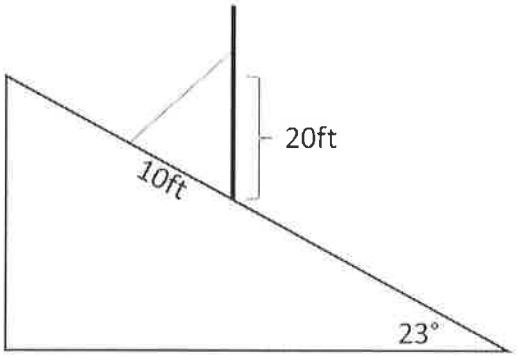
Arkansas Council of Teachers of Mathematics

2017 State Exam

Pre-Calculus

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tie breaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreakers.

1. A utility pole stands vertically on a hill that makes an angle of 23° with the horizontal. If a wire is attached to the pole 20 feet off the ground and is anchored in the ground on the uphill side of the pole 10 feet from the base of the pole, then how long is the wire?



   A. 18.4 feet
   B. 11.5 feet
   C. 7.8 feet
   D. 18.5 feet
   E. None of these

2. Find the polar form of the complex number $v = -2 - 2\sqrt{3}i$

   A. $v = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

   B. $v = 2\sqrt{10}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

   C. $v = 4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

   D. $v = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

   C. None of these

3. Which of the following polynomials has a root of $3 - 2i$?

   A. $x^2 - 6x + 13$
   B. $x^2 - 6x + 21 - 4(x-3)i$
   C. $x^2 - 13 + 4xi$
   D. All of these
   E. None of these

4. Find all solutions of $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$ over the interval from $-\pi$ to π.

A. $\pm\frac{\pi}{4}$

B. $\pm\frac{3\pi}{4}$

C. $-\frac{\pi}{6}$

D. $-\frac{5\pi}{6}$

E. All of these.

5. Find the area of the circle given by the equation $x^2 - 8x + y^2 + 6y - 11 = 0$

A. 11π

B. 50π

C. 36π

D. 25π

E. None of these

6. Which of the following vector operations on the vectors $\mathbf{u} = \langle 9,6 \rangle$ and $\mathbf{v} = \langle 8,12 \rangle$ will produce the vector $\langle 12,13 \rangle$?

A. $\frac{2}{3}\mathbf{u} + \frac{3}{4}\mathbf{v}$

B. $3\mathbf{v}$

C. $\frac{1}{2}\mathbf{v} + \mathbf{u}$

D. All of these

E. None of these

7. A function has a period of 4 and an amplitude of 6. The function is an even function. Which of the following functions could be described these characteristics?

A. $f(x) = 6\sin\left(\frac{\pi}{2}x\right)$

B. $f(x) = 6\cos(4x)$

C. $f(x) = 6\cos\left(\frac{\pi}{2}x\right)$

D. $f(x) = 4\sin(6x)$

E. None of these

8. Compute the magnitude of the vector $\mathbf{x} = \langle 1,2,-1 \rangle$

A. 2
B. 4
C. 6
D. $\sqrt{6}$
E. None of these

9. Find the modulus of a zero of the function $y = x^2 - 4x + 13$.

A. $\sqrt{13}$
B. $\frac{\sqrt{13}}{2}$
C. $\sqrt{5}$
D. $2\sqrt{10}$
E. None of these.

10. Given: the $\sin\theta = -\frac{2}{5}$ and the $\cos\theta$ is positive. Find the $\tan(\pi + \theta)$

A. $\frac{5}{2}$
B. $-\frac{2\sqrt{21}}{21}$
C. $-\frac{\sqrt{21}}{2}$
D. 9.01
E. None of these

11. Given the complex number $z = \sqrt{12}\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right]$, write its conjugate in standard form ($a + bi$).

A. $-\sqrt{3} - 3i$
B. $\sqrt{3} - 3i$
C. $3 - \sqrt{3}i$
D. $-3 - \sqrt{3}i$
E. None of these

12. A woman looks out the window of her apartment. Across the street there is an office building. Using a laser range finder, she determines the distance to the base of the office building to be 42.5 ft. She then finds the distance to the roof of the office building to be 61.8 feet. She calculates the angle of depression to the base of the office building to be 32° and the angle of elevation to the roof of the office building to be 40°. What is the height of the office building?

