Arkansas Council of Teachers of Mathematics
2019 Trigonometry & Precalculus State Exam

Work the multiple choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tie breaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1. At the end of the semester, students' math grades are listed in a table which gives each student's ID number in the left column and the student's grade in the right column. Let N represent the ID number and G represent the grade. Which quantity, N or G, must necessarily be a function of the other?

    a. N must be a function of G because N depends on G.
    b. N must be a function of G because each grade is only related to one ID number.
    c. G must be a function of N because G depends on N.
    d. G must be a function of N because each ID number is only related to one grade.
    e. Either. G can be a function of N AND N can be a function of G.

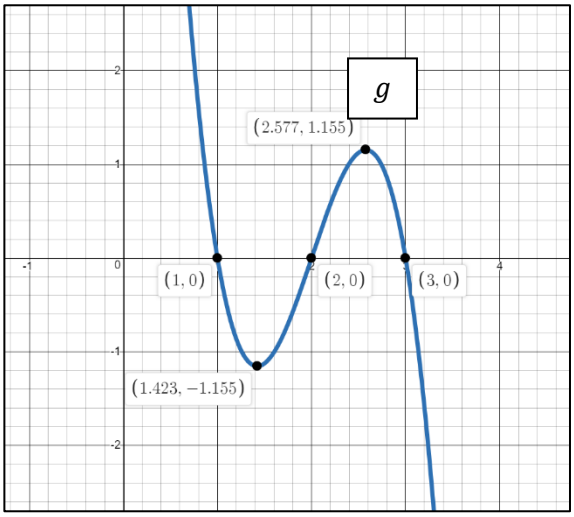
2. A price increases 5% due to inflation and is then reduced 10% for a sale. Express the final price as a function of the original price, p. Don't round.

    a. $f(p) = .95p$
    b. $f(p) = -0.05p$
    c. $f(p) = .945p$
    d. $f(p) = 1.05p$
    e. None of the above.

3. A candle stick is burning in such a way that the height of the candle is decreasing at a constant rate of $\frac{1}{2}$ an inch every 10 minutes. If the candle is 6.25 inches tall after burning for 45 minutes, how tall was the candle before it was lit?

    a. 4 inches
    b. 8 inches
    c. 7.25 inches
    d. 26.25 inches
    e. 8.5 inches

4. Find an interval on the graph of g, shown to the right, for which the average rate of change is positive over the entire interval.



   a. $(-\infty, 2)$
   b. $(-\infty, 1)$
   c. $(-1.155, 1.155)$
   d. $(2,3)$
   e. $(1.423, 2.577)$

5. Find an interval on the graph of g, shown to the right, for which the change in the average rate of change is positive over the entire interval.

   a. $(-\infty, 2)$
   b. $(-\infty, 1)$
   c. $(-1.155, 1.155)$
   d. $(2,3)$
   e. $(1.423, 2.577)$

6. Your height above the ground (in feet) on the London Eye after boarding at $t = 0$ minutes is

$$f(t) = 200 \sin\left(\frac{\pi}{15}(t - 7.5)\right) + 250$$

   Find the period of this function and explain what it represents in the context of this problem. (Note that the London Eye is a famous Ferris wheel in London.)

   a. The period is 2π because every circle completes one revolution in 2π radians.
   b. The period of this function is 250. This means that the center of the London Eye is 250 feet above the ground.
   c. The period of this function is 30. This means that it takes 30 minutes for the London Eye to make one full revolution.
   d. The period of this function is 200. This means that your maximum height is 200 feet above the center of the London Eye.
   e. The period of this function is $\frac{\pi}{15} \approx 0.209$. This means that it takes 20.9% longer to ride the London Eye than any other Ferris wheel.

7. Consider the polynomial function below:

$$P(a) = -3a^5 + 6a^2 - a - 2$$

Find the end behavior of P. That is, describe the behavior of $P(a)$ as a increases without bound, and describe the behavior of $P(a)$ as a decreases without bound.

a. As a increases without bound, $P(a)$ increases without bound. As a decreases without bound, $P(a)$ decreases without bound.
b. As a increases without bound, $P(a)$ increases without bound. As a decreases without bound, $P(a)$ increases without bound.
c. As a increases without bound, $P(a)$ decreases without bound. As a decreases without bound, $P(a)$ increases without bound.
d. As a increases without bound, $P(a)$ decreases without bound. As a decreases without bound, $P(a)$ decreases without bound.
e. None of the above.

