

Arkansas Council of Teachers of Mathematics

2020 Algebra II Regional Exam

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet and the tie-breaker pages when you are finished. You may keep the pages with the multiple-choice questions. Figures are not necessarily drawn to scale.

1. Find the term that should be added to the expression to form a perfect square trinomial:

$$x^2 + 18x + \underline{\hspace{2cm}}$$

- a. 9
- b. 18
- c. 36
- d. 81
- e. 324

2. Find the product of the polynomials: $(3y^2 + 3y - 5)(y^2 - 2y - 2)$

- a. $3y^4 - 6y^3 - 12y^2 + 4y + 10$
- b. $3y^4 - 3y^3 - 17y^2 + 4y + 10$
- c. $3y^4 - 3y^3 - 12y^2 + 4y + 10$
- d. $3y^4 - 6y^3 - 17y^2 + 4y + 10$
- e. $3y^4 + 6y^3 + 12y^2 + 4y + 10$

3. Find the zeros of the polynomial function and state the multiplicity of each.

$$f(x) = 3(x + 6)^2(x - 4)^3$$

- a. 0, multiplicity 3; 2, multiplicity 6; 3, multiplicity 4
- b. 3, multiplicity 1; -6, multiplicity 2; 4, multiplicity 3
- c. 3, multiplicity 1; 6, multiplicity 2; -4, multiplicity 3
- d. 6, multiplicity 2; -4, multiplicity 3
- e. -6, multiplicity 2; 4, multiplicity 3

4. A polynomial $P(x)$ and a divisor $d(x)$ are given. Identify the quotient $Q(x)$ and remainder $R(x)$ when $P(x)$ is divided by $d(x)$.

$$P(x) = 2x^4 - x^3 - 15x^2 + 3x$$

$$d(x) = x + 3$$

- a. $Q(x) = 2x^3 - 7x^2 + 6x - 15$; $R(x) = -45$
- b. $Q(x) = 2x^3 - 7x^2 + 6x - 15$; $R(x) = 45$
- c. $Q(x) = 2x^3 - 5x^2 + 3$; $R(x) = -9$
- d. $Q(x) = 2x^3 + 5x^2 + 3$; $R(x) = 9$
- e. $Q(x) = 2x^3 + 5x^2$; $R(x) = 3x$

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5. Jim wants to plan a meal with 82 grams of carbohydrates and 1260 calories. If green beans have 7 grams of carbohydrates and 30 calories per half cup serving and if fried shrimp have 9 grams of carbohydrates and 190 calories per three-ounce serving, how many servings of green beans and shrimp should he use?
- 4 half cups of beans and 6 three-ounce helpings of shrimp
 - 6 half cups of beans and 4 three-ounce helpings of shrimp
 - 7 half cups of beans and 9 three-ounce helpings of shrimp
 - 9 half cups of beans and 7 three-ounce helpings of shrimp
 - 12 half cups of beans and 0 three-ounce helpings of shrimp
6. What is the coefficient of the x^2y term in the expansion of $(3x - y)^3$?
- 27
 - 18
 - 9
 - 18
 - 27
7. Find the solutions to the quadratic equation $0.3x^2 + 0.7x = -0.5$.
- $\frac{-7 \pm \sqrt{11}}{3}$
 - $\frac{7 \pm \sqrt{11}}{3}$
 - $\frac{-7 \pm i\sqrt{11}}{6}$
 - $\frac{7 \pm i\sqrt{11}}{6}$
 - $\frac{-7 \pm i\sqrt{1.1}}{6}$
8. A model rocket is launched with an initial velocity of 100 ft/sec from the top of a hill that is 20 feet high. Its height, in feet, t seconds after it has been launched is given by the function $s(t) = -16t^2 + 100t + 20$. For what times is the rocket at a height of at least 168 ft?
- Between 0 and 2.41 seconds
 - Between 2.41 and 3.12 seconds
 - Between 3.12 and 3.84 seconds
 - Between 3.84 and 6.44 seconds
 - Between 2.41 and 3.84 seconds

