Arkansas Council of Teachers of Mathematics **Name ______________________________**

2023 Regional Geometry Competition

Begin by removing the three tie breaker sheets at the end and writing your name on all three pages. Work the multiple-choice questions first, choosing the single ***best*** (most detailed and complete correct) response from the choices available. Indicate your answer here and on your answer sheet. Make sure you attempt the tie-breaker questions at the end of the test starting with tie breaker 1, then 2, and then 3 if you have time. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Notations and Definitions:

- All questions on this test are in **Euclidean Geometry**.
- All angles are measured in **radians**. **π radians = 180°**.
- *AB* indicates the distance between points *A* and *B*.
- ***A-B-C*** indicates that *B* is ***between*** *A* and *C* that is: *A, B*, and *C* are collinear and *AB + BC = AC*.
- A ***kite*** is a quadrilateral with at least two non-overlapping pairs of congruent consecutive sides. Its ***major diagonal*** has endpoints where the congruent sides meet.
- A ***trapezoid*** is a quadrilateral with *at least* one pair of parallel sides.
- An ***isometry*** (rigid transformation) is a transformation mapping every preimage to a congruent image.
- ***Z Property***: Alternate interior angles formed by a transversal to lines *l* and *m* are congruent if and only if *l* and *m* are parallel.

1. Given ΔABC with m∢A=$\frac{\pi}{12}$ and with m∢B=$\frac{\pi}{4}$ what is m∢C?

   A. $\frac{\pi}{3}$ B. $\frac{2\pi}{3}$ C. $\frac{5\pi}{12}$ D. $\frac{\pi}{4}$ E. Each of the other answers is incorrect.

2. The sum of the measures of the exterior angles of a convex *n*-gon is _____.
   A. 2π
   B. *n*π
   C. *n*π – π
   D. (*n* – 2) π
   E. Each of the other answers is incorrect.

3. Which of the following conditions is NOT sufficient to conclude that two triangles are congruent? (Conditions indicate corresponding pairs of congruent parts.)

   A. SSS B. SAS C. AAS D. ASA E. SSA

4. Consider ΔABC and ΔA'B'C' such that m∢A = π/3, m∢B = π/6, m∢A' = π/3, m∢B' = π/6. Which of the following best describes this situation?
   A. These are two congruent triangles.
   B. These are two similar triangles.
   C. These are two congruent right triangles.
   D. These are two similar right triangles.
   E. Each of the other answers is incorrect

5. Let the coordinates of three points be given as A = (2,1), B = (0,2), and C = (1, 4). Which of the following best describes ΔABC?
   A. Right Triangle
   B. Isosceles Triangle
   C. Right Isosceles Triangle
   D. Scalene Triangle
   E. Obtuse Triangle

6. Let the coordinates of three points be given as A = (3, 0), B = (5, 3), and C = (11, -1). Which of the following best describes ΔABC?
   A. Right Triangle
   B. Isosceles Triangle
   C. Right Isosceles Triangle
   D. Acute Triangle
   E. Obtuse Triangle

7. Which of the following does not have to have a pair of congruent diagonals?
   A. Parallelogram B. Isosceles Trapezoid C. Square D. Rectangle E. Regular Quadrilateral

8. How many different (non-congruent) parallelograms can be constructed with sides of length 4 cm, 4 cm, 6 cm, and 6 cm?
   A. 0
   B. Exactly 1
   C. Exactly 2
   D. Infinitely many
   E. It cannot be determined from the information given.

9. Given quadrilateral $ABCD$ such that $\Delta ABC \cong \Delta CDA$ the quadrilateral must be a _______.
   A. Rhombus B. Parallelogram C. Kite D. Square E. Trapezoid

10. Given quadrilateral $ABCD$ such that one diagonal is a perpendicular bisector of the other diagonal. Quad $ABCD$ must be a ________.
    A. Rhombus B. Parallelogram C. Kite D. Square E. Trapezoid

11. Given quadrilateral $ABCD$ such that both pair of opposite angles are congruent. Quad $ABCD$ must be a __________.
    A. Rhombus B. Parallelogram C. Kite D. Square E. Trapezoid

