Arkansas Council of Teachers of Mathematics **Name _____**

2022 Geometry Regional Competition

Begin by removing the three tie breaker sheets at the end of the exam and writing your name on all three pages. Work the multiple-choice questions first, choosing the single ***best*** (most detailed and complete correct) response from the choices available. Indicate your answer here and on your answer sheet. Make sure you attempt the tie-breaker questions at the end of the test starting with tie breaker 1, then 2, and then 3 if you have time. Turn in your answer sheet and the tie breaker pages when you are finished.

Notations and Definitions:

- All questions on this test are in **Euclidean Geometry**.
- All angles are measured in **radians** unless marked otherwise. **π radians = 180°**.
- *AB* indicates the distance between points *A* and *B*.
- ***A-B-C*** indicates that *B* is ***between*** *A* and *C* that is: *A, B*, and *C* are collinear and *AB + BC = AC*.
- A ***kite*** is a quadrilateral with at least two non-overlapping pairs of congruent consecutive sides. Its ***major diagonal*** has endpoints where the congruent sides meet.
- A ***trapezoid*** is a quadrilateral with *at least* one pair of parallel sides.
- An ***isometry*** (rigid transformation) is a transformation mapping every preimage to a congruent image.
- ***Z* Property**: Alternate interior angles formed by a transversal to lines *l* and *m* are congruent if and only if *l* and *m* are parallel.

1. Given ΔABC and point D such that A-B-D with $m\measuredangle A = \frac{\pi}{6}$ and $m\measuredangle CBD = \frac{\pi}{4}$what is $m\measuredangle C$?
   A. $\frac{5\pi}{12}$
   B. $\frac{\pi}{10}$
   C. $\frac{\pi}{12}$
   D. $\frac{\pi}{3}$
   E. Each of the other answers is incorrect.

2. We are given AB = 6, BC = 8, and AC = 12, DE = 3, EF = 4, and $\measuredangle$ABC ≅ $\measuredangle$DEF. What is FD?
   A. 12
   B. 6
   C. 4
   D. There is not enough information to determine FD.
   E. Each of the other answers is incorrect.

3. We are given AB = 6, BC = 4, m$\measuredangle$CAB = $\frac{\pi}{6}$. What is AC rounded to 2 decimal places?
   A. 2.55
   B. 7.84
   C. 10.00
   D. There is not enough information to determine AC.
   E. Each of the other answers is incorrect.

4. Which of the following sets of triangle side measures is impossible? All are in centimeters.
   A. 3-4-6
   B. 2-5-4
   C. 3-4-5
   D. 4-8-10
   E. 3-6-2

5. Let $A = (2, 2)$, $B = (6, 1)$, $C = (5, 5)$, and $D = (2, 5)$. Which of the following *best* describes polygon ABCD?
   A. Quadrilateral
   B. Parallelogram
   C. Rectangle
   D. Kite
   E. There is no such quadrilateral.

6. Let $A = (2, 5)$, $B = (5, 8)$, $C = (9, 4)$, and $D = (6, 1)$. Which of the following *best* describes polygon ABCD?
   A. Quadrilateral
   B. Parallelogram
   C. Rectangle
   D. Kite
   E. There is no such quadrilateral.

7. Which of the following has to have rotational symmetry?
   A. Parallelogram
   B. Isosceles Trapezoid
   C. Kite
   D. Isosceles Triangle
   E. Right Triangle

8. If a quadrilateral has two pair of consecutive supplementary angles, then the quadrilateral must be a _____.
   A. Parallelogram
   B. Trapezoid
   C. Kite
   D. Rectangle
   E. Rhombus

9. Given quadrilateral *ABCD* such that one of its diagonals is a perpendicular bisector of the other. Quad *ABCD* must be a _____.
   A. Rhombus
   B. Parallelogram
   C. Rectangle
   D. Square
   E. Kite

10. How many triangle shapes can be constructed with sides of length 4 cm, 2 cm, and 5 cm?
    A. 0
    B. Exactly 1
    C. Exactly 2
    D. Infinitely many
    E. It cannot be determined from the information given.

11. Let $A = (2, 1)$, $B = (6, 1)$, $C = (8, 5)$, and $D = (3, 5)$. What is the area of Quad ABCD?
    A. 12.5
    B. 15
    C. 16.5
    D. 18
    E. Each of the other answers is incorrect.