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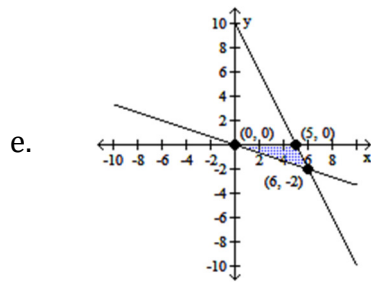
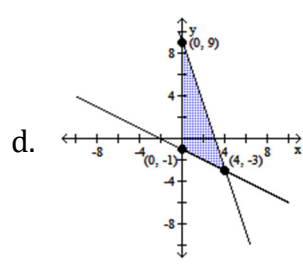
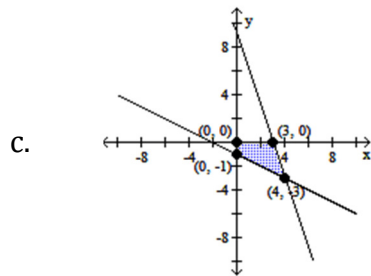
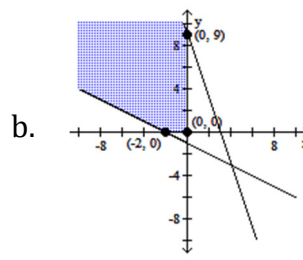
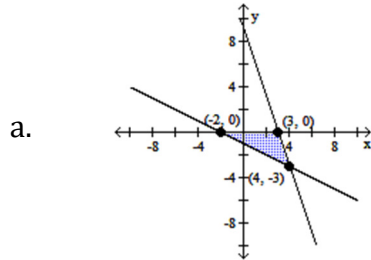
9. Choose the graph that represents the following system of inequalities.

$$2y + x \geq -2$$

$$y + 3x \leq 9$$

$$y \leq 0$$

$$x \geq 0$$

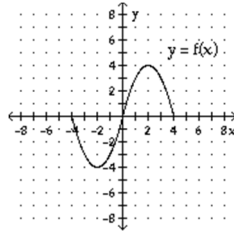


10. For the equation $kx^2 - 21x + 34 = 0$, one solution is 2. Find the second solution for this equation.

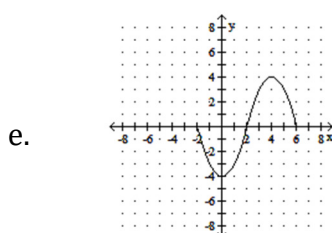
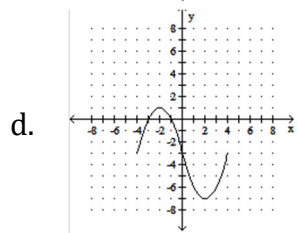
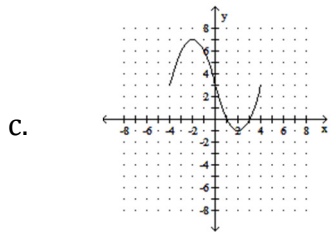
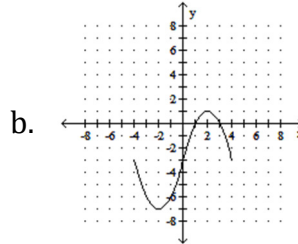
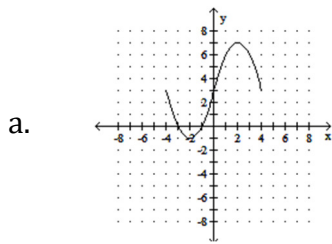
- a. -2
- b. $\frac{3}{2}$
- c. 2
- d. 5
- e. $\frac{17}{2}$

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11. The graph of $f(x)$ is shown below.



Use transformations to graph the function $g(x)$ if $g(x) = -f(-x) - 3$.



12. For the pair of functions, find $(g \circ f)(x)$.

$$f(x) = 4x^2 + 3x + 8$$

$$g(x) = 3x - 6$$

- a. $4x^2 + 9x + 18$
- b. $4x^2 + 3x + 2$
- c. $12x^2 + 9x + 30$
- d. $12x^2 + 9x + 18$
- e. $12x^2 + 3x + 2$

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13. Find the equation of a parabola with a focus at $(-6, 10)$ and a directrix of $x = 2$.

- a. $(x - 10)^2 = -16(y + 2)$
- b. $(x + 10)^2 = -16(y + 2)$
- c. $(y - 10)^2 = -16(x + 2)$
- d. $(y + 10)^2 = -16(x + 2)$
- e. $(y - 10)^2 = -16(x - 2)$

14. Which expression represents the solution of the equation $\frac{x}{y} = \frac{c}{a+b}$ solved for a ?

- a. $\frac{c}{b} - \frac{x}{y}$
- b. $\frac{yc}{a+b}$
- c. $\frac{yc}{x} + b$
- d. $\frac{xc - xb}{yc - xb}$
- e. $\frac{xc + xb}{x}$

15. Find the 4th term of the sequence (a_n) if the sequence is defined as $a_n = 2a_{n-1} + 3$ and $a_1 = 3$.

- a. 3
- b. 6
- c. 11
- d. 45
- e. 93

16. For matrices A, B, and C, determine which pair of expressions are not equal.

$$A = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 \\ -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

- a. $0 \cdot A$ and $A \cdot 0$
- b. AB and BA
- c. $A(BC)$ and $(AB)C$
- d. $A(B + C)$ and $AB + AC$
- e. $(B + C)A$ and $BA + CA$

