Arkansas Council of Teachers of Mathematics **Name** _____

2023 Regional Algebra I Competition

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tiebreaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tiebreaker pages when you are finished. You may keep the pages with the multiple-choice questions.

1. Line l and line k are perpendicular. Line l has a slope of 3. Line k contains the points (5,8) and $(2, y)$. What is the value of y?

   a. -1
   b. 7
   c. 9
   d. 17
   e. None of the above

2. Which expression is NOT equivalent to $32x + 28y$?

   a. $4(8x + 7y)$
   b. $4(5x + 7y + 3x)$
   c. $6(6x + 4y)$
   d. $2(16x + 14y)$
   e. None of the above

3. A phone company charges a base fee of \$12 per month plus an additional charge per minute. The monthly phone cost can be represented by this equation: $C = 12 + a \cdot m$, where a is the additional charge per minute, and m is the number of minutes used.

   Which equation can be used to find the number of minutes a customer used if we know a and C?

   a. $m = \frac{(C-12)}{a}$
   b. $m = (C - 12) - a$
   c. $m = C - 12a$
   d. $m = \frac{C}{a} - 12$
   e. None of the above

4. If $8y - 14 = -8$, find the value of $2y$.

   a. $\frac{3}{4}$
   b. $\frac{6}{8}$
   c. $\frac{4}{6}$
   d. $\frac{44}{8}$
   e. None of the above

5. Which of these points does not lie on the graph of $y = -2x + 7$?

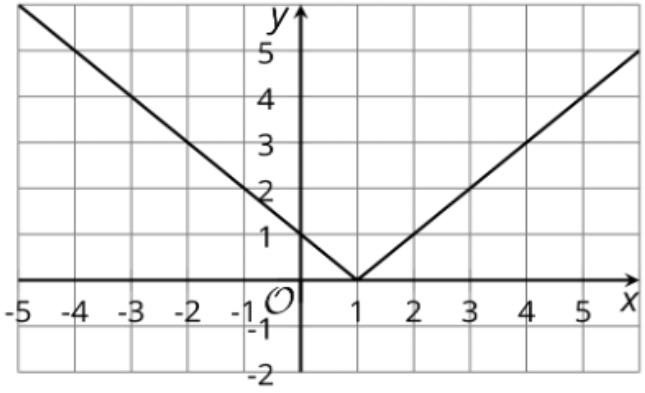
   a. $(-1,9)$
   b. $(4,-1)$
   c. $(-3,1)$
   d. $(-2,11)$
   e. None of the above

6. Consider the inequality $\frac{x+3}{6} > \frac{x}{4} + 1$. Which value of x satisfies the inequality?

   a. $x = -6$
   b. $x = 6$
   c. $x = 10$
   d. $x = -10$
   e. None of the above

7. On the right is a graph that represents a function. Which equation represents the function?

   a. $g(x) = |x - 1|$
   b. $(x) = |x| - 1$
   c. $h(x) = |x + 1|$
   d. $j(x) = |x| + 1$
   e. None of the above



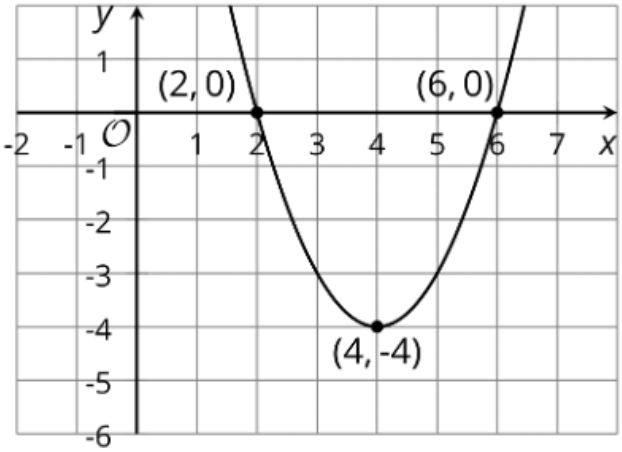
8. For which function does f decrease by 15% every time x increases by 1?

   a. $f(x) = 0.15^x$
   b. $f(x) = 0.85^x$
   c. $f(x) = 15^x$
   d. $f(x) = 85^x$
   e. None of the above

9. The function f is given by $f(x) = x^2 - 2x$. Which statement is true about the graph of f?

   a. The graph has a y-intercept of $(2, 0)$.
   b. The coordinates of the vertex are $(1, 1)$.
   c. The graph of the function opens downward.
   d. The x-intercepts are at $(0, 0)$ and $(0, 2)$.
   e. None of the above

