Arkansas Council of Teachers of Mathematics **Name _____**

2023 Regional Trigonometry & Precalculus Competition

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1) While traveling in a car, the centrifugal force a passenger experiences as the car drives in a circle varies jointly as the mass of the passenger and the square of the speed of the car. If a passenger experiences a force of 225 newtons when the car is moving at a speed of 50 kilometers per hour and the passenger has a mass of 100 kilograms, find the force a passenger experiences when the car is moving at 60 kilometers per hour and the passenger has a mass of 60 kilograms.
   a. 252 newtons
   b. 216 newtons
   c. 194.4 newtons
   d. 172.8 newtons
   e. None of the above

2) A fossilized leaf contains 21% of its normal amount of carbon 14. How old is the fossil (to the nearest hundred years)? Use 5600 years as the half-life of carbon 14.
   a. 24,600
   b. 1,900
   c. 35,200
   d. 12,600
   e. None of the above

3) Give the equation(s) of any vertical asymptotes for $h(x) = \frac{4x-2}{x^2+2x-48}$.
   a. $x = -6, x = 8$
   b. $y = 6, y = -8$
   c. $y = 4$
   d. $x = 6, x = -8$
   e. None of the above

4) Write an equation for the line parallel to $9x + 7y = -45$ and passing through $(2, -5)$ in standard form, $Ax + By = C$, where $A, B, \& C$ are real numbers with $A > 0$.
   a. $7x + 9y = -5$
   b. $9x - 7y = -17$
   c. $2x + 7y = -45$
   d. $9x + 7y = -17$
   e. None of the above

5) During its initial year of operation, 200,000 people visited Rave Amusement Park. In year 6, the number had grown to 834,000. If the number of visitors to the park obeys the law of uninhibited growth, find the exponential growth function that models this data.
   a. $f(t) = 634{,}000e^{0.238t}$
   b. $f(t) = 200{,}000e^{0.238t}$
   c. $f(t) = 634{,}000e^{0.248t}$
   d. $f(t) = 200{,}000e^{0.248t}$
   e. None of the above

6) The vector $\boldsymbol{v}$ has initial position $P = (3, 1)$ and terminal point $Q = (-1, -4)$. Write $\boldsymbol{v}$ in the form $ai + bj$; that is, find its position vector.
   a. $\boldsymbol{v} = -4i - 5j$
   b. $\boldsymbol{v} = 4i + 5j$
   c. $\boldsymbol{v} = -5i - 4j$
   d. $\boldsymbol{v} = 5i + 4j$
   e. None of the above

7) A bridge is built in the shape of a parabolic arch. The bridge arch has a span of 164 feet and a maximum height of 25 feet. Find the height of the arch at 10 feet from its center.
   a. 13.1 ft
   b. 0.1 ft
   c. 1.5 ft
   d. 24.6 ft
   e. None of the above

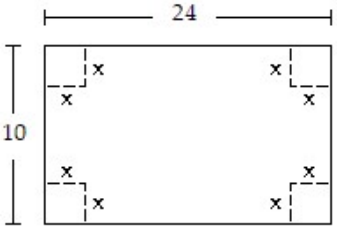
8) Find the product of the value $z = -4 + \sqrt{-9}$ and its conjugate.
   a. -8
   b. 5
   c. 25
   d. $\frac{7}{25} - \frac{24}{25}i$
   e. No product is possible.

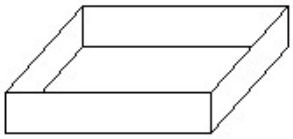
9) A discount warehouse offers two types of annual memberships. Plan A has an annual membership fee of \$300 and the member pays 60% of the manufacturer's recommended list price. Plan B has an annual membership fee of \$60 and the member pays 90% of the manufacturer's recommended list price. How many dollars of merchandise would a member have to purchase in a year to pay the same amount under both plans?
   a. \$1600.00
   b. \$160.00
   c. \$800.00
   d. \$80.00
   e. None of the above

10) Find the equation of the parabola with a vertex at $(5, 1)$ and a focus at $(3, 1)$.
   a. $(y-1)^2 = -8(x-5)$
   b. $(x-5)^2 = -8(y-1)$
   c. $(x-5)^2 = 8(y-1)$
   d. $(y-1)^2 = 8(x-5)$
   e. None of the above

