Arkansas Council of Teachers of Mathematics **Name _____**

2022 Pre-Calculus/Trigonometry Regional Competition

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet, your tie-breaker pages, and your scratch work when you are finished. Figures are not necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1. If $f(x)$ is a polynomial having only real coefficients and $1-4i$ is a zero of $f(x)$, then $-1+4i$ is also a zero.
   A. True
   B. False
   C. Sometimes True, Sometimes False.
   D. Not enough information to determine.

2. Find the slant asymptote, if any, of the graph of $f(x)=\frac{x^2-8x+9}{x+4}$.
   A. $y=x+17$
   B. $y=x-12$
   C. $x=y+8$
   D. $x=y+4$
   E. No slant asymptote

3. If $f(x)=x^2-36$ and $g(x)=x^2$, find all zeros of $f\big(g(x)\big)$.
   A. $\sqrt{6}, -\sqrt{6}, \sqrt{6}i, -\sqrt{6}i$
   B. $\sqrt{6}$ (multiplicity 2), $-\sqrt{6}$ (multiplicity 2)
   C. $6, -6, 0$ (multiplicity 2)
   D. 6 (multiplicity 2), -6(multiplicity 2)
   E. None of the above.

4. The rectangular coordinates of a point are $\left(5\sqrt{3}, 5\right)$. Find polar coordinates of the point using radians.
   A. $\left(5, \frac{\pi}{6}\right)$
   B. $\left(10, \frac{\pi}{6}\right)$
   C. $\left(5, \frac{\pi}{3}\right)$
   D. $\left(10, \frac{\pi}{3}\right)$
   E. None of the above.

5. Two tugboats pull a barge. Tugboat A pulls the barge in a due east direction with a force of 67 Newtons. Tugboat B pulls due north with a force of 54 Newtons. Find the magnitude of the resultant force on the barge.
   A. 13 Newtons
   B. 86 Newtons
   C. 121 Newtons
   D. 3618 Newtons
   E. None of the above.

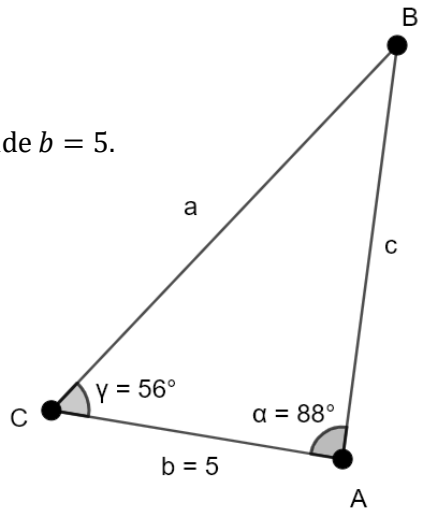
6. A merry-go-round takes 29 seconds to make one complete revolution. It has a radius of 18 feet. What is the linear speed of the merry-go-round, in feet per second, at a point on its outside edge?
   A. 3.9 feet per second
   B. 113.1 feet per second
   C. 0.62 feet per second
   D. 10.12 feet per second
   E. None of the above.

7. Seismographs are devices that can detect earthquakes. Three seismographs are located at locations named A, B, and C. These locations are mapped to a coordinate plane at the points $(0, 9)$, $(-12, 17)$, and $(-8, -7)$, respectively. Suppose the epicenter an earthquake is determined to be 5 units from A, 15 units from B, and 13 units from C. What is the location of the epicenter?
   A. $(-2, 4)$
   B. $(-4, 7)$
   C. $(-3, 5)$
   D. $(-2, 5)$
   E. None of the above.

8. Spacely Space Sprockets is a large business that manufactures various models of sprockets. Its Data Analytics department has developed a mathematical model describing the sales of one sprocket model, $S(x)$, (in thousands of dollars) as a function of the number of months the sprocket has been on the market, x. The sales model is $S(x) = 3000 - 15000e^{-x/9}$. After how many months will the sales reach \$2,300,000?
   A. 18 months
   B. 28 months
   C. 21 months
   D. 25 months
   E. None of the above.