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17. How can you write the three equations below as a matrix equation for a system?

$$2x - 3y + z + 10 = 0$$

$$x + 4y = 2z + 11$$

$$-2y + 3z + 7 = 3x$$

- a. $\begin{bmatrix} 2 & -3 & 10 \\ 4 & 2 & 11 \\ -2 & 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$
- b. $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & 2 \\ -2 & 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 3 \end{bmatrix}$
- c. $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ -3 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10 \\ 11 \\ -7 \end{bmatrix}$
- d. $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ 3 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 7 \end{bmatrix}$
- e. $\begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 4 \\ 2 & 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 7 \end{bmatrix}$

18. In a random sample of toner cartridges, the mean number of pages a toner cartridge can print is 4302 and the standard deviation is 340. Assuming that the distribution of the data are bell shaped, approximate the percentage of toner cartridges who print total is between 3622 and 3962 pages.

- a. 68%
- b. 47.5%
- c. 34%
- d. 15.85%
- e. 13.5%

19. Boat builders share an old rule of thumb for sailboats. The maximum speed K in knots is 1.35 times the square root of the length L in feet of the boat's waterline. How much longer would the waterline have to be to achieve a maximum speed of 15 knots compared to a maximum speed of 12 knots?

- a. About 0.55 ft
- b. About 5.47 ft
- c. About 29.54 ft
- d. About 44.44 ft
- e. About 79.01 ft

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20. Tatyana has $x + 2$ pens in the pocket of her backpack. Samuel has $2x - 1$ pens in the pocket of his backpack. Tatyana has 2 blue pens and Samuel has $x - 3$ blue pens. Find the probability that either Tatyana or Samuel pulls out a blue pen at random.

- a. $\frac{2}{x+2}$
- b. $\frac{x^2+3x-8}{2x^2+3x-2}$
- c. $\frac{x-3}{2x-1}$
- d. $\frac{x^2-x-6}{4x^2-2}$
- e. $\frac{x-1}{3x+1}$

21. Which pair of words make this sentence FALSE?

The product of any two _____(I) numbers is always a(n) _____ (II) number.

- a. (I) complex; (II) complex
- b. (I) complex; (II) real
- c. (I) irrational; (II) real
- d. (I) rational; (II) rational
- e. (I) rational; (II) real

22. What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?

- a. 2
- b. 3
- c. 4
- d. 5
- e. 6

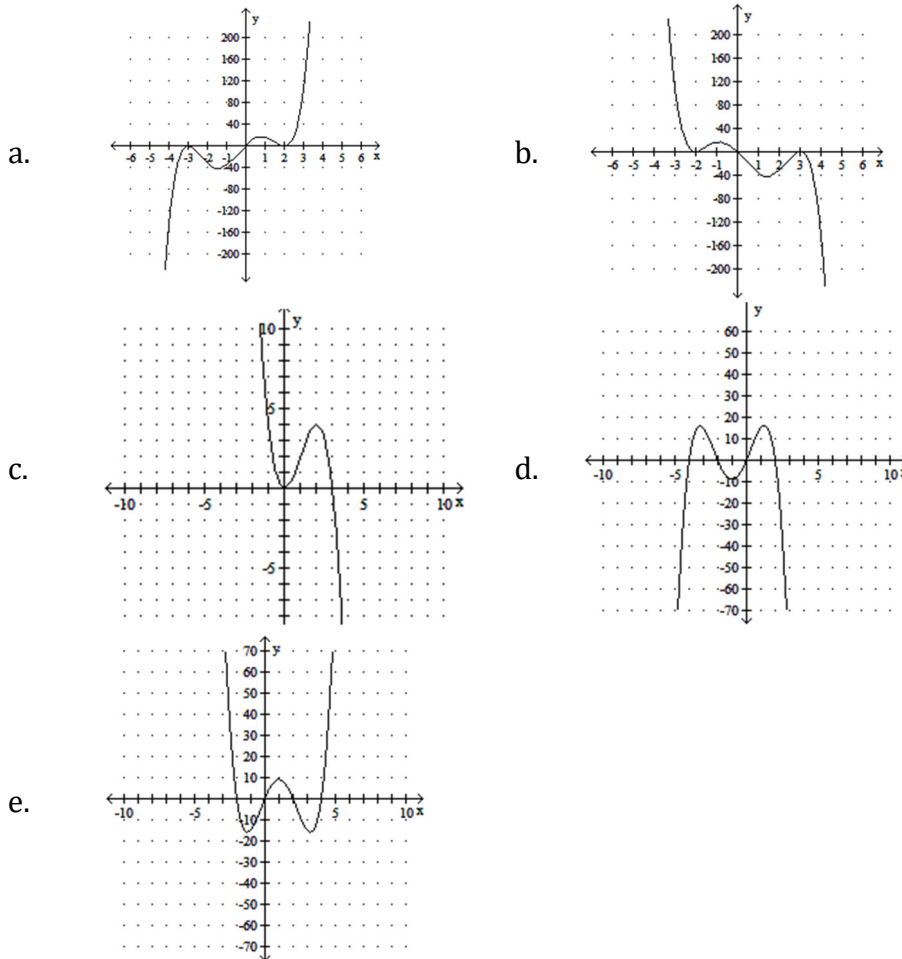
23. Using the data in the table below, use regression to determine which function model would be considered the best fit.