8. Consider the rational function below:

$$R(x) = \frac{1}{(2t - a)(t + b)}$$

If the domain of R is $(-\infty, 4) \cup (4,5) \cup (5, \infty)$, find all possible values for a and b.

a. $a = 8, b = -5$
b. a could be 8 or 10. b could be -4 or -5.
c. $a = 10, b = -4$
d. Both a and b could be -4 or -5.
e. Not enough information to determine.

9. Find the y-value of the hole in the rational function $r(x) = \frac{x-2}{x^3-8}$.

a. $\frac{1}{4}$
b. $\frac{1+i\sqrt{3}}{2}$
c. $\frac{1}{12}$
d. 2
e. There is not a hole in the function.

10. Consider the two exponential functions below:

$$f(x) = 3^x \text{ and } g(x) = e^{kx}$$

Find a value for k so that $f(x) = g(x)$.

a. No solution
b. $k = \frac{3}{e}$
c. $k = 3e$
d. $k = \log 3$
e. $k = \ln 3$

11. Write the following exponential function in standard form, $f(t) = ab^t$. Round to two decimal places if needed.

$$f(t) = \frac{60}{5 \cdot 2^{t/11.2}}$$

a. $f(t) = 12(0.940)^t$
b. $f(t) = 60(1.228)^t$
c. $f(t) = 12(1.064)^t$
d. $f(t) = 60(0.814)^t$
e. None of the above.

12. Based on the table below, select a true statement from the answer choices.

x	-3	-2	-1	0	1	2	3
$f(x)$	-8.1	-2.4	-0.3	0	0.3	2.4	8.1

a. $f(x)$ is symmetric about the y-axis and is therefore an even function.
b. $f(x)$ is symmetric about the origin and is therefore an odd function.
c. $f(x)$ is symmetric about the y-axis and is therefore an odd function.
d. $f(x)$ is symmetric about the origin and is therefore an even function.
e. $f(x)$ has no symmetry and is neither even nor odd.

13. Find a possible formula for the function $Q(n)$ in terms of the function $P(n)$ given the following domains and ranges.

Domain of P: $(-3,8]$	Domain of Q: $(-10,1]$
Range of P: $[-6,12)$	Range of Q: $[-2,4)$

a. $Q(n) = 3P(n+7)$
b. $Q(n) = \frac{1}{3}P(n-7)$
c. $Q(n) = P(n+7)+4$
d. $Q(n) = \frac{1}{3}P(n-7)+4$
e. $Q(n) = \frac{1}{3}P(n+7)$

14. Let θ be an angle in the first quadrant with $\cos\theta = a$. Evaluate $\cos(180° - \theta)$ in terms of a.

a. a
b. $1-a^2$
c. $\sqrt{1-a^2}$
d. $-a$
e. $1-a$

15. What angle in radians corresponds to 0.75 rotations around the unit circle? Round to four decimal places.

a. 0.0131
b. 2.3562
c. 4.5000
d. 4.7124
e. None of the above

16. A line passing through the origin and the point $P = (16, b)$ forms the angle $\theta = 50°$. Find the value of b. Round to four decimal places.

a. $b \approx 19.0681$
b. $b \approx 13.4256$
c. $b \approx 12.2567$
d. $b \approx 10.2846$
e. $b \approx 20.8865$

17. Find the slope of a line that is perpendicular to a second line that passes through the origin at an angle of $\frac{5\pi}{8}$. Round to four decimal places.

    a. 0.5093
    b. -0.5093
    c. -29.1691
    d. 0.4142
    e. None of the above

18. Consider the two compositions shown below:

    I. $\sin(\arcsin y) = y$
    II. $\sin^{-1}(\sin\theta) = \theta$

    Select the true statement below.

    a. Statement I is true for all values in the domain of the composition.
    b. Statement II is true for all values in the domain of the composition.
    c. Both statements, I and II, are true for all values in the domain of the composition.
    d. Neither statement is true for all values in the domain of the composition.
    e. There is not enough information given to determine if the statements are true or not.

19. For the equation below, find all solutions in the interval $[0, 2\pi)$.

$$\cot^2 x + 3\csc x = -3$$

    a. $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$
    b. $x = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$
    c. $x = \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$
    d. $x = \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$
    e. None of the above.