9. Will was sitting in a treehouse. He looked down and saw a very rare ivory-billed woodpecker on the ground. If the angle of depression from Will's line of sight is $41°24'$, and the woodpecker is 20 feet from the base of the treehouse, how tall is the treehouse?
   A. 24 ft
   B. 18 ft
   C. 20 ft
   D. 22 ft
   E. None of the above.

10. Consider the figure to the right. The measure of angle $\alpha = 88°$, $\gamma = 56°$, and side $b = 5$. Find the length of side a. Round to one decimal place.
    A. 0.3 units
    B. 2.9 units
    C. 6.0 units
    D. 8.5 units
    E. Not enough information given.



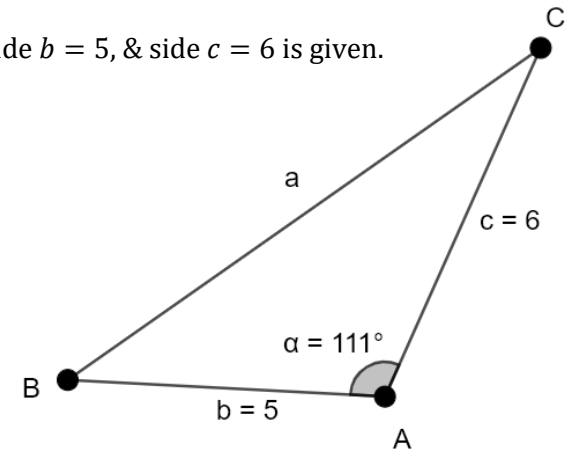
11. You want to make an open-topped box using a piece of cardboard. The cardboard starts as a rectangle measuring 22 inches by 30 inches. Its four corners are removed, with each corner forming the shape of a square. The remaining sides are folded upward to form a box. Let x represent the length of a side of each removed corner square. For what value of x will the volume be a maximum?
    A. 13.2 in.
    B. 26.3 in.
    C. 16.2 in.
    D. 4.2 in.
    E. None of the above.

12. Given two coordinates, $(2, y)$ and $(-10, 3)$. Find all values of y so that the distance between these coordinates is 13.
    A. $-2, 8$
    B. $-8, -2$
    C. $2, 8$
    D. $-8, 2$
    E. None of the above.

13. Find the rectangular coordinate equivalent to the polar coordinate $(-5, -120°)$.
    A. $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$
    B. $\left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$
    C. $\left(\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$
    D. $\left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$
    E. None of the above.

14. Find the zeros of $f(x) = 2x^3 + 4x^2 - 126x$ and state the multiplicity of each.
    A. -9 (multiplicity 1), 7 (multiplicity 2)
    B. -9 (multiplicity 1), 9 (multiplicity 1), 7 (multiplicity 1)
    C. -9 (multiplicity 1), 0 (multiplicity 1), 7 (multiplicity 1)
    D. -9 (multiplicity 1), 7 (multiplicity 1)
    E. None of the above.

15. Consider the figure to the right. The measure of angle $\alpha = 111°$, side $b = 5$, & side $c = 6$ is given. Find the length of side a. Round to one decimal place.
    A. 2.2 units
    B. 7.8 units
    C. 9.1 units
    D. 9.5 units
    E. Not enough information given.



16. A circle has a diameter with endpoints $(-2, 1)$ and $(10, 13)$. Find the coordinates of the center.
    A. $(22, 25)$
    B. $(4, 7)$
    C. $(8, 14)$
    D. $(-8, -5)$
    E. Unable to determine.

17. The time, in hours, for an artificial satellite to complete a single orbit around Earth varies directly as the radius of the orbit (measured from the center of Earth) and inversely as the orbital velocity. If a satellite completes an orbit 880 miles above the surface of Earth in 19 hours at a velocity of 39,000 mph, how long would it take a satellite to complete an orbit if it is at 1300 miles above the earth at a velocity of 39,000 mph? (Use 3960 miles as the radius of the Earth.)
    A. 20.6 hours
    B. 206.5 hours
    C. 5.1 hours
    D. 28.1 hours
    E. None of the above.

