Arkansas Council of Teachers of Mathematics 2020
Precalculus & Trigonometry Regional Exam

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tiebreaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tiebreaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1. If $f(2x-1) = 4x^2 - 10x + 16$, then $f(x) =$
   A. $x^2 + 3x + 18$
   B. $x^2 - 3x + 18$
   C. $x^2 - 6x + 12$
   D. $x^2 - 7x + 18$
   E. $x^2 - 3x + 12$

2. If $\sec\alpha = -3$ and α is in the second quadrant of the xy-plane, then $\cos\frac{\alpha}{2} =$

   A. $-\frac{\sqrt{3}}{3}$
   B. $\frac{\sqrt{3}}{3}$
   C. 0
   D. 1
   E. 2

3. The number intersection points of the graphs $y = |x|$ and $y = |x^2 - 4|$ is:
   A. 0
   B. 1
   C. 2
   D. 3
   E. 4

4. If $2^{2\log_2 x} = \left(2^{(\log_2 x)+1}\right) - 1$, find x.

   A. $x = 2$
   B. $x = 3$
   C. $x = 1$
   D. $x = -1$
   E. $x = 4$

5. Solve the equation $\sin 2x + \cos 2x + \sin x + \cos x + 1 = 0$ for $0 \leq x \leq 2\pi$.

   A. $x = {}^{2\pi}/_{3}\,, {}^{4\pi}/_{3}\,, {}^{3\pi}/_{4}\,, {}^{7\pi}/_{4}$
   B. $x = {}^{\pi}/_{3}\,, {}^{2\pi}/_{3}\,, {}^{3\pi}/_{4}\,, {}^{5\pi}/_{4}$
   C. $x = \pi\,, {}^{4\pi}/_{3}\,, {}^{3\pi}/_{2}\,, {}^{7\pi}/_{4}$
   D. $x = {}^{\pi}/_{4}\,, {}^{2\pi}/_{3}\,, {}^{3\pi}/_{4}\,, {}^{5\pi}/_{4}$
   E. $x = {}^{\pi}/_{6}\,, {}^{4\pi}/_{3}\,, {}^{5\pi}/_{4}\,, {}^{7\pi}/_{4}$

6. When $f(x) = x^3 - nx^2 - 10nx + 25$ is divided by $x - 2$ the remainder is 9. The value of n is

   A. $n = 25$
   B. $n = -\frac{1}{2}$
   C. $n = \frac{7}{4}$
   D. $n = 1$
   E. $n = -\frac{13}{8}$

7. The expression $\cos 3\theta$ is equivalent to

   A. $3\cos^3\theta - 4\cos\theta$
   B. $3\cos^3\theta + 4\cos\theta$
   C. $4\cos^3\theta - 3\cos\theta$
   D. $4\cos^3\theta + 3\cos\theta$
   E. $4\cos^2\theta - 3\cos\theta$

8. If $\log_{(a^2)} b + \log_{(b^2)} a = \frac{5}{2}$ with $a, b > 0$ then the number of values of b for a given value of a is

   A. 1
   B. 2
   C. 3
   D. 4
   E. 5

9. If a planar figure has a configuration of SSA (Side, Side, Angle) where side $a = 30$, side $b = 40$, and angle $A = 20°$ then this figure produces

   A. No triangle
   B. One right triangle
   C. One oblique triangle
   D. Two triangles
   E. Cannot determine from the given information

10. A shop has 600 shirts for sale. Of these 100 are red, 200 are blue, and the remainder are yellow. Not all the shirts have collars and/or pockets: 400 have collars and 150 have pockets. What is the maximum number of blue shirts with pockets and without collars?