20. Ten inches of snow contains an equivalent amount of water as one inch of rain. Write an equation for the amount of precipitation, measured in inches of rain, $r = f(s)$, as a function of the number of inches of snow, s.

    a. $f(s) = 10s$
    b. $f(s) = \frac{s}{10}$
    c. $f(s) = \frac{10}{s}$
    d. $f(s) = 10 + s$
    e. $f(s) = s - 10$

21. The population of your home town, in thousands of people, is $P = f(t)$, where t is the number of years since you were born. Interpret the expression $f(20) - f(10)$.

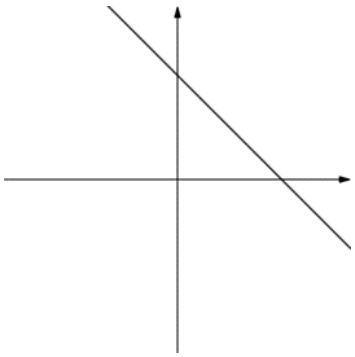
    a. $f(20) - f(10)$ is the population of your home town when you are 10 years old.
    b. $f(20) - f(10)$ indicates the ten years that pass from when you are age 10 to when you are age 20.
    c. $f(20) - f(10)$ is the change in population as your age increases from 10 to 20.
    d. $f(20) - f(10)$ is your age when the population is 10,000 people.
    e. None of the above.

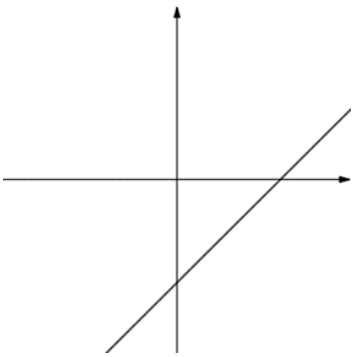
22. Find the domain of the following function:

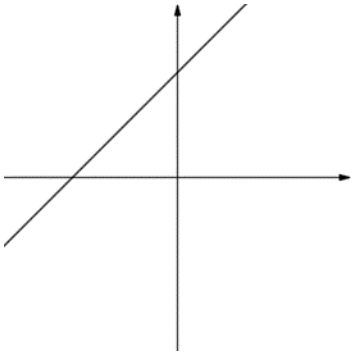
$$g(x) = \sqrt{12x^2 - 11x - 5}$$

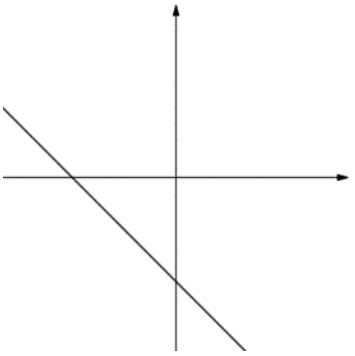
    a. $(-\infty, \infty)$
    b. $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \frac{5}{4}\right) \cup \left(\frac{5}{4}, \infty\right)$
    c. $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{5}{6}, \infty\right)$
    d. $\left(-\infty, -\frac{5}{4}\right) \cup \left(\frac{1}{3}, \infty\right)$
    e. None of the above

23. Assume A, B, and C are all positive constants satisfying the equation $Ax + By = C$. Which of the following could be the graph of this equation?

a. 

b. 

c. 

d. 

e. None of the above.

24. Which of the following is true?

a. $e^{\ln x} = x$
b. $\sqrt{x^2} = x$
c. $\sqrt{x^2 + y^2} = x + y$
d. $\frac{1}{x} + \frac{1}{y} = \frac{1}{x+y}$
e. None of the above are true.

25. A drop of water lands in a pond and creates an expanding circular ripple. Define $r = f(t)$ as the radius in centimeters of the circle at time t seconds, and define $A(r)$ is the area in square centimeters of a circle of radius r centimeters. Give the meaning and units of $A\big(f(t)\big)$.

    a. $A\big(f(t)\big)$ is the radius of the circle, in square centimeters, at time t seconds.
    b. $A\big(f(t)\big)$ is the time t, in seconds, it takes for the area to be a certain amount (in square centimeters).
    c. $A\big(f(t)\big)$ is the radius of the circle in centimeters in terms of the area in square centimeters.
    d. $A\big(f(t)\big)$ is the area of the circle, in square centimeters, in terms of t, the number of seconds that have elapsed.
    e. $A\big(f(t)\big)$ is the area of the circle, in centimeters, in terms of the radius of the circle, in centimeters.