11) If $P = (9, 7)$ and $Q = (x, 142)$, find all numbers x such that the vector represented by $\overrightarrow{\boldsymbol{PQ}}$ has length 225.
   a. $\{-180, 180\}$
   b. $\{-189, 171\}$
   c. $\{-171, 189\}$
   d. $\{-171, -189\}$
   e. None of the above

12) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 10 inches by 24 inches by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.





a. $V(x) = x(10 - 2x)(24 - 2x)$
b. $V(x) = x(10 - x)(24 - x)$
c. $V(x) = (10 - x)(24 - x)$
d. $V(x) = (10 - 2x)(24 - 2x)$
e. None of the above

13) Given $\tan(\theta) = -\frac{4}{7}$, where θ is in quadrant II. Find the exact value of $\cos(\theta)$.

a. $-\frac{7\sqrt{65}}{65}$
b. $\frac{\sqrt{65}}{4}$
c. $-\frac{\sqrt{65}}{7}$
d. $\frac{7\sqrt{65}}{65}$
e. None of the above

14) The profit that the vendor makes per day by selling x pretzels is given by the function $P(x) = -0.004x^2 + 2.4x - 400$. Find the number of pretzels that must be sold to maximize profit.

a. -40 pretzels
b. 1.2 pretzels
c. 300 pretzels
d. 600 pretzels
e. None of the above

15) The accompanying tables represent a function f that converts seconds to hours and a function g that converts hours to days.

x	86,400	172,800	259,200	345,600	432,000
$f(x)$	24	48	72	96	120

x	24	48	72	96	120
$g(x)$	1	2	3	4	5

Express $(f^{-1} \circ g^{-1})(x)$ symbolically.

a. $(f^{-1} \circ g^{-1})(x) = 86{,}400x^2$
b. $(f^{-1} \circ g^{-1})(x) = \frac{x}{86{,}400}$
c. $(f^{-1} \circ g^{-1})(x) = \frac{x^2}{86{,}400}$
d. $(f^{-1} \circ g^{-1})(x) = 86{,}400x$
e. None of the above

16) Solve the equation:

$$\sin^{-1}(x) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{2\pi}{3}$$

a. $x = 0$
b. $x = \frac{\pi}{6}$
c. $x = \frac{\sqrt{3}}{2}$
d. $x = 1$
e. No solution

17) The table lists the average annual cost (in dollars) of room and board at public four-year colleges in the city of Bookhaven for selected years. Determine a linear function that models the data using the first and last years.

Year	**Room & Board (in $)**
1	1250
2	1505
3	1717.5
4	1967.5
5	2245
6	2505

a. $f(x) = 999x + 251$
b. $f(x) = 1250$
c. $f(x) = 2151 - 901x$
d. $f(x) = 251x + 999$
e. None of the above

18) If $f(x) = (x+2)^3 - 8$ is one-to-one, find its inverse.

a. $f^{-1}(x) = \sqrt[3]{x+6}$
b. $f^{-1}(x) = \sqrt[3]{x+10}$
c. $f^{-1}(x) = \sqrt[3]{x+8} - 2$
d. $f^{-1}(x) = \sqrt[3]{x-2} + 8$
e. Function $f(x)$ is not one-to-one.

19) Given this function $f(x)$, list all values of x in $[1, \infty)$ for which $f(x)$ is undefined.

$$f(x) = \frac{1}{\tan\left(\frac{\pi}{2} - \frac{\pi}{x}\right)}$$

a. $\{1\}$
b. $\{1, 2\}$
c. $\{1, 2, 3, \dots\}$
d. $\{1, 3, 5, \dots\}$
e. None of the above

20) A rocket is launched upward so that its distance (in meters) above the ground after t seconds is $h(t) = -10t^2 + 360t$. What is the average rate of change in the rocket's position between $t = 5$ and $t = 7$ seconds?

a. $240\ m/s$
b. $480\ m/s$
c. $1550\ m/s$
d. $1800\ m/s$
e. None of the above

21) For ΔABC, find the length of AB if $\angle A = 30°$, $AC = 15$, and $CB = 10$. Round to the nearest tenth.

a. $AC = 6.4$
b. $AC = 11.2$
c. $AC = 19.6$
d. Unable to solve as no triangle with these measurements is possible.
e. Unable to solve as these measurements result in ambiguous triangles.