    A. 50
    B. 100
    C. 150
    D. 200
    E. None of these

11. The equation $x^2 + 9y^2 + 16x - 54y + 136 = 0$ represents what type of conic section?

    A. Circle
    B. Parabola
    C. Ellipse
    D. Hyperbola
    E. None of these

12. Solve for x in the given logarithmic equation.

$$2\log(x) = \log 4 + \log(x + 3)$$

    A. $x = -1$
    B. $x = 0$
    C. $x = 2$
    D. $x = 3$
    E. $x = 6$

13. The angle coterminal to $\alpha = \frac{64\pi}{3}$ is

    A. π
    B. $\frac{4\pi}{3}$
    C. $\frac{\pi}{6}$
    D. $-\frac{\pi}{2}$
    E. 2π

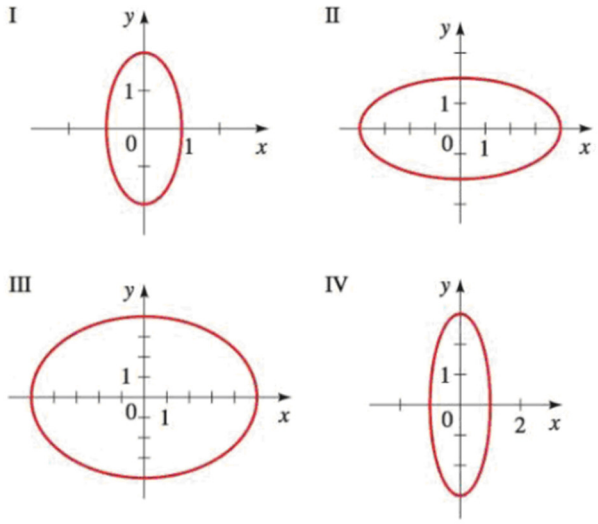
14. Match the following trigonometric function with its equivalently simplified version.

$$\sin x + \cot x \cos x$$

    A. $\cos x$
    B. $\sin x$
    C. $\sec x$
    D. $\csc x$
    E. $\tan x$

15. Which of the following graphs correctly shows the ellipse given in the equation?

$$4x^2 + y^2 = 4$$



A. Graph I
B. Graph II
C. Graph III
D. Graph IV
E. None of these

16. Find a number less than 3000 which when divided by 1 leaves a remainder 0, when divided by 2 leaves a remainder 1, when divided by 3 leaves a remainder 2, … , when divided by 9 leaves a remainder of 8, and when divided by 10 leaves a remainder 9.

A. 1757
B. 2123
C. 2377
D. 477
E. 2519

17. What is the exact value of cos 36°?

A. $\frac{2+\sqrt{5}}{8}$
B. $\frac{1+\sqrt{2}}{5}$
C. $\frac{5+\sqrt{2}}{10}$
D. $\frac{1+\sqrt{5}}{4}$
E. $\frac{\sqrt{5}}{3}$

18. Given that k is an integer, which of the following describes the full set of solutions of θ for the trigonometric property $\cos\theta = \frac{1}{2}$?

    A. $\frac{\pi}{3} + k\pi$
    B. $\frac{\pi}{6} + k\pi$
    C. $\frac{2\pi}{3} + 2k\pi$ and $\frac{4\pi}{3} + 2k\pi$
    D. $\frac{5\pi}{3} + 2k\pi$
    E. $\frac{\pi}{3} + 2k\pi$ and $\frac{5\pi}{3} + 2k\pi$

19. Evaluate the product: $(i-4)^2(i+4)^2$

    A. 169
    B. 289
    C. $128i$
    D. $-16+16i$
    E. $-16-32i$

20. Determine which of the following aspects general graph aspects does not exist in a particular function, given that the function is $f(x) = 2^{x+2} - 1$.

    A. Horizontal asymptote
    B. Vertical asymptote
    C. y-intercept
    D. x-intercept
    E. None of these are absent in this function

21. Which of the following coordinates is the vertex of the given parabola?

$$2y = x^2 + 4x + 4$$

    A. $(2, 4)$
    B. $(2, 0)$
    C. $(0, -2)$
    D. $(-2, 0)$
    E. $(-2, 4)$

22. For the given function, determine the horizontal asymptote.

$$f(x) = \frac{2x+1}{x-3}$$

A. $y = -3$
B. $y = -\frac{1}{3}$
C. $y = 0$
D. $y = 1$
E. $y = 2$

23. A $20ft$ tall extension ladder leaning against a house makes a $70°$ angle with respect to the ground. How far up the house does the ladder reach?