- a. Linear
- b. Quadratic
- c. Cubic
- d. Exponential
- e. Logarithmic

x	y
-3	0.1
-1	0.4
1	1.6
3	6.4
5	25.6
7	52.4

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24. Choose the graph that matches the following description: An odd degree polynomial with a negative leading coefficient and zeros at $x = -2, 0,$ and 3 .



25. The function $T(t) = T_r + (T_i - T_r)e^{kt}$ models Newton's Law of Cooling. $T(t)$ is the temperature of a heated substance t minutes after it has been removed from a heat (or cooling) source. T_i is the substance's initial temperature, k is a constant for that substance, and T_r is room temperature. Once removed from the oven, the initial surface temperature of a beef roast is 236°F and room temperature is 72°F . If $k = -0.041$, how long will it take for this roast to cool to 100°F ?

- About 4.57 minutes
- About 43.11 minutes
- About 52.68 minutes
- About 130.35 minutes
- About 3317.07 minutes

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Tie Breaker #1

Name: _____

School: _____

The equation $\sqrt{\sqrt{x+25}} = \sqrt{x+5}$ yields two solutions. Which one would be considered extraneous?

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Tie Breaker #2

Name: _____

School: _____

A berry farmer needs to separate and enclose two adjacent rectangular fields, one for blueberries and one for strawberries. A lake forms one side of the adjacent fields and requires no fencing.

If the farmer has 240 yd of fencing available, what is the largest total area that can be enclosed?

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Tie Breaker #3

Name: _____

School: _____

Baking a tray of corn muffins takes 4 cups of milk and 3 cups of wheat flour. Baking a tray of bran muffins takes 2 cups of milk and 3 cups of wheat flour. A baker has 16 cups of milk and 15 cups of wheat flour. He makes \$3 profit per tray of corn muffins and \$2 profit per tray of bran muffins. How many trays of each type of muffin should the baker make to maximize his profit?

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Answers

1. D
2. B
3. E
4. B
5. A
6. A
7. C
8. E
9. C
10. E
11. B
12. D
13. C
14. D
15. D
16. B
17. C
18. E
19. D
20. B
21. B
22. D
23. C
24. B
25. B

Tiebreaker #1 -9 would be the extraneous solution

Handwritten work showing the solution to a radical equation:

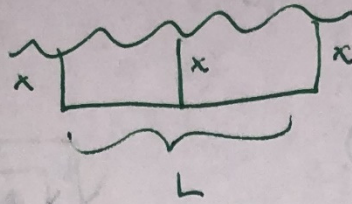
$$\sqrt{x+25} = x+5$$
$$x+25 = x^2+10x+25$$
$$0 = x^2+9x$$
$$x=0 \quad x=-9$$

The solution $x = -9$ is circled and labeled "is extraneous".

$x = 0$ works fine.

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Tiebreaker #2) 4800 yd^2



$$L + 3x = 240$$

$$L = 240 - 3x$$

$$A(x) = x(240 - 3x)$$

max @ $(40, 4800)$

max area: 4800 yd^2

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Tiebreaker #3) 3 trays of corn muffins and 2 trays of bran muffins using linear programming.

$$c = \text{corn} \\ b = \text{bran}$$

$$b \geq 0, c \geq 0$$

$$4c + 2b \leq 16 \rightarrow b \leq -2c + 8$$

$$3c + 3b \leq 15 \rightarrow b \leq -c + 5$$

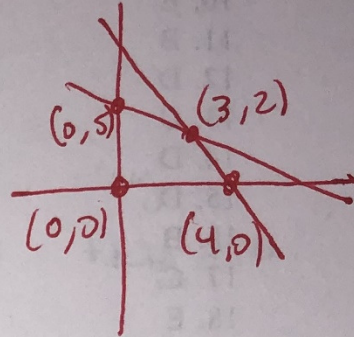
$$P(c, b) = 3c + 2b$$

$$P(0, 0) = 0$$

$$P(4, 0) = 12$$

$$P(0, 5) = 10$$

$$P(3, 2) = 9 + 4 = 13$$



3 corn muffins + 2 bran muffins