18. Suppose you attached a mass to a spring and oriented it perpendicularly to a floor. You then pulled the mass down 2 inches below its baseline position. Determine a trigonometric model that gives the position of the mass at t seconds. Assume the frequency of the system is $\pi/3$ cycles per second. (Ignore all friction.)
    A. $y = 2\cos(6t)$
    B. $y = 2\pi\cos(3t)$
    C. $y = -2\cos(6t)$
    D. $y = -2\cos(3t\pi)$
    E. None of the above.

19. Greg plans to buy a "new" car in 5 years. He expects this car will cost \$15,000. How much should he deposit in a savings account now at 6%, compounded quarterly, so that he will have enough to buy this car in the future?
    A. \$10,574.41
    B. \$12,939.13
    C. \$11,881.40
    D. \$11,137.06
    E. None of the above.

20. Cogswell's Cogs is another large business with a Data Analytics division. They have analyzed years of sales data for their cog products and found seasonal variations. The total sales, in dollars, fluctuates according to the model $S = A + B\sin\left(\frac{\pi x}{6}\right)$, where x is the time in months with $x = 1$ corresponding to January. Their research has found $A = 8500$ & $B = 4200$. Determine the month with the greatest total sales and the sales in that month.
    A. June; \$8500
    B. March; \$12,700
    C. December; \$12,700
    D. September; \$4300
    E. None of the above.

21. If $a, b, c, \& d$ are non-zero constants, then the graph of $y = \frac{ax+b}{cx+d}$ has a horizontal asymptote at:
    A. $y = \frac{a}{c}$
    B. $y = \frac{b}{d}$
    C. $y = -\frac{b}{a}$
    D. $y = -\frac{d}{c}$
    E. None of the above.

22. The pressure of a gas varies jointly as the amount of the gas (measured in moles) and the temperature and inversely as the volume of the gas. If the pressure is 1248 kPa (kiloPascals) when there are 8 moles, the temperature is 320 K (Kelvins), and the volume is 640 cc (cubic centimeters), find the pressure when there are 9 moles, the temperature is 290 K, and the volume is 1080 cc.
    A. 780 kPa
    B. 1456 kPa
    C. 1508 kPa
    D. 754 kPa
    E. None of the above.

23. Carbon-14 is a radioactive form of the element. It exponentially decays over time, with a half-life of 5700 years. If a fossilized leaf sample contains 29% of its original amount of Carbon-14, how old is that sample? Round your answer to the nearest hundred.
    A. 0 years
    B. 8,100 years
    C. 11,400 years
    D. 23,400 years
    E. None of the above.

24. Big Ben's House of Clocks has modeled the cost of manufacturing its clocks by $C(x) = 70 + 37x - x^2$. It has also found that in t hours the number of clocks that can be produced is given by $x = 2t$, where $1 \leq t \leq 12$. Express C as a function of t.
    A. $C(t) = 70 + 37t + t^2$
    B. $C(t) = (70 + 37t - x^2) \cdot (2t)$
    C. $C(t) = 70 + 74t - 4t^2$
    D. $C(t) = 70 + 37t^2$
    E. None of the above.

25. Define the functions f and g according to the following tables of values.

x	1	2	3	4	5
$f(x)$	4	5	2	3	3

x	1	2	3	4	5
$g(x)$	5	1	3	2	4

    Evaluate $(g \circ f)(1) - (f \circ g)(3)$.
    A. 0
    B. 1
    C. 2
    D. 3
    E. Not enough information given.

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TIEBREAKER #1

Name: _____

School: _____

Two people are carrying a box. One person exerts a force of 129 pounds at an angle of 49.4° with the horizontal. The other person exerts a force of 98 pounds at an angle of 52.4°. Find the weight of the box. Show all your work.

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TIEBREAKER #2

Name: _____

School: _____

The arch beneath a bridge is semi-elliptical, a one-way roadway passes under the arch. The width of the roadway is 34 feet and the height of the arch over the center of the roadway is 10 feet. Two trucks plan to use this road. They are both 10 feet wide. Truck 1 has an overall height of 9 feet and Truck 2 has an overall height of 10 feet. Draw a rough sketch of the situation and determine which of the trucks can pass under the bridge.

