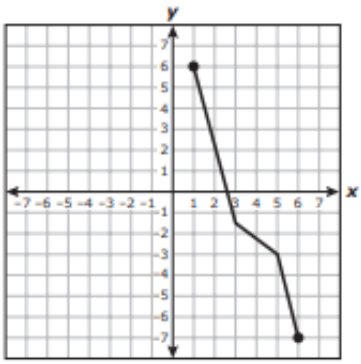
Arkansas Council of Teachers of Mathematics **Name** _____

2023 Regional Algebra II Competition

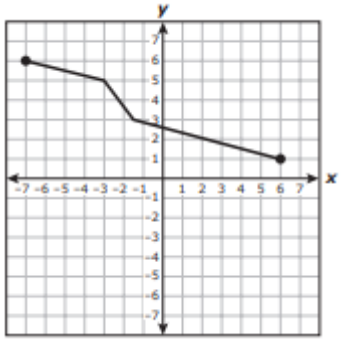
Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet, your tie-breaker pages, and your scratch work when you are finished. Figures are not necessarily drawn to scale.

1) Consider the graph of the function f shown on the right.



Which of the following sketches below shows the graph of f^{-1} ?

a)



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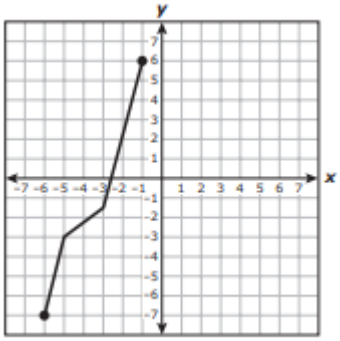

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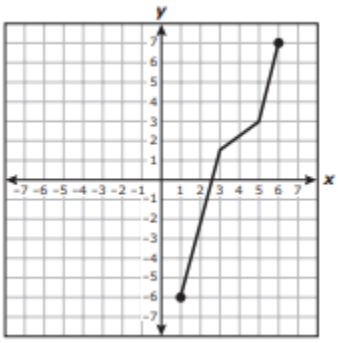
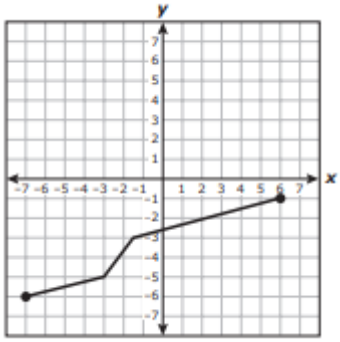


d)



e) None of these options.

2) Consider these values: `3, 6, 2, 1, 7, 5`

James found the mean and standard deviation of the set of numbers given above. If he adds 5 to each number, which of the following will result?

a) The mean will be multiplied by 5.
b) The standard deviation will increase by 5.
c) The mean will not change.
d) The standard deviation will not change.
e) The mean and standard deviation change is unknown.

3) What is the solution to the equation $5^x = 17$?

a) $x = 2$
b) $x = \log_{10}(2)$
c) $x = \log_{10}(17) + \log_{10}(5)$
d) $x = \frac{\log_{10}(17)}{\log_{10}(5)}$
e) Unable to solve for x.

4) The graph of $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{3}\right)^2 = 1$ is a hyperbola. Which set of equations represents the asymptotes of the hyperbola's graph?

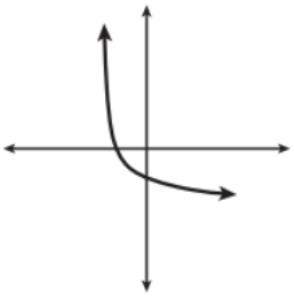
a) $y = \frac{3}{2}x, y = -\frac{3}{2}x$
b) $y = \frac{2}{3}x, y = -\frac{2}{3}x$
c) $y = \frac{1}{2}x, y = -\frac{1}{2}x$
d) $y = \frac{1}{3}x, y = -\frac{1}{3}x$
e) The set of equations are not given.

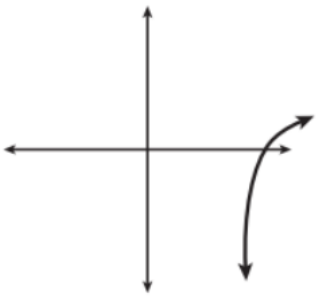
5) Which of the following most accurately describes the translation of the graph $y = (x + 3)^2 - 2$ to the graph of $y = (x - 2)^2 + 2$?