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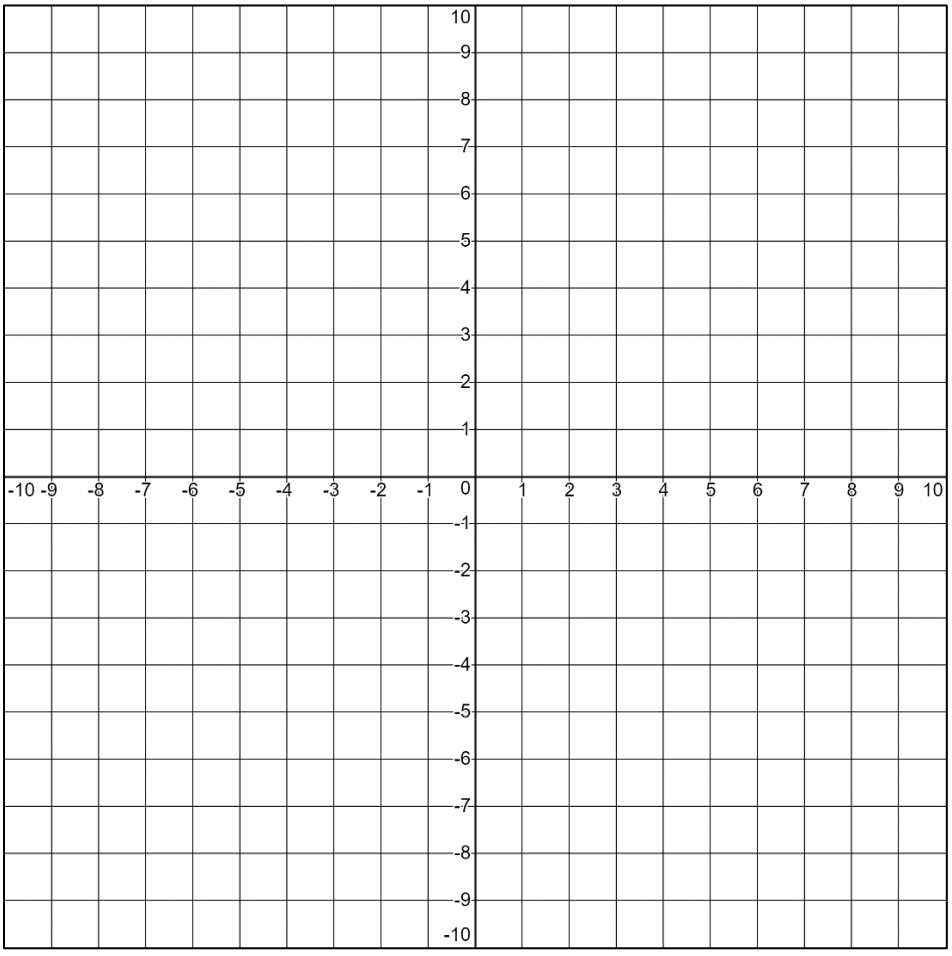
Tie Breaker #1

Name: _____

School: _____

Part 1: Consider the exponential function $Q = r \cdot s^t$. Letting $q = \ln Q$, show that q is a linear function of t by writing it in the form $q = b + mt$. State the values of m and b.

Part 2: Using your values from part 1, let $r = \frac{1}{e}$ and $s = \sqrt[3]{e^2}$. Draw the graph of q on the coordinate plane below.



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Tie Breaker #2

Name: _____

School: _____

Two fire stations are located 56.7 miles apart, at points A and B. There is a forest fire at point C.
If $\angle CAB = 54°$ and $\angle CBA = 58°$, which fire station is closer? How much closer is it?
Support your answers with a sketch and appropriate work.