A. $6.8\ ft$
B. $12.7\ ft$
C. $15.5\ ft$
D. $18.8\ ft$
E. $54.9\ ft$

24. Solve for all possible values of t in the following trigonometric equation on $[0, 2\pi]$.

$$2\cos^2 t - \sqrt{3}\cos t = 0$$

A. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{11\pi}{6}$
B. $\frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}$
C. $0, \pi, 2\pi$
D. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{6}$
E. $\frac{\pi}{6}, \frac{11\pi}{6}$

25. If $xy = 6$, $yz = 9$, and $zx = 24$ then the value of $xyz =$

A. 648
B. 1296
C. 48
D. 1.5
E. 36

Tie Breaker #1

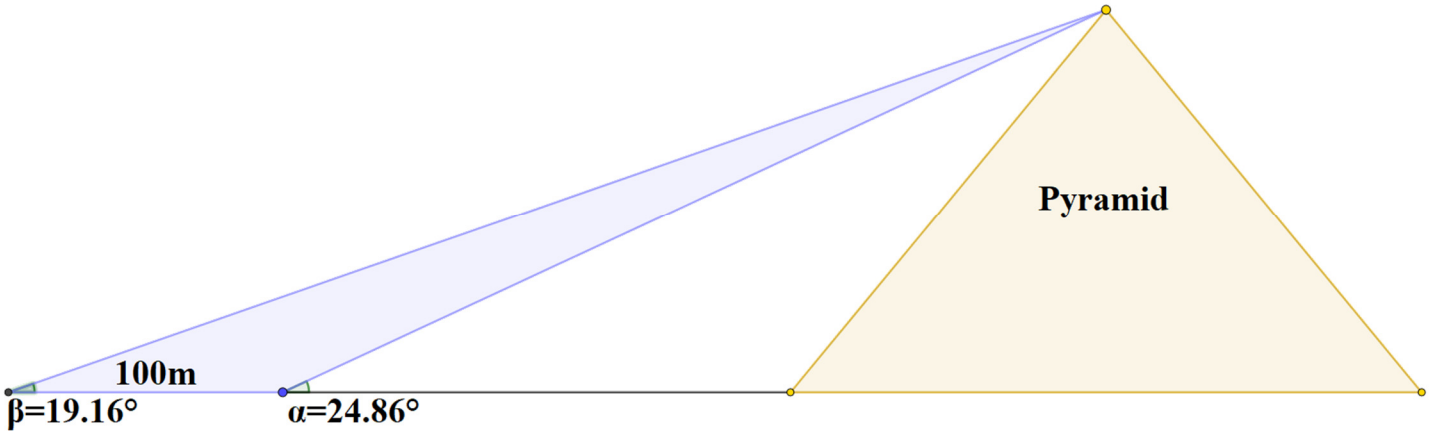
Name: _____

School: _____

The Great Pyramid of Giza is one of the original Seven Wonders of the World. Its original height was 147 meters—which made it the tallest building in the world for nearly four millennia. The height has decreased over the centuries as the outer layers of stone have eroded.

In order to find the current height of the Pyramid, an engineer with an angle finder stood some distance away from the Pyramid and found the angle to the current height of the Pyramid. The angle was $\alpha = 24.86°$. She then stepped backwards by 100m and repeated her angle measurement, finding an angle of $\beta = 19.16°$.

Find the height of the current top of the Pyramid.