A. 75 feet
B. 85.1 feet
C. 63.26 feet
D. 104.3 feet
E. None of these

13. Consider an eye bracket in which two ropes are fastened to the bracket at the same point. If one rope has a force of $F_1 = \langle 1,2 \rangle$ and another rope has a force of $F_2 = \langle 3,1 \rangle$, then what is the magnitude of the resultant vector?

A. $\sqrt{10}$
B. 10
C. 7
D. 4
E. None of these

14. Given the function $y = x^2 - 10x + 16$, find an interval on which an inverse can be defined for y.

A. $(0,\infty)$
B. $(-\infty,\infty)$
C. $(-1,1)$
D. $(2,8)$
E. None of these

15. Determine the area of the parallelogram whose vertices are located at (– 1, – 1), (0, 2), (1, 0), and (2, 3).

A. 5

B. $\frac{\sqrt{10}}{2}$

C. $5\sqrt{10}$

D. $\sqrt{2}$

E. None of these

16. Which of the following functions has an inverse defined on all real numbers?

A. $f(x) = 3x^2 - 1$

B. $f(x) = x^3 + 1$

C. $f(x) = \cos x$

D. All of these

E. None of these

17. Which of the following functions does not have an inverse defined on the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$

A. $y = \tan x$

B. $y = x^3$

C. $y = \sin x$

D. $y = \arccos x$

E. None of these

18. Find the asymptotes of the hyperbola given by $3x^2 - 4y^2 - 16y - 28 = 0$

A. $y = x + 2,\ \ y = -x + 2$

B. $y = \frac{2\sqrt{3}}{3}x - 2,\ \ y = -\frac{2\sqrt{3}}{3}x - 2$

C. $y = x - 2,\ \ y = -x - 2$

D. $y = \frac{\sqrt{3}}{4}x - 2,\ \ y = -\frac{\sqrt{3}}{4}x - 2$

E. None of these

19. Find the focus or foci of the following conic section: $7x^2 - 56x + 4y^2 + 16y + 100 = 0$

A. $(4,-2)$

B. $(4,-2-\sqrt{7})$ and $(4,-2+\sqrt{7})$

C. $(4,-2-\sqrt{3})$ and $(4,-2+\sqrt{3})$

D. $(2,-2)$ and $(6,-2)$

E. None of these

20. Find the magnitude of the resultant vector of **u**+2**v** when **u** = $\langle 1,-2 \rangle$ and **v** = $\langle 2,3 \rangle$

A. $\sqrt{41}$

B. $\sqrt{26}$

C. $\sqrt{13}$

D. $\sqrt{73}$

E. None of these

21. Find all solutions to the equation $2\sin^2(4x) - 1 = 0$

A. $x = \dfrac{\pi(2n-1)}{4}$

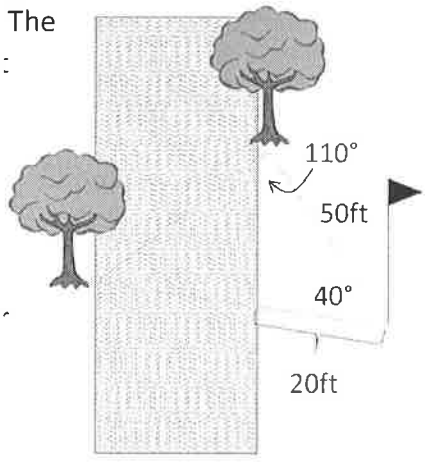
B. $x = \dfrac{\pi(2n-1)}{16}$

C. $x = \dfrac{n\pi}{4}$

D. Both A and B

E. None of these

22. A tree and a flagpole both stand on the east bank of a river. The distance between them is 50ft. Another tree stands on the west bank. When standing at the tree on the east bank, the angle between the line of sight to the flagpole and the other tree is 110°. When standing at the flagpole the angle between the line of sight to each tree is 40°. When measuring the distance from the flagpole to the river, using a line between the flagpole and the tree on the west bank, the distance to the river is 20 ft. How wide is the river (following this same path)?