10. On the right is the graph of a quadratic function f .
Select all equations that could define the function f.

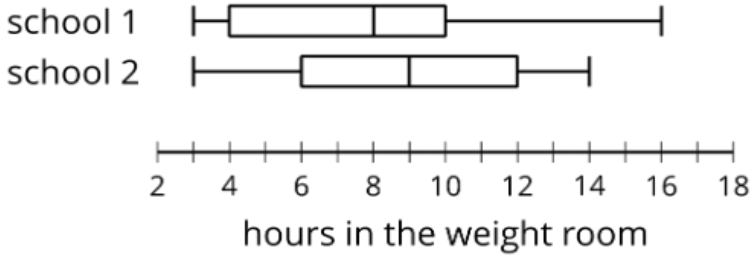


a. $f(x) = -x^2 + 8x - 12$
b. $f(x) = x^2 - 8x + 12$
c. $f(x) = (x + 2)(x + 6)$
d. $f(x) = (x - 4)^2 + 4$
e. None of the above

11. Select the equation that is equivalent to $x^2 + 6x = 16$.

a. $x^2 + 6x + 9 = 0$
b. $(x + 3)^2 = 16$
c. $x^2 + 6x - 9 = 25$
d. $(x + 3)^2 = 25$
e. None of the above

12. The two box plots summarize the number of hours spent in the weight room for all the players on the football team for two different high schools. Which of the statements must be true about the distribution of data represented in the boxplots?



a. Players at school 1 typically spent more time in the weight room than players at school 2.
b. The middle half of the data for school 1 has more variability than the middle half of the data for school 2.
c. The median hours spent in the weight room for school 1 is less than the median for school 2 and the interquartile ranges for both schools are equal.
d. The total number of hours spent in the weight room for players at school 2 is greater than the total number of hours for players at school 1.
e. None of the above

13. In a game show, players play multiple rounds to score points. Each round has 3 times as many points available as the previous round. Which equation shows the number of points available, p, in round n, of the game show?

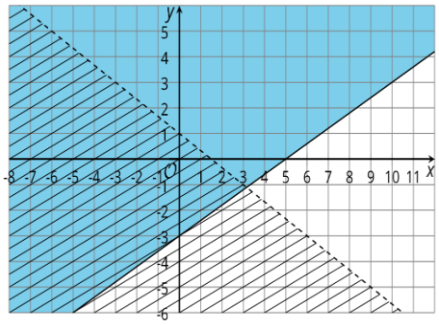
Round	**0**	**1**	**2**	**3**	**n**
Points	25	75	225	675	

a. $p = 3n$
b. $p = 3 \cdot 25^n$
c. $p = 25 + 50n$
d. $p = 25 \cdot 3^n$
e. None of the Above

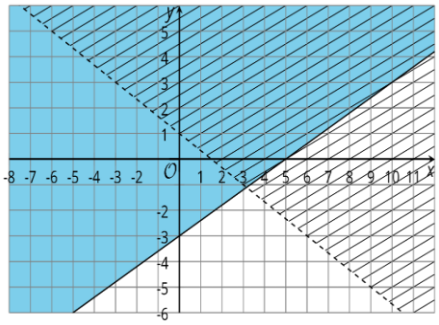
14. Which graph represents the solution to this system of inequalities?

$$\begin{cases} 3x - 5y \leq 15 \\ y > -\frac{2}{3}x + 1 \end{cases}$$

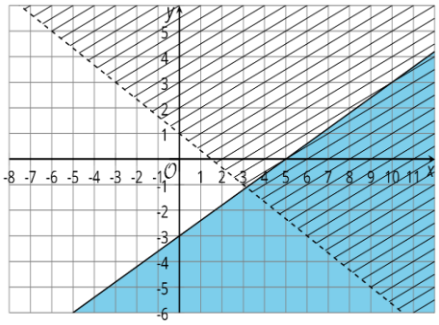
a.



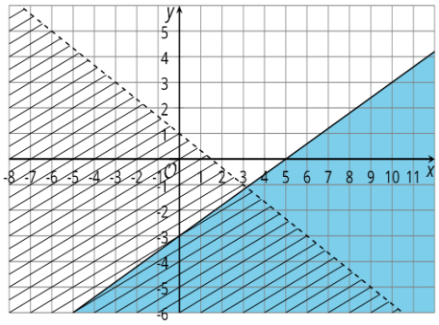
b.



c.



d.



e. None of the above

15. Which of the following is an equivalent expression to $(9x - 4) \cdot (3x - 2)$?

a. $27x^2 - 36x - 6x + 8$
b. $12x - 6$
c. $8 - 12x - 18x + 27x^2$
d. $27x^2 + 30x - 8$
e. None of the above

16. Daniel uploads 4 videos on his YouTube channel every month. Each video consistently gets 200,000 view per month. His likes-to-views ratio is 1:5. How many months will it take for Daniel to reach 12,800,000 views per month?

    a. 16 months
    b. 64 months
    c. 80 months
    d. 32 months
    e. None of the above