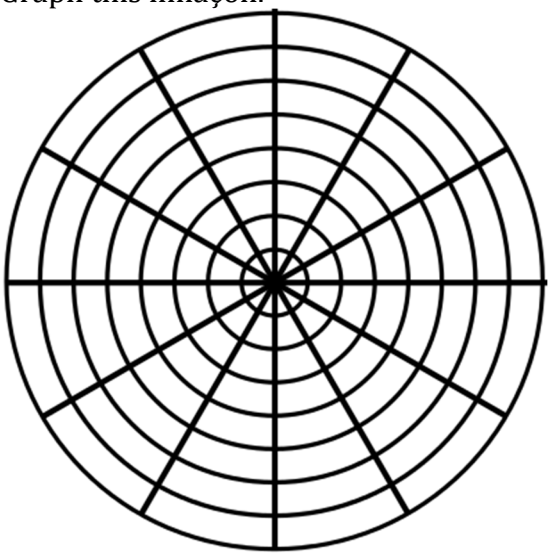
Tie Breaker #2

Name: _____

School: _____

Consider the limaçon with a polar equation of $r = 3 - 4\cos(\theta)$.

A. Graph this limaçon.



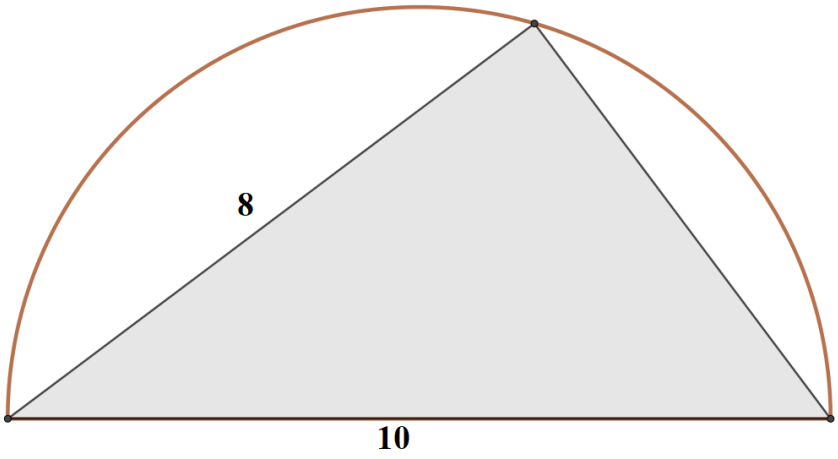
B. This limaçon contains more than one loop. Find the distance between the origin and the outer edge of each loop.

Tie Breaker #3

Name: _____

School: _____

Find the area inside a semicircle with diameter of 10cm, but outside the circumscribed triangle. One edge of the circumscribed triangle is along the diameter of the semicircle. A second edge has a of length 8cm.