12. What is the ratio of the length of an altitude to the length of a side in an equilateral triangle?
    A. 2
    B. $\frac{\sqrt{3}}{2}$
    C. $\frac{\sqrt{2}}{2}$
    D. $\sqrt{3}$
    E. Each of the other answers is incorrect.

13. The composition of two reflections about intersecting lines is always a single _____.
    A. Rotation
    B. Reflection
    C. Translation
    D. Glide-reflection
    E. Each of the other answers is incorrect.

14. A quadrilateral that is both a parallelogram and an isosceles trapezoid must be a _____.
    A. Rhombus
    B. Parallelogram
    C. Kite
    D. Square
    E. Rectangle

15. We are given a right rectangular prism with a base whose diagonal has length 5 and whose width is 4. The height of the prism is 6. A second prism is similar to the first one. The height of the second prism is 12. What is the volume of the second prism?
    A. 144
    B. 240
    C. 480
    D. 576
    E. Each of the other answers is incorrect.

16. If we take quadrilateral ABCD and perform a single isometry, and we make line segments from each of A, B, C, and D to its image and the 4 line segments are parallel and the same length, then the isometry is a_____.
    A. Translation
    B. Rotation
    C. Reflection
    D. Glide-reflection
    E. Dilation

17. Given quadrilateral $ABCD$ containing a pair of opposite right angles and a pair of parallel sides. Quad $ABCD$ must be a _____.
    A. Rhombus
    B. Parallelogram
    C. Rectangle
    D. Square
    E. Right Trapezoid

18. Which of the following is an outline of a correct deduction, given a quadrilateral ABCD such that both pairs of opposite sides are congruent?
    A. $\measuredangle DCA \cong \measuredangle BAC$ and $\measuredangle BCA \cong \measuredangle DAC$ by the Z Property and $\triangle ABC \cong \triangle ADC$ by AAS Triangle Congruence Theorem
    B. $\measuredangle DCA \cong \measuredangle BAC$ and $\measuredangle BCA \cong \measuredangle DAC$ by the Z Property and $\triangle ABC \cong \triangle CDA$ by ASA Triangle Congruence Theorem
    C. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$ and $\overline{AC} \cong \overline{CA}$, thus $\triangle ABC \cong \triangle CDA$ by SSS Triangle Congruence Theorem
    D. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$ and $\overline{AC} \cong \overline{CA}$, thus $\triangle ABC \cong \triangle ADC$ by SSS Triangle Congruence Theorem
    E. Each of the other answers is incorrect.

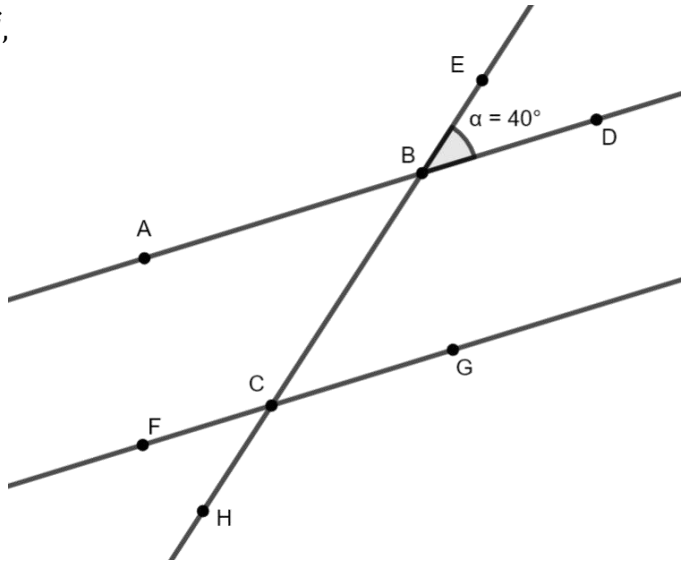
19. A quadrilateral with a pair of opposite interior angles congruent must be a _____.
    A. Rhombus
    B. Parallelogram
    C. Kite
    D. Square
    E. Rectangle

20. Graph the function $f(x) = |x|$. Which of the following is the best description of the graph?
    A. Right Angle
    B. Acute Angle
    C. Obtuse Angle
    D. Line
    E. Each of the other answer is incorrect.