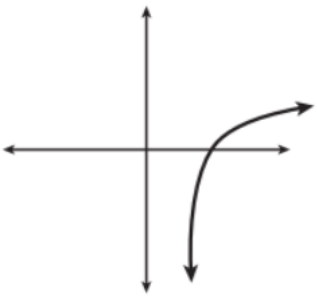
a) Up 4 and 5 to the right
b) Down 2 and 2 to the right
c) Down 2 and 3 to the left
d) Up 4 and 2 to the left
e) None of the above

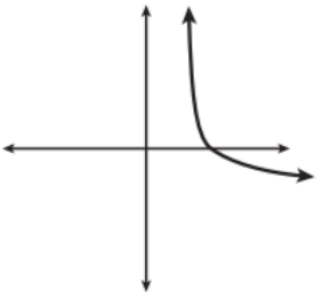
6) What is the inverse of $f(x) = x^3 - 2$?
   a) $f^{-1}(x) = \sqrt[3]{x} + 2$
   b) $f^{-1}(x) = \pm\sqrt[3]{x} + 2$
   c) $f^{-1}(x) = \sqrt[3]{x + 2}$
   d) $f^{-1}(x) = \pm\sqrt[3]{x + 2}$
   e) $f^{-1}(x)$ does not exist

7) Which sketch could represent the function $m(x) = -\log_{100}(x - 2)$?

a) 

b) 

c) 

d) 

   e) None of these choices

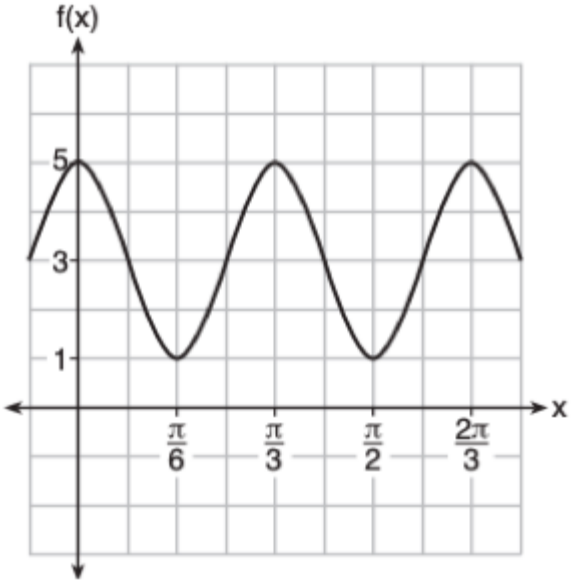
8) What is the y-value of the solution to the matrix equations below?

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

   a) -6
   b) 14
   c) -5
   d) 12
   e) No value exists

9) What is the product of the complex numbers $(3 + i)$ and $(3 - i)$?
   a) 8
   b) 10
   c) $9 - i$
   d) $10 - 6i$
   e) Unable to determine

10) The function $f(x) = a\cos(bx) + c$ is plotted on the beside graph. What are the values of $a, b, \& c$?



a) $a = 2, b = 6, c = 3$
b) $a = 2, b = 3, c = 1$
c) $a = 4, b = 6, c = 5$
d) $a = 4, b = \frac{\pi}{3}, c = 3$
e) None of the above

11) A monthly cell phone plan charges \$5.00 for the first 300 text messages used and \$0.15 for each additional message. On this plan, what is the maximum number of text messages that must be used in a month in order to make the average cost per message \$0.05?

a) 350
b) 400
c) 500
d) 900
e) Not enough information

12) Which polynomial represents $(3x^2 + x + 4)(2x - 5)$?

a) $6x^3 + 13x^2 - 13x - 20$
b) $6x^3 - 13x^2 - 13x + 20$
c) $6x^3 - 13x^2 + 3x - 20$
d) $6x^3 + 13x^2 + 3x + 20$
e) None of the above options.