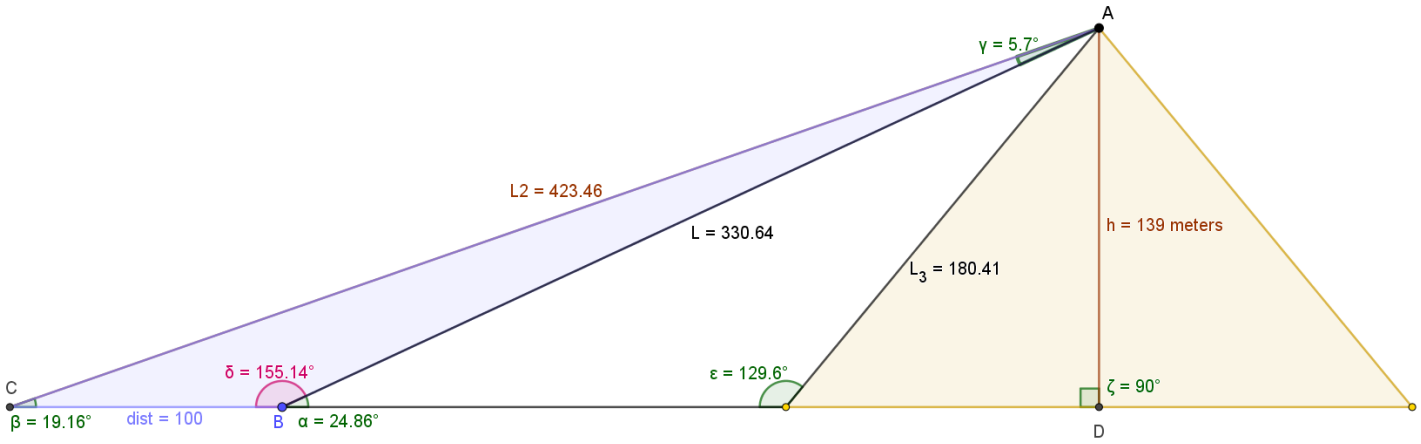


2020 ACTM Regional Precalculus Exam Answer Key

1.	E
2.	B
3.	E
4.	C
5.	A
6.	D
7.	C
8.	B
9.	D
10.	C
11.	C
12.	E
13.	B
14.	D
15.	A
16.	E
17.	D
18.	E
19.	B
20.	B
21.	D
22.	E
23.	D
24.	A
25.	E

Tie-Breaker #1 Solution

An engineer with an angle finder stood some distance away from the Pyramid and found the angle to the current height of the Pyramid. The angle was $\alpha = 24.86°$. She then stepped backwards by 100m and repeated her angle measurement, finding an angle of $\beta = 19.16°$. Find the height of the current top of the Pyramid.



The geometric construction used in this problem with many extra angles and lengths labeled is shown above.

There are many different approaches to this problem. One approach uses the law of sines.

Find angle $\delta = 155.14°$. It is supplementary with angle α.

Angles $\beta + \delta + \gamma = 180°$. Thus $\gamma = 5.70°$.

Use Law of Sines to solve for side L.

$$\frac{sin(5.7°)}{100} = \frac{sin(19.16°)}{L} \quad \rightarrow \quad L = 100 \cdot \frac{sin(19.16°)}{sin(5.7°)} \approx 330.5\ meters \text{ (or so)}$$

Now we can consider ΔABD. We want to solve for side h.

$$sin(24.86°) = \frac{h}{L} = \frac{h}{330.5} \quad \rightarrow \quad h = 138.9\ meters \text{ (or so)}$$

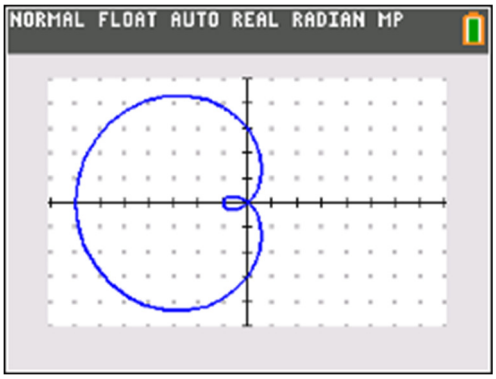
The current height of the pyramid is about 139 meters tall. This is reasonably close to the expected value from the geometric construction.