17. George knows that the volume of a cone is found by the formula $V = \frac{1}{3}\pi r^2$. How can this formula be adapted to find the radius in terms of volume?

    a. $r = 3\pi V$
    b. $r = \sqrt{3\pi V}$
    c. $r = \sqrt{(V - 1/3\pi)}$
    d. $r = V - 1/3\pi$
    e. None of the above

18. Mohamed decided to track the number of leaves on the tree in his backyard each year. The first year, there were 500 leaves. Each year thereafter, the number of leaves was 40 percent more than the year before. Identify the type of sequence this problem forms and its recursive formula.

    a. Arithmetic; $j(n) = j(n-1) + 1.4$
    b. Arithmetic; $h(n) = 500 + (n-1) \cdot 1.4$
    c. Geometric; $g(n) = 500 \cdot 1.4^{(n-1)}$
    d. Geometric; $f(n) = f(n-1) \cdot 1.4$
    e. None of the above

19. Find the line containing Isaac's first mistake when he worked the following problem:

Multiply $(\boldsymbol{x^2 + 17})(\boldsymbol{3x^3 - 6x + 21})$

Step #	**Work**
1	$[x^2 \cdot 3x^3 + x^2 \cdot (-6x) + x^2 \cdot 21] + [17(3x^3 - 6x + 21)]$
2	$[3x^6 - 6x^3 + 21x^2] + [51x^3 - 102x + 357]$
3	$3x^6 + (-6 + 51)x^3 + 21x^2 - 102x + 357$
4	$3x^6 + 45x^3 + 21x^2 - 102x + 357$

    a. Step 1
    b. Step 2
    c. Step 3
    d. Step 4
    e. There are no mistakes in this work

20. Function f is defined as $f(x) = x^2 + 3$. If the function is reflected across the y-axis, translated to the left three, and translated up five units to create function g, which of these is the equation of function g?

    a. $g(x) = (-x - 3)^2 - 8$
    b. $g(x) = (-x + 5)^2$
    c. $g(x) = -(x - 3)^2 + 8$
    d. $g(x) = -(x + 5)^2 - 3$
    e. None of the above

21. Which of these functions will have a non-real result given a value x in the domain $(0, \infty)$?

    a. $\sqrt{-4x^2}$
    b. $\sqrt{4x}$
    c. $(6x^2)^{1/3}$
    d. $(-6x)^{1/3}$
    e. None of the above

22. Sydney buys 4 pairs of pants and 1 jacket for \$97.11. Her friend Cassie buys 2 jackets and 3 pairs of pants for \$112.62. If all the pants cost the same price and all the jackets are set at the same price, how much would it cost for Maria to buy 3 jackets and 1 pair of pants?

    a. \$80.79
    b. \$111.81
    c. \$94.70
    d. \$79.44
    e. None of the above

23. Stacy's backyard has a width of $3x + 2$ and a length of $4x^2 + 32x + 1$. What is the area of Stacy's backyard in terms of x?

    a. $20x + 160x + 5\ units^2$
    b. $12x^3 + 88x^2 - 61x + 2\ units^2$
    c. $4x^2 + 35x + 3\ units^2$
    d. $12x^3 + 104x^2 + 67x + 2\ units^2$
    e. None of the above

24. Jonah went to the store to buy a new baseball for \$4.70. He paid the cashier with 35 coins which were all either nickels or quarters. Select the system of equations which could be used to find the number of nickels Jonah had.

a. $\begin{cases} n + q = 4.70 \\ 35n + 35q = 4.70 \end{cases}$

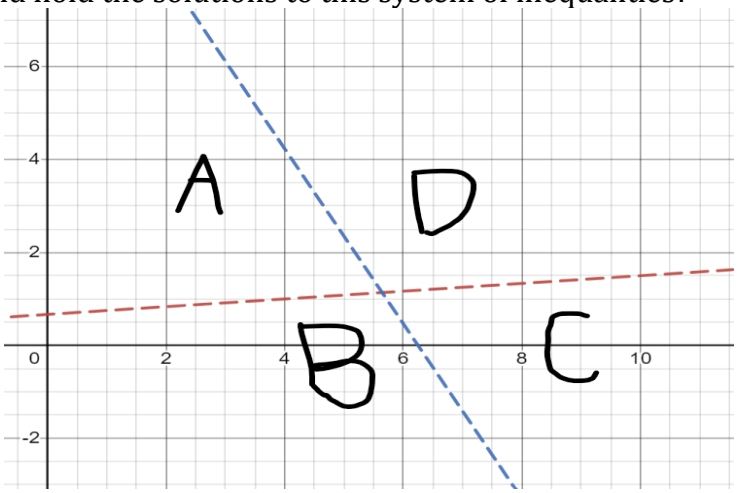
b. $\begin{cases} n + q = 35 \\ 5n + 25q = 4.70 \end{cases}$

c. $\begin{cases} n + q = 35 \\ 0.05n + 0.25q = 4.70 \end{cases}$

d. $\begin{cases} 0.05n + 0.25q = 35 \\ 0.05 + 0.25q = 4.70 \end{cases}$

e. None of the above

25. The system of inequalities $-x + 12y > 8$ *and* $160x + 85y < 1000$ is represented in this graph. Which region of the graph would hold the solutions to this system of inequalities?