13) What are the solutions to the equation $1 + \frac{1}{x^2} = \frac{3}{x}$?

a) $x = \frac{3}{2} + \frac{\sqrt{5}}{2};\ x = \frac{3}{2} - \frac{\sqrt{5}}{2}$
b) $x = 3 + \frac{\sqrt{5}}{2};\ x = 3 - \frac{\sqrt{5}}{2}$
c) $x = \frac{3}{2} + \frac{\sqrt{13}}{2};\ x = \frac{3}{2} - \frac{\sqrt{13}}{2}$
d) $x = 3 + \frac{\sqrt{13}}{2};\ x = 3 - \frac{\sqrt{13}}{2}$
e) No solution

14) The expression $\left(\frac{m^2}{m^{1/3}}\right)^{-1/2}$ is equivalent to

a) $-\sqrt[6]{m^5}$
b) $\frac{1}{\sqrt[6]{m^5}}$
c) $-m\sqrt[5]{m}$
d) $\frac{1}{m\sqrt[5]{m}}$
e) None of the above

15) Which of the following quadratic functions does not have zeros of -15 and 6?

a) $f(x) = \frac{1}{3}x^2 + 3x - 30$
b) $f(x) = -x^2 - 9x + 90$
c) $f(x) = -\frac{2}{3}x^2 - 6x + 60$
d) $f(x) = -x^2 - 9x - 90$
e) All of the above

16) For all values of x for which the expression is defined, $\frac{x^2+3x}{x^2+5x+6}$ is equivalent to

a) $1 - \frac{x}{x+2}$
b) $\frac{x}{x+2}$
c) $\frac{3x}{5x+6}$
d) $1 + \frac{1}{2x+6}$
e) All of the above

17) The formula $P = 2\pi\left(\sqrt{\frac{L}{32}}\right)$ can be used to approximate the period of a pendulum, where L is the pendulum's length in feet and P is the pendulum's period in seconds. If a pendulum's period is 1.6 seconds, which of the following is closest to the length of the pendulum?

a) 1.4 ft
b) 4.2 ft
c) 2.1 ft
d) 3.2 ft
e) Not enough information given

18) On a recent test, Jeremy wrote the equation $\frac{x^2-16}{x-4} = x + 4$. Which of the following statements is correct about the equation he wrote?

a) The equation is always true.
b) The equation is always true, except when $x = 4$.
c) The equation is never true.
d) The equation is sometimes true when $x = 4$.
e) Unable to determine.

19) A table showing values of x, $f(x)$, and $g(x)$ is shown on the right. What is the value of $f(-0.5)$?

a) -3.0
b) -0.5
c) 0
d) 1.5
e) Unable to determine.

x	$f(x)$	$g(x)$
-1.5	-0.5	-3.5
-1.0	0	0.5
-0.5	1.5	-3.0
0	2.4	2
0.5	-4.0	-1.0

20) Use the table given in the previous problem. Evaluate $f\big(g(-1.0)\big)$.

a) -4.0
b) -1.0
c) 0
d) 0.5
e) Unable to determine

21) Given $x \neq -3$, the expression $\frac{2x^3 + 7x^2 - 3x - 25}{x+3}$ is equivalent to

a) $2x^2 + x - 6 - \frac{7}{x+3}$
b) $2x^2 + 13x - 36 + \frac{83}{x+3}$
c) $2x^2 + x - 13$
d) $x^2 + 4x - 15 + \frac{20}{x+3}$
e) None of these options

22) Consider the system of equations below:

$$x + 2y - z = 1$$
$$-x - 3y + 2z = 0$$
$$2x - 4y + z = 10$$

What is the (x, y, z) solution to the given system of equations?

a) $(1, 1, 2)$
b) $(5, -1, 2)$
c) $(3, -1, 0)$
d) $(3, 5, 8)$
e) No solution

23) What are the solutions to the equation $x(x + 2) = -2$?

a) $x = 0\,;\ x = -2$
b) $x = 0\,;\ x = -2i$
c) $x = -1 + \sqrt{3}\,;\ x = -1 - \sqrt{3}$
d) $x = -1 + \sqrt{3}i\,;\ x = -1 - \sqrt{3}i$
e) None of the above

24) The base of a triangle is 3 inches less than twice its own height. If the area of the triangle is 126 square inches, which of the following equations can be used to find h, the height of the triangle in inches?

a) $2h^2 - 3h + 63 = 0$
b) $2h^2 - 3h - 63 = 0$
c) $2h^2 - 3h + 252 = 0$
d) $2h^2 - 3h - 252 = 0$
e) Unable to determine

25) Jenny is solving the equation $x^2 - 8x = 9$ by completing the square. What number should be added to both sides of the equation to complete the square?

a) 2
b) 4
c) 8
d) 16
e) No value exists

Tie Breaker #1

Name: _____

School: _____

Given the following, where $y > 0$,

$$\left(\frac{y^{17/8}}{y^{5/4}}\right)^{-4} = y^n$$

Determine the value of n.

