Arkansas Council of Teachers of Mathematics   Name _____
<u>**2022 Algebra II Regional Competition**</u>

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet, your tie-breaker pages, and your scratch work when you are finished. Figures are not necessarily drawn to scale.

1.   For what value of c will the graphs of $y = 2x^2 - 36x + c$ and $y = 2(x - 9)^2 - 8$ be the same?
   A. -180
   B.   63
   C.   99
   D.   144
   E.   None of the above

2.   The rectangular patio shown in the diagram beside is enclosed on three sides by a fence. The area of this patio is no more than 600 square feet. The total length of the three sides of this fence is no more than 40 feet. Which system of inequalities can be used to find values of l, the patio's length, and w, the patio's width, that will satisfy these conditions?
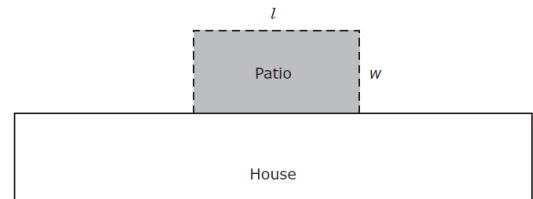
   A. $\begin{cases} l + 2w \le 40 \\ lw \le 600 \end{cases}$
   B. $\begin{cases} 2l + 2w \le 40 \\ lw \le 600 \end{cases}$
   C. $\begin{cases} l + 2w \ge 40 \\ lw \ge 600 \end{cases}$
   D. $\begin{cases} 2l + 2w \ge 40 \\ lw \ge 600 \end{cases}$
   E.   None of the above

3.   The cost of printing a book is $3.49 per book plus a one-time setup fee of $209. All taxes are included. Which of the following functions models $a(x)$, the average cost per book of printing x books?
   A. $a(x) = 3.49x + \dfrac{209}{x}$
   B. $a(x) = \dfrac{3.49x + 209}{x}$
   C. $a(x) = \dfrac{3.49 + 209}{3.49x}$
   D. $a(x) = \dfrac{3.49x + 209}{3.49}$
   E.   None of the above

4.   There were 417 cell phones sold at an electronics store in January. Since then, cell phone sales at this store have increased at a rate of 3.75% per month. At this rate of growth, which function can be used to determine the monthly cell phone sales m months after January?
   A. $p(m) = 417(.0375)^m$
   B. $p(m) = 417(.0375)^{m+1}$
   C. $p(m) = 417(.9625)^{m+1}$
   D. $p(m) = 417(1.0375)^m$
   E.   None of the above

5.   The graph of the function h was obtained from the graph of the function g using a composite transformation, as shown beside.
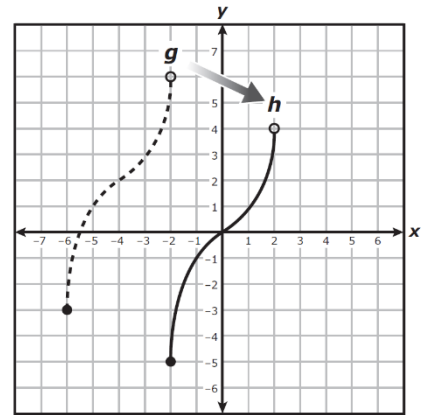
Which equation can be used to describe $h(x)$ in terms of $g(x)$?
A. $h(x) = g(x + 4) + 2$
B. $h(x) = g(x + 4) - 2$
C. $h(x) = g(x - 4) - 2$
D. $h(x) = g(x - 4) + 2$
E. None of the above

6.   The function $A = 36w - w^2$ can be used to model the area of a rectangle in square inches, A, if the rectangle has a perimeter of 72 inches and a width of w inches. In this situation, which of the following best describes the domain of the function?
A. $0 < w < 6$
B. $0 < w < 18$
C. $0 < w < 36$
D. $0 < w < 72$
E. None of the above

7.   What is the inverse of $h(x) = \frac{1}{2}\log_3(x)$?
A. $h^{-1}(x) = 3^{(2x)}$
B. $h^{-1}(x) = 2(3)^x$
C. $h^{-1}(x) = 2(\log_x 3)$
D. $h^{-1}(x) = \log_3(2x)$
E. None of the above

8.   An antibiotic is introduced into a colony of 12,000 bacteria during a laboratory experiment. The function below can be used to model the number of bacteria in the colony after m minutes.
$$n(m) = 12{,}000(0.851)^{m/10}$$
What is the amount of time needed for the population to drop to 8,000 bacteria?
A. 8 min
B. 11 min
C. 25 min
D. 42 min
E. None of the above

9.   The given function models the equivalent resistance in ohms, R, when a fixed 9-ohm resistor is connected in parallel with a variable resistor of x ohms.
$$R = \frac{9x}{9 + x}$$
Which value of x is closest to the number of ohms needed in the variable resistor for there to be an equivalent resistance, R, of 6.4 ohms?
A. 3.7
B. 15.4
C. 22.2
D. 57.6
E. None of the above

10. For the functions h and g, which statement is true if $h(x) = g(x + 14) - 14$?
     A. The graph of h is the result of the graph of g being translated right 14 units and down 14 units.
     B. The graph of h is the result of the graph of g being translated left 14 units and down 14 units.
     C. The graph of h is the result of the graph of g being translated right 14 units and up 14 units.
     D. The graph of h is the result of the graph of g being translated left 14 units and up 14 units.
     E. None of the above

