ACTM Regional Geometry Competition 2020

Begin by removing the three tie breaker sheets at the end of the exam and writing your name on all three pages. Work the multiple-choice questions first, choosing the single ***best*** (most detailed and complete correct) response from the choices available. Indicate your answer here and on your answer sheet. Make sure you attempt the tie-breaker questions at the end of the test starting with tie breaker 1, then 2, and then 3 if you have time. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Notations and Definitions:

- All questions on this test are in **Euclidean Geometry**.
- All angles are measured in **radians**. **π radians = 180°**.
- AB indicates the distance between points A and B.
- ***A-B-C*** indicates that B is ***between*** A and C that is: A, B, and C are collinear and $AB + BC = AC$.
- A ***kite*** is a quadrilateral with at least two non-overlapping pairs of congruent consecutive sides. Its ***major diagonal*** has endpoints where the congruent sides meet.
- A ***trapezoid*** is a quadrilateral with *at least* one pair of parallel sides.
- An ***isometry*** (rigid transformation) is a transformation mapping every preimage to a congruent image.
- ***Z* Property**: Alternate interior angles formed by a transversal to lines l and m are congruent if and only if l and m are parallel.

1. Given ΔABC with $m\measuredangle A = \frac{\pi}{3}$ and $m\measuredangle B = \frac{\pi}{4}$ what is $m\measuredangle C$?

   A. $\frac{5\pi}{12}$ B. $\frac{\pi}{5}$ C. $\frac{6\pi}{7}$ D. $\frac{\pi}{2}$ E. Each of the other answers is incorrect.

2. The sum of the measures of the interior angles of a convex n-gon is _____.

   A. $(n-2)\frac{\pi}{2}$
   B. $n\pi$
   C. $n\pi - \pi$
   D. $(n-2)\,\pi$
   E. Each of the other answers is incorrect.

3. Which of the following conditions is NOT sufficient to conclude that two triangles are congruent? (Conditions indicate corresponding pairs of congruent parts.)

   A. SSS B. SAS C. SSA D. ASA E. AAS

4. Which of the following conditions are sufficient conditions to determine that the two triangles are similar?

   I. SSS: The lengths of all corresponding sides are proportional.
   II. AA: Two pairs of corresponding angles are congruent.
   III. SAS: The lengths of two pairs of corresponding sides are proportional and the pair of corresponding included angles are congruent.

   A. II only B. I and II only C. II and III only D. I, II, and III
   E. Each of the other answers is incorrect

5. Let the coordinates of three points be given as $A = (3, 4)$, $B = (2, 5)$ and $C = (5, 7)$. Which of the following best describes ΔABC ?
   A. Right Triangle
   B. Isosceles Triangle
   C. Right Isosceles Triangle
   D. Scalene Triangle
   E. Obtuse Triangle

6. Let the coordinates of three points be given as $A = (3, 0)$, $B = (5, 3)$ and $C = (11, -1)$. Which of the following best describes ΔABC ?
   A. Right Triangle
   B. Isosceles Triangle
   C. Right Isosceles Triangle
   D. Acute Triangle
   E. Obtuse Triangle

7. Which of the following does not have to have a pair of congruent interior angles?
   A. Parallelogram B. Trapezoid C. Kite D. Rhombus E. Rectangle

8. How many kites can be constructed with sides of length 4 cm, 4 cm, 6 cm, and 6 cm?
   A. 0
   B. Exactly 1
   C. Exactly 2
   D. Infinitely many
   E. It cannot be determined from the information given.