22) Polar coordinates $\left(7, \frac{2\pi}{3}\right)$ are given. Find the rectangular coordinates of the point.

a. $\left(-\frac{7}{2}, -\frac{7\sqrt{3}}{2}\right)$
b. $\left(\frac{7}{2}, \frac{7\sqrt{3}}{2}\right)$
c. $\left(\frac{7}{2}, -\frac{7\sqrt{3}}{2}\right)$
d. $\left(-\frac{7}{2}, \frac{7\sqrt{3}}{2}\right)$
e. None of the above

23) A surveyor is measuring the distance across a small lake. He set up his transit on one side of the lake 110 feet from a piling that is directly across from a pier on the other side of the lake. From his transit, the angle between the piling and the pier is 65°. What is the distance between the piling and the pier to the nearest foot? (Assume this makes a right triangle.)

a. 100 ft
b. 51 ft
c. 46 ft
d. 236 ft
e. None of the above

24) Give the equations of any horizontal asymptotes of $h(x) = \frac{9-7x}{-2x+5}$.

a. $y = -\frac{9}{2}$
b. $y = \frac{7}{2}$
c. $y = \frac{9}{5}$
d. $y = -\frac{7}{5}$
e. None

25) An organization determines that the cost per person of chartering a bus is given $C(x) = \frac{250+3x}{x}$, where x is the number of people in the group and $C(x)$ is in dollars. Find a formula for $C^{-1}(x)$.

a. $C^{-1}(x) = \frac{250}{x+3}$
b. $C^{-1}(x) = \frac{250+x}{3}$
c. $C^{-1}(x) = \frac{250}{x-3}$
d. $C^{-1}(x) = \frac{2503}{x-250}$
e. None of the above

Tie Breaker #1

Name: _____

School: _____

Show all your work.

Expand the polynomial $(x + y)^5$. What is the coefficient of the x^3y^2 term?

Tie Breaker #2

Name: _____

School: _____

Prove or disprove this trig identity:

$$\mathbf{sec}(x) + \mathbf{tan}(x) = \frac{\mathbf{1}}{\mathbf{sec}(x) - \mathbf{tan}(x)}$$

Tie Breaker #3

Name: _____

School: _____

In flying the 78 miles from Champaign to Peoria, a student pilot sets a heading that is 11° off course and maintains an average speed of 108 miles per hour. After 15 minutes, the instructor notices the course error and tells the student to correct his heading. Through what angle will the plane move to correct the heading and how many miles away is Peoria when the plane turns?

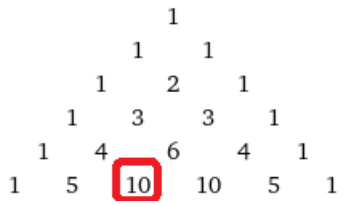
KEY

1) C	**11) C**	**21) E**
2) D	**12) A**	**22) D**
3) D	**13) A**	**23) D**
4) D	**14) C**	**24) B**
5) B	**15) D**	**25) C (corrected)**
6) A	**16) D**	
7) D	**17) D**	
8) C	**18) C**	
9) C	**19) B**	
10) A	**20) A**	

Tie Breaker 1 Suggested answer

$$(x+y)^5 = x^5 + 5x^4y + \boxed{10x^3y^2} + 10x^2y^3 + 5xy^4 + y^5$$

We could use Pascal's Triangle to evaluate this coefficient.



Tie Breaker 2 Suggested answer

We can prove this trig identity.