A. 94 ft

B. 146 ft

C. 126 ft

D. 74 ft

E. None of these

23. Simplify the following expression: $\sin(\theta)(\csc(\theta)-\sin(\theta))$

A. $\cos^2(\theta)$

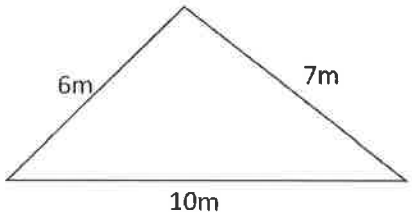
B. $\tan(\theta)-\sin^2(\theta)$

C. $\tan(\theta)-\sin(\theta)$

D. 1

E. None of these

24. Find the area of the given triangle:



A. $21m^2$

B. $27.5m^2$

C. $20.66m^2$

D. $23m^2$

E. None of these

25. Find an equation for the parabola that contains the points (– 6, 5), (6, 2) and (2, 1) and has an axis parallel to the y-axis.

A. $16(y-1)=(x-2)^2$

B. $y=\frac{1}{16}x^2-\frac{1}{4}x+\frac{5}{4}$

C. $16y=x^2-4x+20$

D. All of these

E. None of these

Tie Breaker 1

Name:_____

The foci of an ellipse are at (0,0) and (0,8). The major axis has a length of 16 units. Find the following:

a) The center of the ellipse is at:

b) The length of the minor axis:

c) The equation of the ellipse is:

Tie Breaker 2

Name:_____

Find $f(11)$ for the recursive sequence $f(x)=3\cdot f(x-3)+6$ where $f(2)=8$.

Tie Breaker 3

Name:_____

Write an equation in base α (for $\alpha > 0$) that is equivalent to the equation $y = 3^t$.

Answer Key:

1. D
2. C
3. D
4. E
5. C
6. A
7. C
8. D
9. A
10. B
11. A
12. C
13. E
14. C
15. A
16. B
17. D
18. B
19. C
20. A
21. B
22. D
23. A
24. C
25. D

Tie Breaker 1:

Solution:

a) The center is at (0,4). It is found by finding the midpoint between (0,0) and (0,8). This gives $\left(\frac{0+0}{2},\frac{0+8}{2}\right)=\left(\frac{0}{2},\frac{8}{2}\right)=(0,4)$

b) The distance from the center to the foci is 4 units, which is c. The length of the major axis is 16, so a=8. $a^2-b^2=c^2 \Rightarrow 64-b^2=16 \Rightarrow b^2=48$

c) These, along with the center can be put into the equation of an ellipse to get:

$$\frac{(x-0)^2}{48}+\frac{(y-4)^2}{64}=1$$

Tie Breaker 2:

Find $f(11)$ for the recursive sequence $f(x)=3\cdot f(x-3)+6$ where $f(2)=8$.

Solution:

$f(11)=3\cdot f(11-3)+6=3\cdot f(8)+6$

Now looking at $f(8)$:

$f(8)=3\cdot f(8-3)+6=3\cdot f(5)+6$

Now looking at $f(5)$:

$f(5)=3\cdot f(5-3)+6=3\cdot f(2)+6$

Since we know $f(2)=8$, we can substitute to get:

$f(5)=3\cdot f(2)+6=3\cdot 8+6=30$

Since we know $f(5)=30$, we can substitute to get:

$f(8)=3\cdot f(5)+6=3\cdot 30+6=96$

Since we know $f(8)=96$, we can substitute to get:

$f(11)=3\cdot f(8)+6=3\cdot 96+6=294$

Tie Breaker 3:

Write an exponential equation in base α (for $\alpha > 0$) that is equivalent to the equation $y = 3^t$.

Solution:

Create a general exponential equation. $f(t) = \phi \cdot \alpha^{bt}$

We know that when *t*=0 that y=1. That gives $1 = \phi \cdot \alpha^{b \cdot 0}$ or $1 = \phi \cdot 1$ and therefore, $\phi = 1$.

Now $f(t) = \alpha^{bt}$.
In the original function, when *t* = 1, y = 3. Substituting those values in we get:

$3 = \alpha^b$ which can be solved for b by: $\ln 3 = b \ln \alpha$

This gives us that $b = \frac{\ln 3}{\ln \alpha}$.

Substituting back into $f(t) = \alpha^{bt}$ gives:

$$f(t) = \alpha^{\left(\frac{\ln 3}{\ln \alpha} t\right)}$$