9. Given quadrilateral *ABCD* such that $\Delta ABC \cong \Delta ADC$ the quadrilateral must be a _____.
   A. Rhombus B. Parallelogram C. Kite D. Square E. Trapezoid

10. Given quadrilateral *ABCD* such that its diagonals bisect each other. Quad *ABCD* must be a _____.
   A. Rhombus B. Parallelogram C. Kite D. Square E. Trapezoid

11. Given quadrilateral *ABCD* such that a pair of consecutive angles forms a supplementary pair. Quad *ABCD* must be a _____.
   A. Rhombus B. Parallelogram C. Kite D. Square E. Trapezoid

12. Which of the following is an outline of a correct deduction, given a quadrilateral ABCD such that both pairs of opposite sides are congruent?
   A. $\measuredangle DCA \cong \measuredangle BAC$ and $\measuredangle BCA \cong \measuredangle DAC$ by the Z Property and $\triangle ABC \cong \triangle CDA$ by ASA Triangle Congruence Theorem
   B. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$ and $\overline{AC} \cong \overline{CA}$, thus $\triangle ABC \cong \triangle ADC$ by SSS Triangle Congruence Theorem
   C. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$ and $\overline{AC} \cong \overline{CA}$, thus $\triangle ABC \cong \triangle CDA$ by SSS Triangle Congruence Theorem
   D. $\measuredangle DCA \cong \measuredangle BAC$ and $\measuredangle BCA \cong \measuredangle DAC$ by the Z Property and $\triangle ABC \cong \triangle ADC$ by AAS Triangle Congruence Theorem
   E. Each of the other answers is incorrect.

13. A quadrilateral with both pair of opposite sides congruent must be a _____.
    A. Rhombus B. Parallelogram C. Kite D. Square E. Rectangle

14. A trapezoid with a pair of supplementary opposite interior angles is _____.
    A. a Right Trapezoid
    B. a Rectangle
    C. an Isosceles Trapezoid
    D. a Square
    E. Each of the other answers is incorrect.

15. Which of the following is not an isometry?
    A. Translation B. Rotation C. Reflection D. Glide-reflection E. Dilation

16. $\overline{CD}$ is an altitude of $\triangle ABC$ with A-D-B, AD = 9, AC = 15, BC = 13. What is the area of $\triangle ABC$?
    A. 64 B. 84 C. 97.5 D. 168 E. Each of the other answers is incorrect.

17. In an equilateral triangle, if we divide the length of the altitude by the length of a side what do we obtain?
    A. $\frac{1}{\sqrt{2}}$ B. $\sqrt{2}$ C. $\sqrt{3}$ D. $\frac{\sqrt{3}}{2}$ E. Each of the other answers is incorrect.

18. The composition of two reflections about intersecting lines is a single _____.
    A. Rotation B. Reflection C. Translation D. Glide-reflection
    E. Each of the other answers is incorrect.

19. Take the point (3, 4) rotate it $\pi/2$ about the origin and then translate the intermediate image by <2, 5>. What is the final image?
    A. (6, 9) B. (-2, 8) C. (6, 2) D. (-5, 9) E. Each of the other answers is incorrect.

20. What is the equation of the circle with center (1, 2) and containing (3, 4)?

A. $(x-1)^2+(y-2)^2=8$

B. $(x+1)^2+(y+2)^2=52$

C. $(x-3)^2+(y-4)^2=8$

D. $(x+3)^2+(y+2)^2=52$

E. Each of the other answers is incorrect.

21. $\triangle ABC \sim \triangle DEF$, $AB = 12$, $DE = 3$, and the area of $\triangle DEF = 12$. What is the area of $\triangle ABC$?

A. 3 B. 4 C. 48 D. 192 E. Each of the other answers is incorrect.

22. Given an arbitrary quadrilateral we form a second quadrilateral whose vertices are the midpoints of the original quadrilateral. The second quadrilateral must be a _____.

A. Parallelogram B. Rectangle C. Kite D. Rhombus

E. Each of the other answers is incorrect.

23. $A = (1,2)$, $B = (4, 4)$, $C = (8,-2)$ What is the area of $\triangle ABC$?

A. 6 B. 12 C. 13 D. 26 E. Each of the other answers is incorrect.

24. We have a right square pyramid with height 12 and base edge of length 3. What is its volume? Round to the nearest whole number.