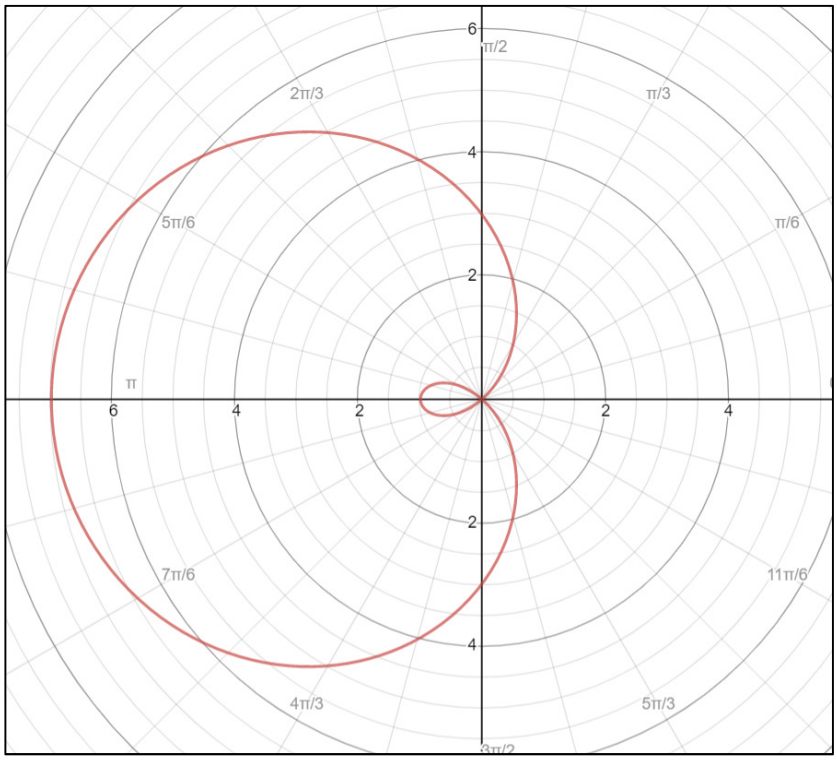
Keep in mind that rounding may cause values to vary somewhat.

Tie-Breaker #2 Solution

Consider the limaçon with a polar equation of $r = 3 - 4\cos(\theta)$.

A. Graph this limaçon.





B. This limaçon contains more than one loop. Find the distance between the origin and the outer edge of each loop.

There are two loops in this limaçon. They correspond to the extreme values of the cosine function.

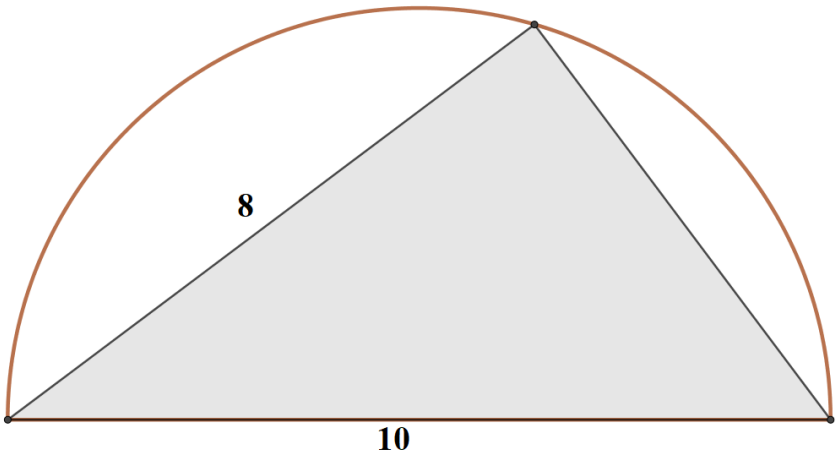
$$\mathbf{3 - 4(1) \le 3 - 4cos(\theta) \le 3 - 4(-1)}$$

$$\mathbf{-1 \le 3 - 4cos(\theta) < 7}$$

The inner loop has a maximum distance of 1 unit from the origin.
The outer loop has a maximum distance of 7 units from the origin.

Tie-Breaker #3 Solution

Find the area inside a semicircle with diameter of 10cm, but outside the circumscribed triangle. One edge of the circumscribed triangle is along the diameter of the semicircle. A second edge has a of length 8cm.



Find the area of the semicircle:

$$A = \frac{\pi r^2}{2} = \pi \cdot \frac{5^2}{2} = \frac{25\pi}{2}$$

Now find the area of the Triangle. We know that the inscribed triangle where the hypotenuse is a diameter will be a right triangle from Geometry. We can use the regular area of a triangle formula. There is one missing base of the triangle.

$$10^2 = 8^2 + b^2 \quad \rightarrow \quad b = 6$$

$$A = \frac{1}{2}bh = \frac{1}{2}8 \cdot 6 = 24$$

Subtract to find the indicated area

$$\Delta A = \frac{25\pi}{2} - 24 \approx 15.27 cm^3$$