 could represent a function, but $p = \frac{Q-1,999}{7}$ could not.
- B. $p = \frac{Q-1,999}{7}$ could represent a function, but Q = 7p + 1,999 could not.
- C. The two equations represent two functions that are inverses of one another.
- D. If Q = 7p + 1,999 represents a function, then the inverse function can be defined by p = 7Q 1,999.
- E. None of the above.



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Tie Breaker #1



a. Find the average rate of population growth between 2010 and 2015.

b. The average rate of population growth between 2013 and 2015 is 0.35 million people per year. If the population continues to grow at this rate, in which year will it reach 40 million? Explain your reasoning.

c. How does the average rate of change of the population from 2013 to 2015 compare to that from 2011 to 2013? Explain what this means in terms of the population of the country.

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Tie Breaker #2

- 2. The milligrams of aspirin in a person's body is given by the equation $a = 500 \cdot \left(\frac{3}{4}\right)^t$, where *t* is the number of hours since the patient took the medicine.
 - A. How much aspirin will be in the patient's body after two hours?
 - B. In the equation, what does the 500 tell us about the situation?
 - C. In the equation, what does the $\frac{3}{4}$ tell us about the situation?



D. Graph the equation for values of *t* between 0 and 4.

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A school is constructing a rectangular play area against an exterior wall of the school building. It uses 120 feet of fencing

A. Complete the table by giving the length and area for

material to enclose three sides of the play area.

building.)

3.

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Tie Breaker #3		
ol is constructing a rectangular play area against an r wall of the school building. It uses 120 feet of fencing al to enclose three sides of the play area.	w	l
Complete the table by giving the length and area for each width. (The width is the side perpendicular to the building.)		school building

Width (feet)	Length (feet)	Area (square feet)
10		
25		
40		
W		

- B. Write an equation to represent ℓ , the length of the play area in feet, as a function of *w*, the width in feet.
- C. Write an equation to represent *A*, the area in square feet, as a function of *w*, the width in feet.
- D. What dimensions of the play area would yield the maximum area?

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1. E	11. D	21. C
2. D	12. C	22. D
3. C	13. C	23. B
4. C	14. E	24. A
5. C	15. C	25. C
6. A	16. C	
7. A	17. B	
8. B	18. C	
9. C	19. D	
10. B	20. D	

Multiple Choice Answers:

Tie-Breaker Question Answers:

- 1. Answers for Question 1:
 - a. A 0.52 million people per year, or 520,000 people per year 2.
 - b. The average rate of change between 2013 and 2015 is 0.35 million people per year. If the population continues to grow at this rate, the population will be 39.45 million in 2016, 39.80 million in 2017, and 40.15 million in 2018. So the population will reach 40 million between 2017 and 2018.
 - c. The two average rates of change are equal, both are 0.35 million people per year. This means during those periods the population grew at the same rate each year.
- 2. Answers for Question 2:
 - a. 281.25 milligrams (or equivalent)
 - b. There were 500 mg of aspirin in the person's body when she took the aspirin (so the aspirin dose was probably 500 mg).
 - c. Each hour, the amount of aspirin in the person's body is multiplied by $\frac{3}{4}$ (or decreases by $\frac{1}{4}$ of its previous value).
 - d. Graph:



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- 3. Answers for Question 3:
 - a. Table of values:

Width (feet)	Length (feet)	Area (square feet)
10	100	1,000
25	70	1,750
40	40	1,600
W	120 - 2w	w(120 - 2w)

- b. $\ell = 120 2w$ (or equivalent)
- c. A = w(120 2w) (or equivalent)
- d. 30 ft by 60 ft