A. 18 B. 36 C. 54 D. 108 E. Each of the other answers is incorrect.

25. We have a right square pyramid with height 12 and base edge of length 3. What is its surface area? Round to the nearest whole number.

A. 73 B. 81 C. 82 D. 83 E. Each of the other answers is incorrect.

ACTM Regional Geometry Competition 2020
Tie Breaker 1

Name: _____

School:_ _____

Prove the following. Provide a sketch to accompany your proof.

If quad*ABCD* exists with $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \| \overline{CD}$, then quad*ABCD* is a parallelogram.

ACTM Regional Geometry Competition 2020
Tie Breaker 2

Name: _____

School:_ _____

Prove the following. Provide a sketch to accompany your proof.
The sum of the measures of the interior angles of a triangle is π.

ACTM Regional Geometry Competition 2020
Tie Breaker 3

Name: _____

School:_ _____

Prove the following. Provide a sketch to accompany your proof.

If quad*ABCD* exists with $\overline{BD}$ perpendicularly bisected by $\overleftrightarrow{AC}$, then quad*ABCD* is a kite.

ACTM Regional Geometry Exam 2020 Solutions

Answers

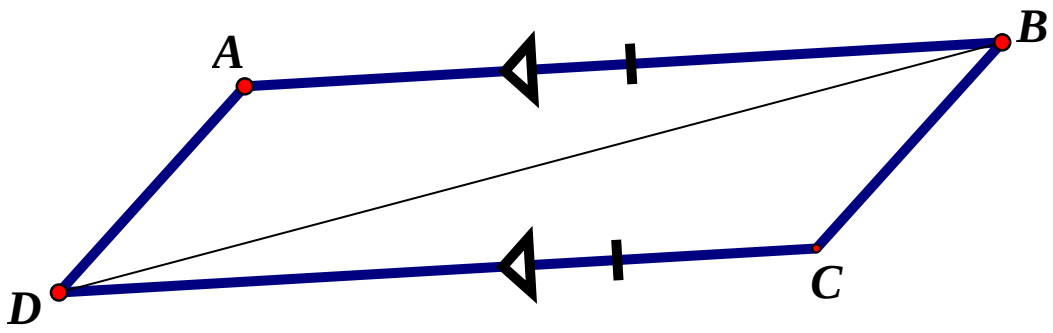
1	**A**
2	**D**
3	**C**
4	**D**
5	**B**
6	**A**
7	**B**
8	**D**
9	**C**
10	**B**
11	**E**
12	**C**
13	**B**
14	**C**
15	**E**
16	**B**
17	**D**
18	**A**
19	**B**
20	**A**
21	**D**
22	**A**
23	**C**
24	**B**
25	**C**

ACTM Regional Geometry Competition 2020
Tie Breaker 1

Name: **Solution Key**

Prove the following. Provide a sketch to accompany your proof.
If quad*ABCD* exists with $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \| \overline{CD}$, then quad*ABCD* is a parallelogram.
Proof.

In the hypothesis we are given quad*ABCD* exists with $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \| \overline{CD}$. Given 2 points there exists a unique line segment with those points as endpoint, so construct $\overline{BD}$. By definition we now have $\triangle ABD$ and $\triangle CDB$. Since $\overline{AB} \| \overline{CD}$, by definition $\overleftrightarrow{AB} \| \overleftrightarrow{CD}$ and $\overleftrightarrow{BD}$ is a transversal to this pair of parallel lines. By the *Z*-Property alternate interior angles are congruent so $\measuredangle ABD \cong \measuredangle CDB$. $\overline{DB} \cong \overline{BD}$ as well, so by the SAS Postulate $\triangle ABD \cong \triangle CDB$. By definition of triangle congruence corresponding parts of congruent triangles are congruent, so $\measuredangle ADB \cong \measuredangle CBD$. These are alternate interior angles formed by transversal $\overleftrightarrow{BD}$ to $\overleftrightarrow{AD}$ and $\overleftrightarrow{CB}$. By the *Z*-Property $\overleftrightarrow{AD} \| \overleftrightarrow{CB}$. By definition, quad*ABCD* is a parallelogram. ■