12. Which of the following is an outline of a correct deduction, given a quadrilateral ABCD such that both pairs of opposite sides are congruent?
    A. ∢DCA ≅ ∢BAC and ∢BCA ≅ ∢DAC by the Z Property and Δ ABC ≅ ΔCDA by ASA Triangle Congruence Theorem
    B. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$ and $\overline{AC} \cong \overline{CA}$, thus $\Delta ABC \cong \Delta ADC$ by SSS Triangle Congruence Theorem
    C. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$ and $\overline{AC} \cong \overline{CA}$, thus $\Delta ABC \cong \Delta CDA$ by SSS Triangle Congruence Theorem
    D. ∢DCA ≅ ∢BAC and ∢BCA ≅ ∢DAC by the Z Property and $\Delta ABC \cong \Delta ADC$ by AAS Triangle Congruence Theorem
    E. Each of the other answers is incorrect.

13. A quadrilateral with a pair of opposite sides that are both congruent and parallel must be a(n) ______________.
    A. Rhombus B. Parallelogram C. Isosceles Trapezoid D. Square E. Rectangle

14. A trapezoid with a pair of congruent opposite interior angles is a(n)________.
    A. Right Trapezoid
    B. Rectangle
    C. Isosceles Trapezoid
    D. Parallelogram
    E. Each of the other answers is incorrect.

15. Which of the following is not an isometry?
    A. Translation B. Rotation C. Reflection D. Glide-reflection E. Dilation

16. $\overline{CD}$ is an altitude of ΔABC with A-D-B, AD = 5, DB = 7, CD = 4. What is the area of ΔABC?
    A. 6 B. 14 C. 24 D. 48 E. Each of the other answers is incorrect.

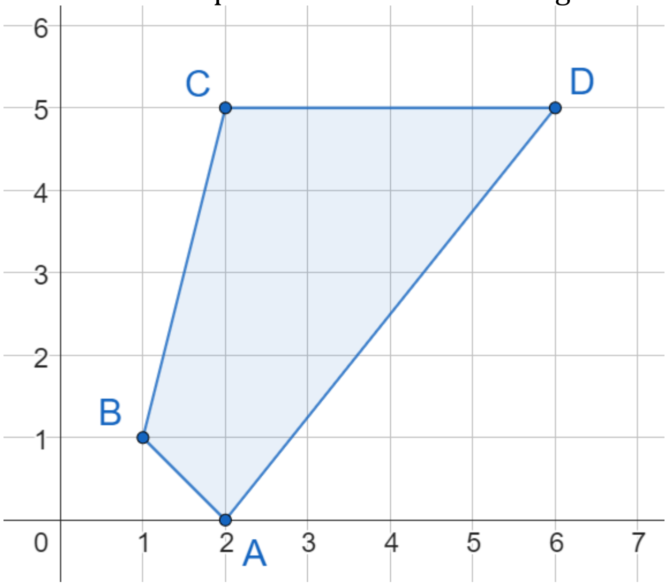
17. In a right isosceles triangle, if we divide the length of the hypotenuse by the length of a leg what do we obtain?
    A. $\frac{1}{\sqrt{2}}$ B. $\sqrt{2}$ C. $\sqrt{3}$ D. $\frac{\sqrt{3}}{2}$ E. Each of the other answers is incorrect.

18. The composition of two reflections about parallel lines is a single ________.
    A. Rotation B. Reflection C. Translation D. Glide-reflection
    E. Each of the other answers is incorrect.

19. Take the point (3, 4) rotate it $-\pi/2$ about the origin and then translate the intermediate image by $\langle 2, 3\rangle$. What is the final image?

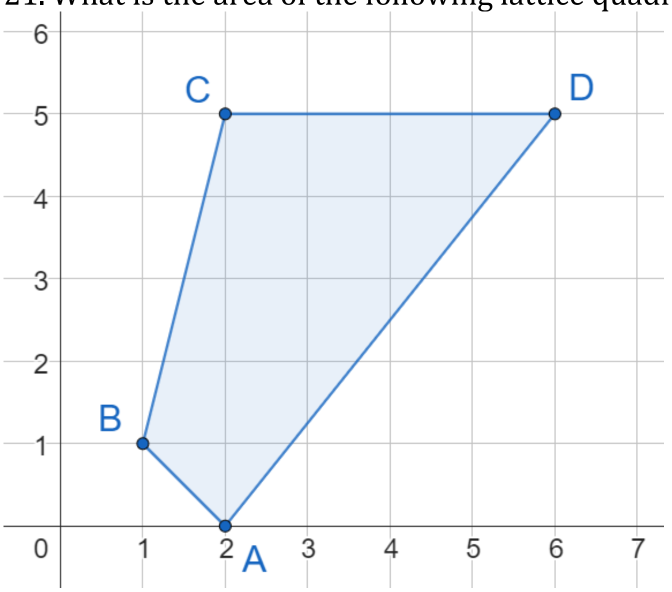
A. (-1,7) B. (-1,-1) C. (2,-6) D. (6, 0) E. Each of the other answers is incorrect.