$\mathbf{sec}(x) + \mathbf{tan}(x) = \dfrac{1}{\mathbf{sec}(x) - \mathbf{tan}(x)}$	Given
$\mathbf{sec}(x) + \mathbf{tan}(x) = \dfrac{1}{\mathbf{sec}(x) - \mathbf{tan}(x)} \cdot \left(\dfrac{\mathbf{sec}(x) + \mathbf{tan}(x)}{\mathbf{sec}(x) + \mathbf{tan}(x)}\right)$	Multiplicative identity
$\mathbf{sec}(x) + \mathbf{tan}(x) = \dfrac{\mathbf{sec}(x) + \mathbf{tan}(x)}{\mathbf{sec^2}(x) - \mathbf{tan^2}(x)}$	Multiplication
$\mathbf{sec}(x) + \mathbf{tan}(x) = \dfrac{\mathbf{sec}(x) + \mathbf{tan}(x)}{\mathbf{1}}$	Pythagorean Identity, $1 + \tan^2(x) = \sec^2(x)$
$\mathbf{sec}(x) + \mathbf{tan}(x) = \mathbf{sec}(x) + \mathbf{tan}(x)$	Proven

There are undoubtedly other steps that would prove this statement.

Tie Breaker 3 Suggested answer

The expected answer is a course correction of $16.7°$ and a final leg of $51.8mi$.

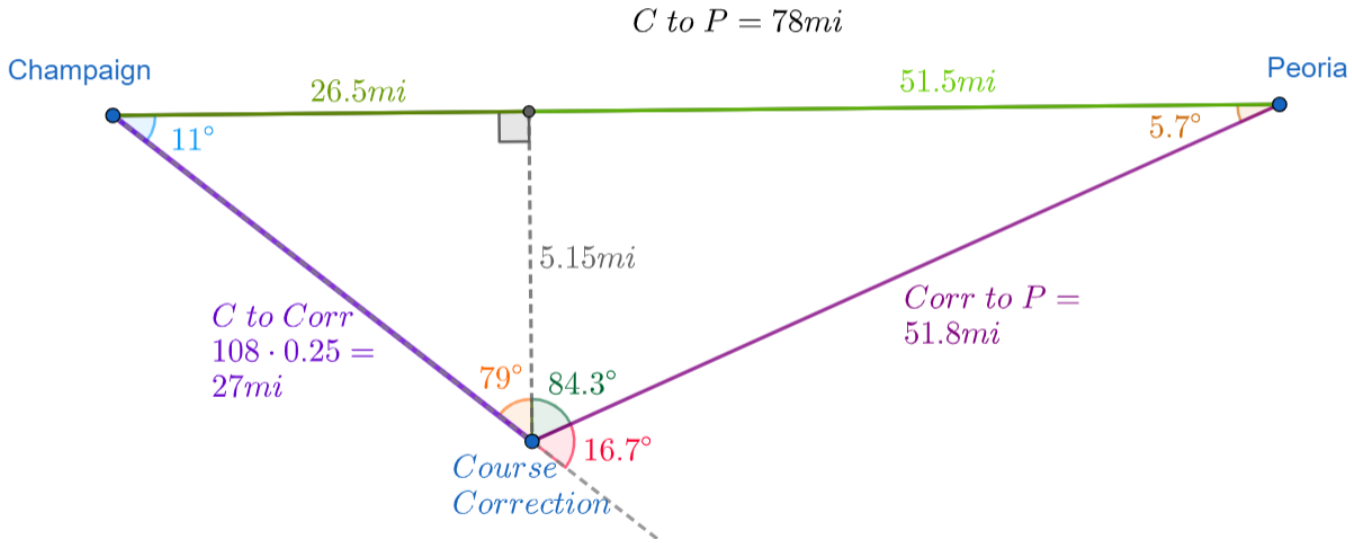
There are many ways to approach this problem. One way is to draw a perpendicular line at the course correction.

For the leftmost triangle, you can then find the other values using $27\sin(11°) = 5.15$, $27\cos(11°) = 26.5$, and $90° - 11° = 79°$.

For the rightmost triangle, we can define the horizontal leg as $78 - 26.5 = 51.5$ and $\sqrt{51.5^2 + 5.15^2} = 51.8$. We then calculate angles $\tan^{-1}\left(\frac{5.15}{51.5}\right) = 5.7°$ and $90° - 5.7° = 84.3°$.

The course correction angle is $180° - 79° - 84.3° = 16.7°$.

Here is a rough sketch of this geometric construction. No sides or angles are drawn to scale.



A more geographically and geometrically correct sketch would be