21. How many lines of symmetry does a regular pentagon have?
    A. 0
    B. 1
    C. 5
    D. 10
    E. Each of the other answers is incorrect.

22. Which of the following can be directly applied to prove that the base angles of an isosceles triangle are congruent?
    A. SAS Postulate
    B. ASA Theorem
    C. SSS Theorem
    D. SSA Theorem
    E. Each of the other answers is incorrect.

23. In the beside diagram there are three lines with $\overleftrightarrow{AD} \parallel \overleftrightarrow{FG}$, and $m\measuredangle EBD = 40°$. What is $m\measuredangle FCB$?
    A. 40°
    B. 120°
    C. 140°
    D. It cannot be determined from this information.
    E. Each of the other answers is incorrect.



24. A regular polyhedron with pentagonal faces has how many faces?
    A. 6
    B. 8
    C. 12
    D. 20
    E. Each of the other answers is incorrect.

25. An isosceles trapezoid has legs of length 4 and bases of length 5 and 9. What is the length of the line segment with endpoints at the midpoints of the legs?
    A. 6
    B. 7
    C. 7.5
    D. It cannot be determined from this information.
    E. Each of the other answers is incorrect.

Tie Breaker 1

Name: _____

School: _____

Prove the following. Provide a sketch to accompany your proof.
The major diagonal of a kite bisects the interior angle at both ends.

Tie Breaker 2

Name: _____

School: _____

Prove the following. Provide a sketch to accompany your proof.

A rhombus is a parallelogram.

2022 Geometry Regional Competition

Tie Breaker 3

Name: _____

School: _____

Prove the following. Provide a sketch to accompany your proof.

A quadrilateral whose diagonals bisect each other is a parallelogram.

ACTM Regional Geometry Exam 2022 Solutions

Answers

1	**C**
2	**B**
3	**D**
4	**E**
5	**D**
6	**C**
7	**A**
8	**B**
9	**E**
10	**B**
11	**D**
12	**B**
13	**A**
14	**E**
15	**D**
16	**A**
17	**C**
18	**C**
19	**B**
20	**A**
21	**C**
22	**A**
23	**C**
24	**C**
25	**B**

Tie Breaker 1

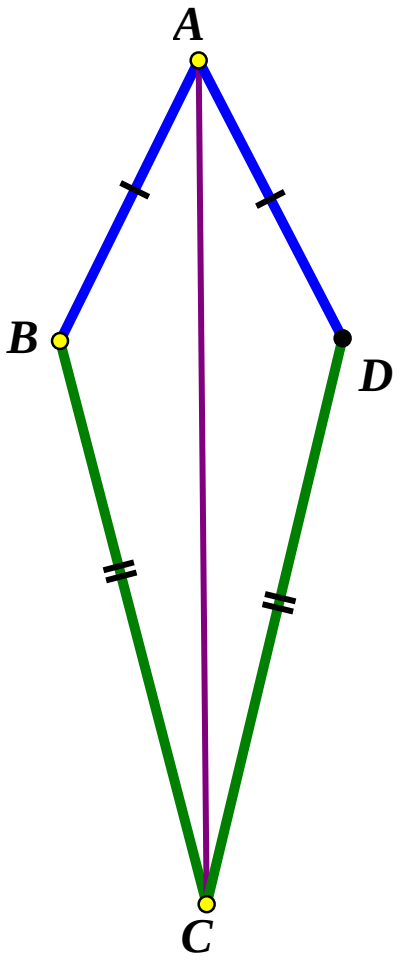
Name: **Key**

School: _____

Prove the following. Provide a sketch to accompany your proof.
The major diagonal of a kite bisects the interior angle at both ends.