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TIEBREAKER #3

Name: _____

School: _____

Define the function $f(x) = \frac{1}{x}$ and $a = 2$. Compute and simplify the following expression:

$$\frac{f(a+h) - f(a)}{h}$$

ANSWER KEY

1) B	11) D	21) A
2) B	12) A	22) D
3) A	13) A	23) E
4) B	14) C	24) C
5) B	15) C	25) A
6) A	16) B	
7) C	17) A	
8) B	18) C	
9) B	19) D	
10) D	20) B	

Tie Breaker Suggested Answers.

1. $\vec{v}_1 = \langle 129\cos(49.4°),\ 129\sin(49.4°)\rangle$

$\vec{v}_2 = \langle 98\cos(52.4°),\ 98\sin(52.4°)\rangle$

$\vec{v}_1+\vec{v}_2 = \langle 143.744,\ 175.59\rangle$

$\text{wt} = \sqrt{143.744^2 + 175.59^2} \sim \boxed{226.9}$

2. **Model the bridge**

The arch bridge is semi-elliptical, so we can use the equation of an ellipse to define it. The major axis (along the x-direction) has a radius of $r_x = \frac{34}{2} = 17$ feet. The minor axis (along the y-direction) has a radius of $r_y = 10$ ft. This gives us a formula of

$$\left(\frac{x}{17}\right)^2 + \left(\frac{y}{10}\right)^2 = 1$$

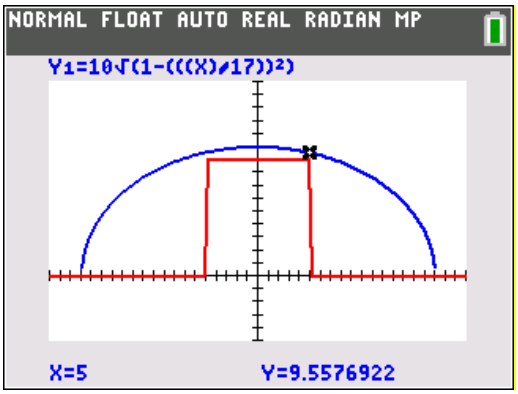
We can solve for y to graph this formula. We only use the principle square root because it's a semi-ellipse.

$$y = 10 \cdot \sqrt{1 - \left(\frac{x}{17}\right)^2}$$

Model the trucks

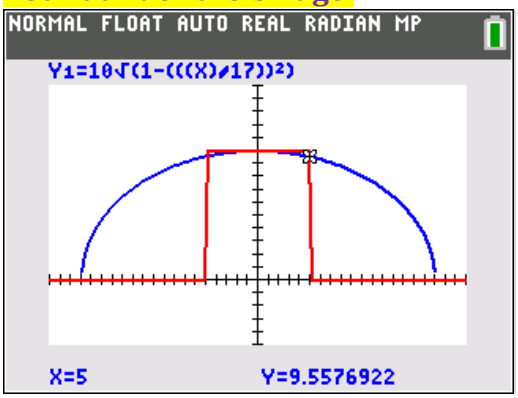
We can represent the two trucks with a rectangle centered at the origin. Both trucks have a width of 10 ft, so we extent 5-units to either side of the origin. We can check the "edges" of the truck at $x = \pm 5$ to see if it hits the bridge. If the height of the bridge is higher than the height of the truck, it will fit.

Check Truck 1. At $x = \pm 5$, the height of the bridge is about 9.6ft. Since 9<9.6, this truck will fit under the bridge.



Check Truck 2.

At $x = \pm 5$, the height of the bridge is lower than the height of the truck, 9.6<10. Truck 2 will not fit under the bridge.



3. This problem evaluates the difference quotient.

$$\frac{f(2+h)-f(2)}{h} = \frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \frac{\left(\frac{2}{2\cdot(2+h)}-\frac{2+h}{2\cdot(2+h)}\right)}{h}$$

$$= \frac{\left(\frac{2-(2+h)}{2\cdot(2+h)}\right)}{h} = \frac{\left(\frac{2-2-h}{2\cdot(2+h)}\right)}{h} = \frac{\left(\frac{-h}{2\cdot(2+h)}\right)}{h/1}$$

$$= \left(\frac{-h}{2\cdot(2+h)}\right)\cdot\left(\frac{1}{h}\right) = \frac{-h}{2\cdot(2+h)\cdot h}$$

$$= \frac{-1}{2\cdot(2+h)}$$