20. What is the perimeter of the following lattice quadrilateral ABCD, rounded to the nearest hundredth?



A. 10.50 B. 12.25 C. 14.75 D. 15.94 E. Each of the other answers is incorrect.

21. What is the area of the following lattice quadrilateral ABCD?



A. 12 B. 12.5 C. 13 D. 14 E. Each of the other answers is incorrect.

22. What is the image of the point (2, 5) after applying a glide-reflection along the vector $\langle 3, 3\rangle$ and over line $y = x$?
    A. (8,5) B. (2,-1) C. (5, 8) D. (1,-2) E. Each of the other answers is incorrect.

23. What is the area of a trapezoid with midline of length 4 and height of length 8?
    A. 12 B. 16 C. 32 D. 64 E. Each of the other answers is incorrect.

24. What is the surface area of a regular tetrahedron with edge length 4?
    A. $8\sqrt{3}$ B. $16\sqrt{3}$ C. $16\sqrt{2}$ D. $8\sqrt{2}$ E. Each of the other answers is incorrect.

25. Let A = (2,0), B = (0,0), C = (2,2), and D = (0,2).
    What is the best description of $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{DA}$?
    A. Square B. Rectangle C. Parallelogram D. Rhombus
    E. Each of the other answers is incorrect.

Tie Breaker 1

Name: ______________________________

School: ______________________________

Prove the following. Provide a sketch to accompany your proof.

Suppose quad$ABCD$ exists with $\Delta ABC \cong \Delta ADC$.

What kind of quadrilateral is this?

Prove your result.

Tie Breaker 2

Name: ______________________________

School: ______________________________

Prove the following. Provide a sketch to accompany your proof.

Suppose quad$ABCD$ exists with $\Delta ABC \cong \Delta CDA$.

What kind of quadrilateral is this?

Prove your result.

Tie Breaker 3

Name: ______________________________

School: ______________________________

Prove the following. Provide a sketch to accompany your proof.

Suppose quad$ABCD$ is a cyclic kite.

What can we say about one pair of opposite interior angles? What can we say about the other pair?

Prove your result.

 Name ______________________________

ACTM Regional Geometry Competition 2023 Solutions

Answers

1	**B**		11	**B**		21	**B**
2	**A**		12	**C**		22	**A**
3	**E**		13	**B**		23	**C**
4	**D**		14	**D**		24	**B**
5	**C**		15	**E**		25	**E**
6	**A**		16	**C**			
7	**A**		17	**B**			
8	**D**		18	**C**			
9	**B**		19	**D**			
10	**C**		20	**D**			

Note on #25: This is not a polygon. It is not simple.

ACTM Regional Geometry Competition 2023
Tie Breaker 1

Name: **Solution Key**

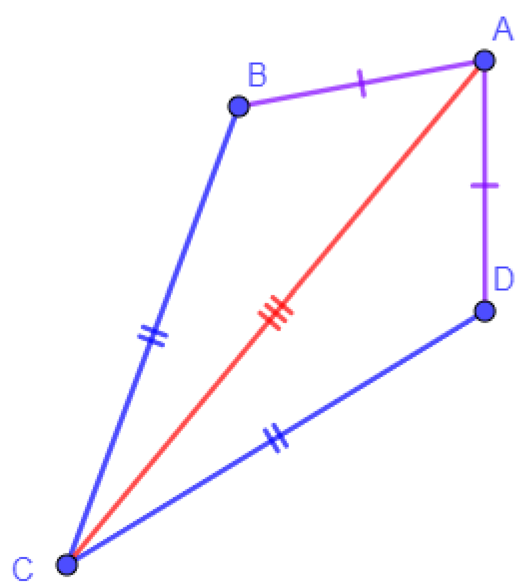
Prove the following. Provide a sketch to accompany your proof.
Suppose quad*ABCD* exists with ΔABC ≅ ΔADC.