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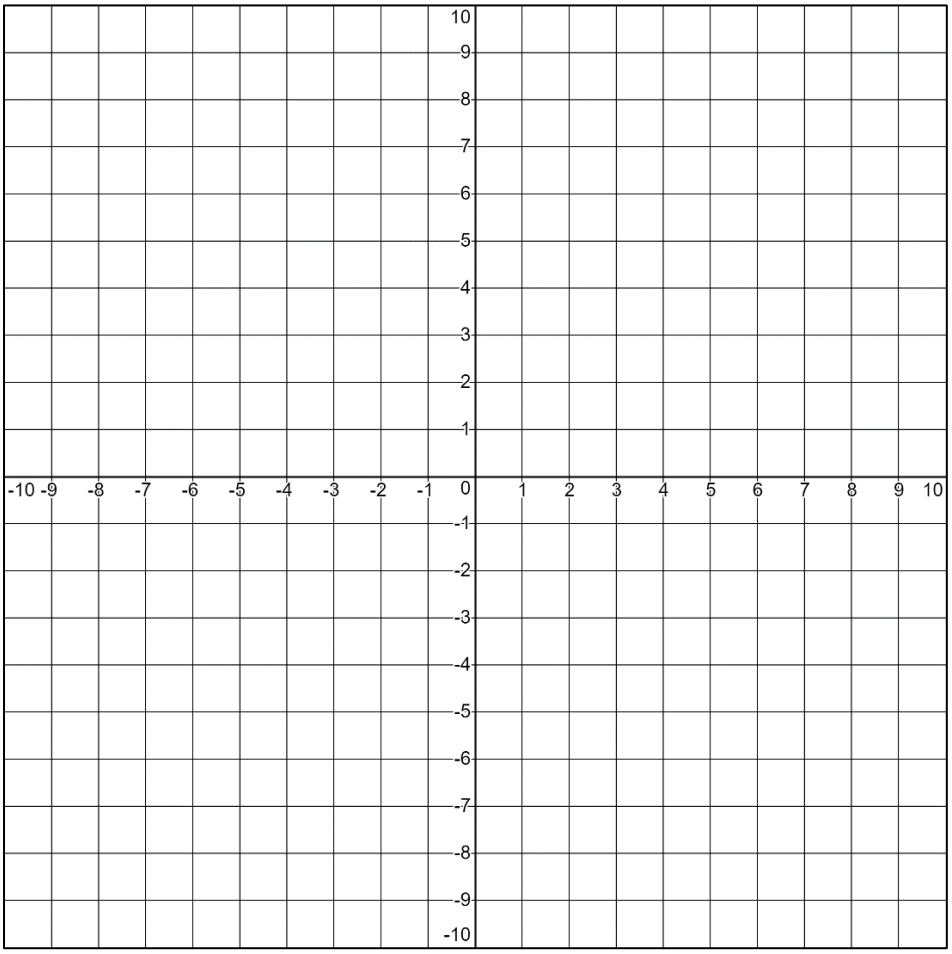
Tie Breaker #3

Name: _____

School: _____

Draw the graph of the piecewise function below:

$$f(x) = \begin{cases} 1/x & \text{for} \quad x < -1 \\ x^2 & \text{for} \quad -1 \leq x \leq 1 \\ \sqrt{x} & \text{for} \quad x > 1 \end{cases}$$



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Multiple Choice Answer Key

1. D
2. C
3. E
4. E
5. A
6. C
7. C
8. B
9. C
10. E
11. A
12. B
13. E
14. D
15. D
16. A
17. D
18. A
19. B
20. B
21. C
22. E
23. A
24. A
25. D