a. A
b. B
c. C
d. D
e. None of the above

Tie Breaker #1

Name: _____

School: _____

Function c is defined by the equation $c(n) = 50 + 4n$. It gives the monthly cost, in dollars, of visiting a gym as a function of the number of visits, n.

a) In this situation, what information would the inverse function give us?

b) Find the inverse function.

c) Find the value of $c(7)$. Show your reasoning and explain what the value means in this situation.

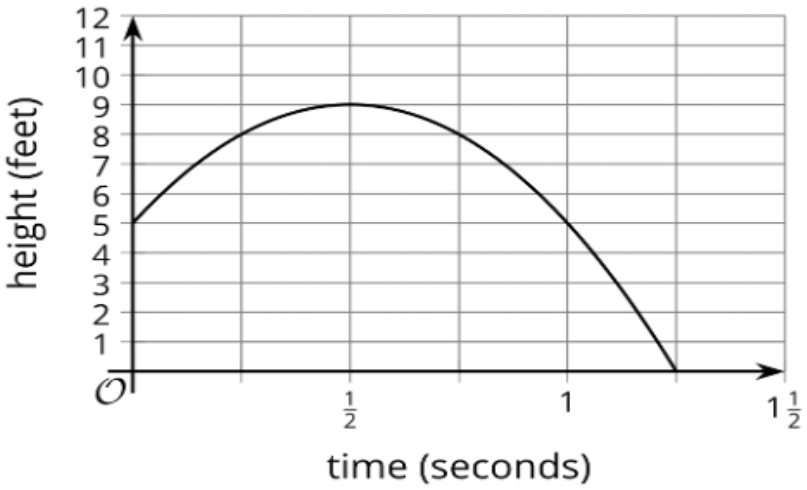
Tie Breaker #2

Name: _____

School: _____

Lin and Clare were standing in the woods near a stream, and each threw a stone up in the air at the same time so that it would land in the water. The graph represents the height of a stone t seconds after it was tossed up by Lin.

The function f given by $f(t) = (-16t - 8)(t - 1)$ represents the height of a stone t seconds after it was thrown by Clare. In both functions, height is measured in feet.



a) Whose stone hit the water first? When did that happen?

b) Clare's stone reached its maximum height $\frac{1}{4}$ second after it was tossed. Did Clare's stone reach a higher maximum than Lin's stone?

c) What was the maximum height of Clare's stone?

Tie Breaker #3

Name: _____

School: _____

Claire was exploring Pythagorean's theorem and noticed a pattern. If she picked any two integers, found the sum of their squares, the difference of their squares, and 2 times their product, she would have the three sides of a right triangle.

a) Does the pattern Claire found always work? Justify your answer.

b) If Claire's conjecture is true, write a concise statement of the conjecture, use variables to represent the integers, and prove it. If it is false, find a way to modify the conjecture to make it true, and prove the new statement.

c) Then use the conjecture to find the sides of a triangle where one side equals exactly 61 units.

Answer Key

1. C
2. C
3. A
4. E
5. C
6. D
7. A
8. B
9. E
10. B
11. D
12. C
13. D
14. B
15. C
16. A
17. E
18. D
19. B
20. E
21. A
22. B
23. D
24. C
25. A

Tie-Breaker Questions

1.a. It gives us the number of gym visits as a function of the cost.
1.b. $n = \frac{c-50}{4}$
1.c. $c(7) = 50 + 4(7) = 78$. This means that 7 visits to the gym cost \$78.

2.a. Clare's. It happened 1 second after the stone was tossed.
2.b. No. Both functions have the same maximum value.
2.c. 9 feet

3.a. False
3.b. $(m^2 + n^2), (m^2 - n^2), (2mn)\ form\ the\ sides\ of\ a\ right\ triangle, if\ m > n$ and m and n are positive integers. To prove this use the sides in Pythagorean theorem:

$$(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$$
$$m^4 + 2m^2n^2 + n^4 = (m^2 - n^2)^2 + (2mn)^2 \quad \text{Expand Left Side}$$
$$m^4 + 2m^2n^2 + n^4 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \quad \text{Expand Right Side}$$
$$m^4 + 2m^2n^2 + n^4 = m^4 + 2m^2n^2 + n^4$$

Therefore the statement is true by Pythagorean Theorem
3.c. $m = 6\ n = 5$, Sides of the triangle are 61, 11, 60