What kind of quadrilateral is this? Quad*ABCD* is a kite.

Prove your result.

Proof:

In the hypothesis we are given quad*ABCD* exists with ΔABC ≅ ΔADC. By definition of triangle congruence, corresponding parts of congruent triangles are congruent, so $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$. By definition, quadABCD is a kite (with major diagonal AC). ■



ACTM Regional Geometry Competition 2023
Tie Breaker 2

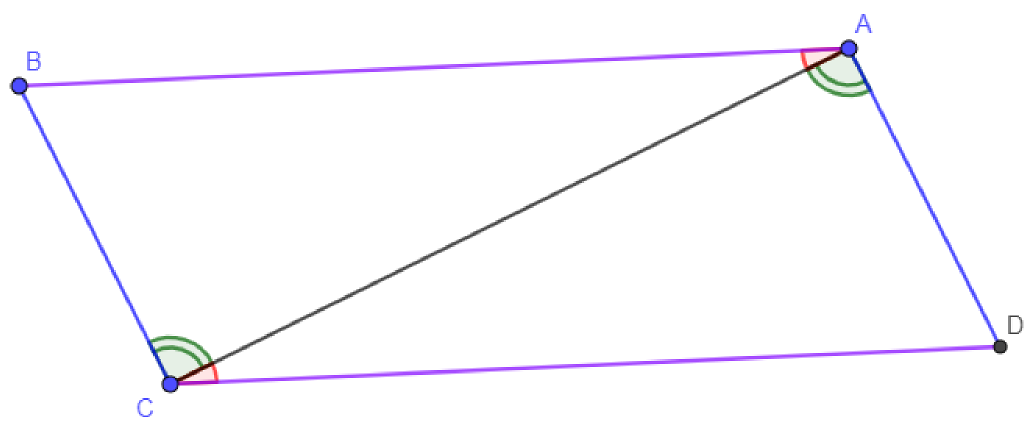
Name: **Solution Key**

Prove the following. Provide a sketch to accompany your proof.
Suppose quad$ABCD$ exists with ΔABC ≅ ΔCDA.

What kind of quadrilateral is this?

Prove your result.

Proof:
In the hypothesis we are given quad$ABCD$ exists with ΔABC ≅ ΔCDA. Construct diagonal $\overline{AC}$. By definition of triangle congruence, corresponding parts of congruent triangles are congruent, so ∢ACB ≅ ∢CAD, and ∢BAC ≅ ∢DCA. Since ∢ACB ≅ ∢CAD the Z Property says that a pair of lines cut by a transversal with congruent alternate interior angles are parallel, so $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Similarly, since ∢BAC ≅ ∢DCA the Z Properties tells us that $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$. By definition, quadABCD is a parallelogram.
■



ACTM Regional Geometry Competition 2023
Tie Breaker 3

Name: **Solution Key**

Prove the following. Provide a sketch to accompany your proof.
Suppose quad$ABCD$ is a cyclic kite.

What can we say about one pair of opposite interior angles? What can we say about the other pair?

The opposite pair of interior angles both formed by congruent sides are supplementary. The other pair of angles are both right angles (congruent and supplementary).

Prove your result.

Proof:
By definition, a kite has two non-overlapping pair of congruent adjacent sides. Without loss of generality, assume that $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$. Construct the major diagonal $\overline{AC} \cong \overline{AC}$. By the SSS Triangle Congruence Theorem, ΔABC ≅ ΔADC. By definition of congruent triangles, corresponding parts of congruent triangles are congruent. ∢ABC ≅ ∢ADC. So in any kite, there is a pair of opposite congruent angles. The measure of any inscribed angle is ½ of the measure of its intercepted arc. Since the kite is cyclic it is inscribed in a circle, and interior angle A intercepts minor arc BAD. The opposite interior angle C intercepts major arc BCD. The measure of major arc BCD = 2π -measure of minor arc BCD. So 2m∢BAC = 2π - 2m∢BCD so m∢BAC = π - m∢BCD so m∢BAC + m∢BCD = π, and m∢BAC and m∢BCD are supplementary. Similarly, ∢ABC and ∢ADC are supplementary. ∢ABC and ∢ADC are supplementary and congruent, so they are right angles. ■