Tie Breaker #2

Name: _____

School: _____

According to a study done at a hospital, the average weight of a newborn baby is 3.4 kg, with a standard deviation of 0.6 kg. The weights of all the newborns in this hospital closely follow a normal distribution.

Last year, 9256 babies were born at this hospital. Determine, to the *nearest integer*, approximately how many babies weighed more than 4 kg.

Tie Breaker #3

Name: _____

School: _____

While experimenting with her calculator, Candy creates the sequence:

4, 9, 19, 39, 79, …

Write a recursive formula for Candy's sequence.

Determine the eighth term in Candy's sequence.

ANSWER KEY

1) A	11) B	21) A
2) D	12) C	22) C
3) D	13) A	23) E
4) A	14) B	24) D
5) A	15) D	25) D
6) C	16) B	
7) D	17) C	
8) A	18) B	
9) B	19) D	
10) A	20) A	

TB1:

Given $y > 0$,

$$\left(\frac{y^{17/8}}{y^{5/4}}\right)^{-4} = y^n$$

$$\frac{y^{(17/8)\cdot(-4)}}{y^{(5/4)\cdot(-4)}} = y^n$$

$$\frac{y^{-17/2}}{y^{-5}} = y^n$$

$$\frac{y^5}{y^{17/2}} = y^n$$

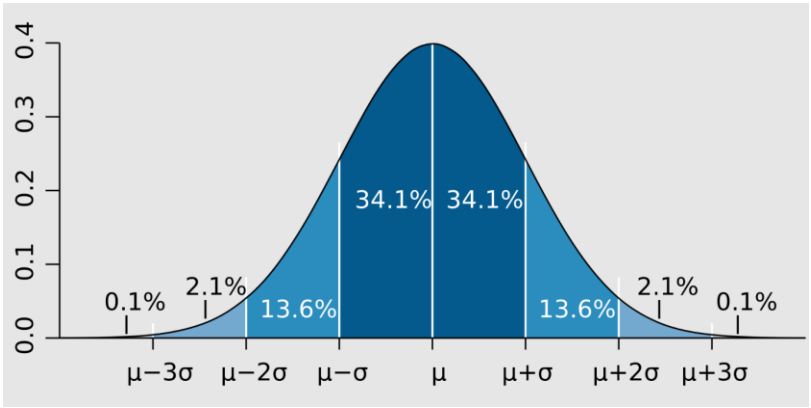
$$y^{5-17/2} = y^n$$

$$\frac{10}{2} - \frac{17}{2} = n$$

$$-\frac{7}{2} = n$$

TB2: This problem relies on the empirical rule (The 68–95–99.7 rule). Note that 4kg=3.4kg+0.6kg, which means we want the proportion of babies born that are above one standard deviation of the mean.

According to the empirical rule, 13.6+2.1+0.1%=15.8% of babies will be found above one standard deviation.



15.8% of 9256 babies = 1462.4 or 1462 babies. (These values may vary slightly due to rounding.)

TB3:

We can analyze the sequence and revise it into a consistent formula.

n	value	Revision
1	4	$a_1 = 4$
2	9	$a_2 = 9 = a_1 + 5$ $= a_1 + 5 \cdot 1$ $= a_1 + 5 \cdot 2^0$ $= a_1 + 5 \cdot 2^{1-1}$
3	19	$a_3 = 19 = a_2 + 10$ $= a_2 + 5 \cdot 2$ $= a_2 + 5 \cdot 2^1$ $= a_2 + 5 \cdot 2^{2-1}$
4	39	$a_4 = 39 = a_3 + 20$ $= a_3 + 5 \cdot 4$ $= a_3 + 5 \cdot 2^2$ $= a_3 + 5 \cdot 2^{3-1}$
5	79	$a_5 = 79 = a_4 + 40$ $= a_4 + 5 \cdot 8$ $= a_4 + 5 \cdot 2^3$ $= a_4 + 5 \cdot 2^{4-1}$

We can generalize the sequence into a recursive formula.

$$a_{n+1} = a_n + 5 \cdot 2^{n-1}$$

There are alternate forms, including:

$$f(n) = f(n-1) + 5 \cdot 2^{n-2}$$

Determine the eighth term in Candy's sequence.

We can

n	$f(n)$
1	4
2	9
3	19
4	39
5	79
6	159
7	319
8	639
9	1279

The eighth term is 639.