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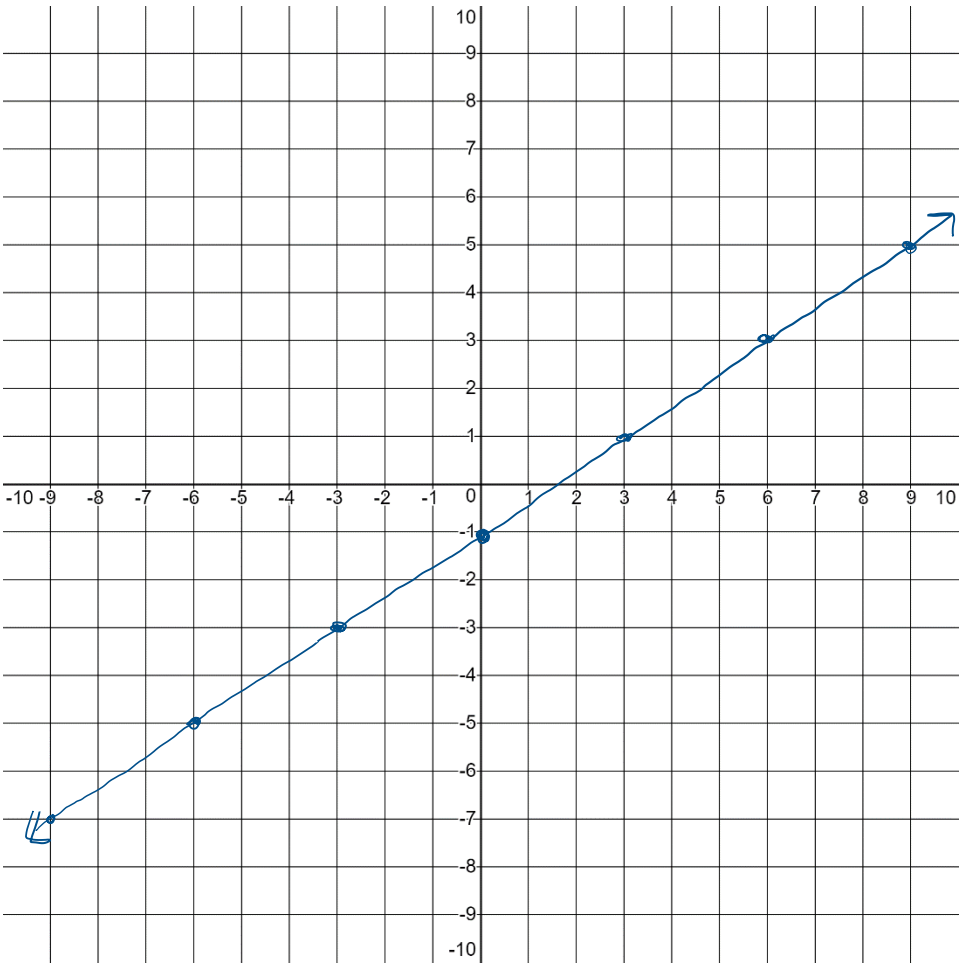
Solution Tie Breaker #1

Part 1: Consider the exponential function $Q = r \cdot s^t$. Letting $q = \ln Q$, show that q is a linear function of t by writing it in the form $q = b + mt$. State the values of m and b.

$q = \ln Q = \ln(r \cdot s^t) = \underbrace{\ln(r)}_{b} + t\underbrace{\ln(s)}_{m}$

Part 2: If $r = \frac{1}{e}$ and $s = \sqrt[3]{e^2}$, draw the graph of q on the coordinate plane below.



$b = \ln(r) = \ln\left(\frac{1}{e}\right) = -1$

$m = \ln(s) = \ln\left(e^{2/3}\right) = 2/3$

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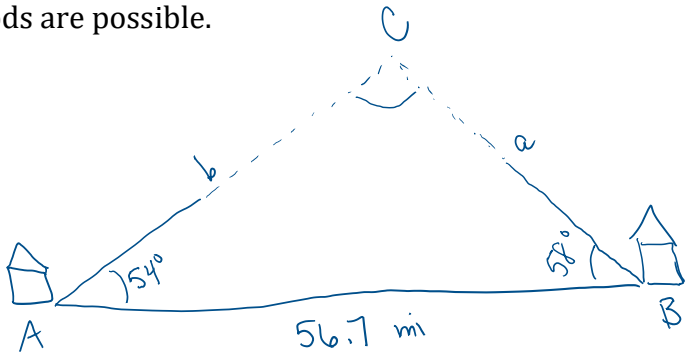
Solution Tie Breaker #2

Two fire stations are located 56.7 miles apart, at points A and B. There is a forest fire at point C. If $\angle CAB = 54°$ and $\angle CBA = 58°$, which fire station is closer? How much closer? Support your answers with a sketch and appropriate work.

Other methods are possible.



$C = 180° - 54° - 58°$

$C = 68°$

$\frac{a}{\sin 54°} = \frac{56.7}{\sin 68°}$

$a = \frac{56.7 \sin 54°}{\sin 68°} \approx 49.4738$

$\frac{b}{\sin 58} = \frac{56.7}{\sin 68°}$

$b = \frac{56.7 \sin 58°}{\sin 68°}$

$b \approx 51.8606$

Fire station B is closer by approximately 2.3869 miles.

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Solution Tie Breaker #3

Draw the graph of the piecewise function below:

$$f(x) = \begin{cases} 1/x & \text{for} \quad x < -1 \\ x^2 & \text{for} \quad -1 \leq x \leq 1 \\ \sqrt{x} & \text{for} \quad x > 1 \end{cases}$$

The graph resembles the following. Note the open and closed end points.