ACTM Regional Geometry Competition 2020
Tie Breaker 2

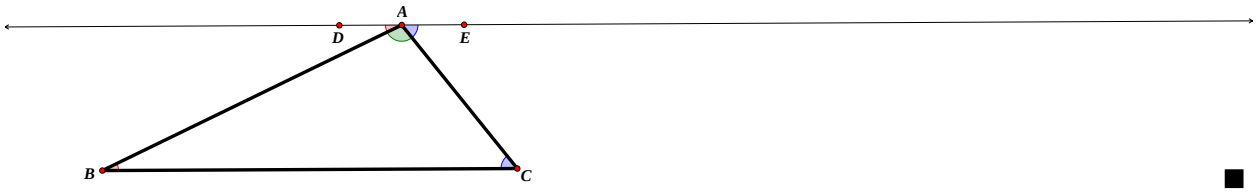
Name: **Solution Key**

Prove the following. Provide a sketch to accompany your proof.
The sum of the measures of the interior angles of a triangle is π.

Proof.
By hypothesis, we are given a triangle $\triangle ABC$. There is exactly one plane containing $\triangle ABC$. By the Parallel Postulate, there exists exactly one line in this plane which contains A and is parallel to $\overleftrightarrow{BC}$. There are points D and E on this line with D-A-E such that ∢EAC is adjacent to ∢CAB and ∢CAB is adjacent to ∢BAD. By the *Z*-Property ∢DAB ≅ ∢CBA and ∢EAC ≅ ∢BCA. The measure of a straight angle is π. By the Angle Addition Postulate, definition of congruent angles, and substitution, we see that

$$m\sphericalangle B + m\sphericalangle C + m\sphericalangle BAC = m\sphericalangle DAB + m\sphericalangle EAC + m\sphericalangle CAB = \pi.$$

Therefore, the sum of the measures of the interior angles of any triangle is π.



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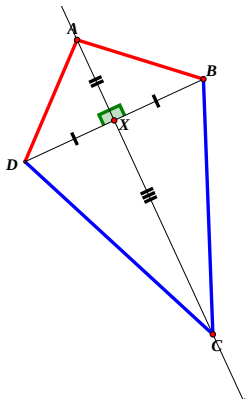
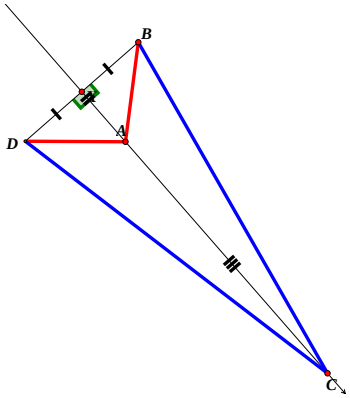
ACTM Regional Geometry Competition 2020
Tie Breaker 3

Name: **Solution Key**

Prove the following. Provide a sketch to accompany your proof.
If quad*ABCD* exists with $\overline{BD}$ perpendicularly bisected by $\overleftrightarrow{AC}$, then quadABCD is a kite.

Proof.
By hypothesis we are given quad*ABCD* exists with $\overline{BD}$ perpendicularly bisected by $\overleftrightarrow{AC}$. Let *X* be the point of intersection of $\overleftrightarrow{AC}$ with $\overline{BD}$. By definition of bisector we see that $\overline{XD} \cong \overline{XB}$. By definition of perpendicular there are four right angles at *X* and $\measuredangle AXD \cong \measuredangle AXB \cong \measuredangle CXB \cong \measuredangle CXD$. $\overline{AX} \cong \overline{AX}$ and $\overline{CX} \cong \overline{CX}$. By the SAS Postulate $\triangle ABX \cong \triangle ADX$ and $\triangle CBX \cong \triangle CDX$. By definition of congruent triangles, corresponding parts of congruent triangles are congruent so $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$. By definition, quad*ABCD* is a kite.





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