Proof:

Let Quadrilateral be a kite. By the definition of a kite we may assume that AB = AD and BC =DC. Construct the major diagonal $\overline{AD}$. AD = AD. By the SSS Triangle Congruence Theorem we have $\Delta ABC \cong \Delta ADC$. By the definition of congruent triangles, corresponding parts of congruent triangles are congruent so: ∢CAB≅ ∢CAD and ∢ACB≅ ∢ACD. By the definition of bisecting an angle, the major diagonal bisects the interior angle at both ends.
■



Tie Breaker 2

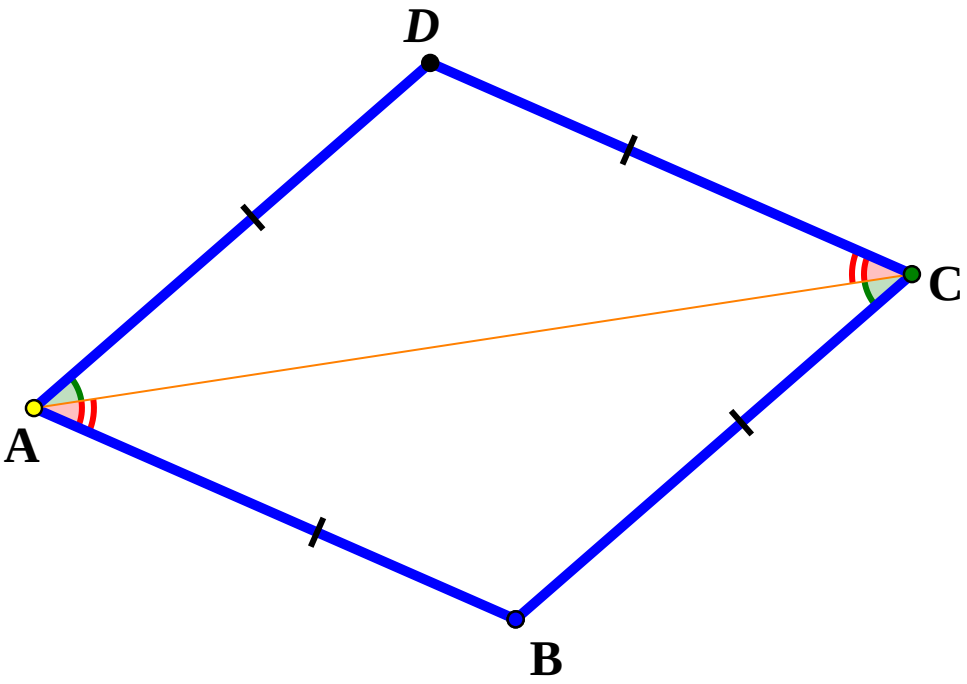
Name: **Key**

School: _____

Prove the following. Provide a sketch to accompany your proof.
A rhombus is a parallelogram.

Proof:

Let quadrilateral ABCD be a rhombus. By definition all of its sides are congruent, so AB = BC = CD = DA. Construct the diagonal $\overline{AC}$. AC = AC. By the SSS Triangle Congruence Theorem $\Delta ABC \cong \Delta CDA$. By the definition of congruent triangles, corresponding parts of congruent triangles are congruent so: ∢BAC≅ ∢DCA and ∢CAD≅ ∢ACB. Since ∢BAC≅ ∢DCA, the Z Property says that these congruent alternate interior angles make $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Similarly, since ∢CAD≅ ∢ACB the Z-Property says that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$. Since both pair of opposite sides of the rhombus are parallel, the rhombus is a parallelogram by definition.
■



Tie Breaker 3

Name: **Key**

School: _____

Prove the following. Provide a sketch to accompany your proof.
A quadrilateral whose diagonals bisect each other is a parallelogram.

Proof:

We are given Quadrilateral ABCD with diagonals that bisect each other. By definition, the diagonals are $\overline{AC}$ and $\overline{BD}$. By the definitions of a bisector of a line segment the intersection of the diagonals is a point X so that CX = AX and DX = BX. Since they are vertical angles, we know that ∢BXC ≅ ∢DXA. By the SAS Postulate we have $\Delta BXC \cong \Delta DXA$. By the definition of congruent triangles, corresponding parts of congruent triangles are congruent so: ∢BCX≅ ∢DAX and ∢CBX≅ ∢ADX. Since ∢BCX≅ ∢DAX is a pair of congruent alternate interior angles, the Z-Property tells us that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Similarly, since ∢CBX≅ ∢ADX is a pair of congruent alternate interior angles, the Z-Property tells us that $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$. Since both pair of opposite sides of the quadrilateral are parallel, the quadrilateral is a parallelogram by definition.
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