11. Which equation has the same graph as $2x^2 - 4x - y + 11 = 0$?
     A. $y = (2x - 1)^2 + 9$
     B. $y = (2x - 1)^2 + 13$
     C. $y = 2(x - 1)^2 + 9$
     D. $y = 2(x - 1)^2 + 13$
     E. None of the above

12. The hypotenuse of a right triangle is 17 inches long. A leg of this triangle, y, is 1 inch less than twice the length of the other leg, x. Which system of equations can be used to determine the lengths of the 2 legs of this right triangle in inches?
     A. $\begin{cases} x^2 + y^2 = 289 \\ y = 2x - 1 \end{cases}$
     B. $\begin{cases} x^2 + y^2 = 289 \\ 2x + y = 1 \end{cases}$
     C. $\begin{cases} x + y = 17 \\ y = 2x - 1 \end{cases}$
     D. $\begin{cases} x + y = 17 \\ 2x + y = 1 \end{cases}$
     E. None of the above

13. What is the solution set for $x^2 + 4 = 6x$?
     A. $\{-3 \pm i\sqrt{14}\}$
     B. $\{-3 \pm \sqrt{5}\}$
     C. $\{3 \pm i\sqrt{14}\}$
     D. $\{3 \pm \sqrt{5}\}$
     E. None of the above

14. What is the solution set for the following equation? $3\left|1 - \frac{1}{4}x\right| = 9$
     A. $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$
     B. $\left\{-\frac{1}{2}, 1\right\}$
     C. $\{-8, 16\}$
     D. $\{-8, -16\}$
     E. No solution

15. Which function is equivalent to its inverse?
     A. $f(x) = \frac{2x-1}{2}$
     B. $g(x) = 7 - x$
     C. $h(x) = \frac{3-x}{3}$
     D. $j(x) = x - 4$
     E. None of the above

16. Rewrite using common logarithms: $\log_6 40$
    A. $\log_{10} 6 + \log_{10} 40$
    B. $\log_{10} 6 - \log_{10} 40$
    C. $(\log_{10} 6) \cdot (\log_{10} 40)$
    D. $\frac{\log_{10} 40}{\log_{10} 6}$
    E. None of the above

17. A rectangular parking lot measures 150 feet by 230 feet. The owner of the parking lot will expand the lot's size by adding x feet and $2x$ feet to its dimensions, as shown beside. If the total area of the parking lot cannot exceed 40,000 square feet, which inequality can be used to find all possible values of x?
    A. $(230 + 2x)(150 + x) \leq 40{,}000$
    B. $(230 + x)(150 + 2x) \leq 40{,}000$
    C. $(230 + 2x)(150 + x) \geq 40{,}000$
    D. $(230 + x)(150 + 2x) \geq 40{,}000$
    E. None of the above
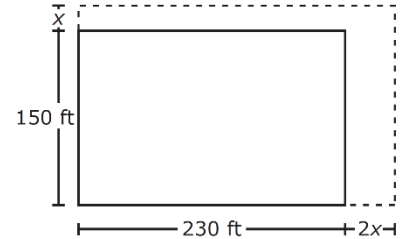
18. What is the solution to the system of equations shown below?
$$\begin{cases} 2x - y + 3z = 8 \\ x - 6y - z = 0 \\ -6x + 3y - 9z = 24 \end{cases}$$
    A. $(0, 4, 4)$
    B. $\left(1, 4, \frac{10}{3}\right)$
    C. $(8, 0, 24)$
    D. Infinitely many solutions
    E. No solution

19. The graph of $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 = 1$ is a hyperbola. Which set of equations represents the asymptotes of the hyperbola's graph?
    A. $y = \frac{3}{2}x, y = -\frac{3}{2}x$
    B. $y = \frac{2}{3}x, y = -\frac{2}{3}x$
    C. $y = \frac{1}{2}x, y = -\frac{1}{2}x$
    D. $y = \frac{1}{3}x, y = -\frac{1}{3}x$
    E. None of the above

20. Abel, Betty, Cain, and Drew each worked the same math problem at the chalkboard. Each student's work is shown. Their teacher said that while two of them had the correct answer, only one of them had arrived at the correct conclusion using the correct steps. Which is a completely correct solution?

    <u>Abel's work</u>: $x^3 x^{-7} = \frac{x^3}{x^{-7}} = x^{10}, x \neq 0$ <u>Cain's work</u>: $x^3 x^{-7} = \frac{x^3}{x^7} = \frac{1}{x^4}, x \neq 0$

    <u>Betty's work</u>: $x^3 x^{-7} = \frac{x^3}{x^{-7}} = x^{-4}, x \neq 0$ <u>Drew's work</u>: $x^3 x^{-7} = \frac{x^3}{x^7} = x^4, x \neq 0$

    A. Abel's work
    B. Betty's work
    C. Cain's work
    D. Drew's work
    E. The teacher is incorrect (Gasp!)

21. The average speed of sound in air at t degrees Celsius can be found using the function below. The speed, s, is measured in meters per second.
$$s = 20.1\sqrt{t + 273.1}$$
If sound is traveling in air at an average speed of 357 meters per second, which temperature is closest to value of t?
   A. 33°C
   B. 75°C
   C. 357°C
   D. 505°C
   E. None of the above

22. The sum of a number, n, and its square root can be represented by the equation $y = n + \sqrt{n}$. If $y = 20$ which of the following is true?
   A. $n = 4$
   B. $n = 16$
   C. $n = 4$ and $n = 5$
   D. $n = 16$ and $n = 25$
   E. None of the above

23. What is the sum of the infinite geometric series $\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \cdots$?
   A. 2
   B. 2.5
   C. 3
   D. 3.5
   E. None of the above

24. Which expression represents $f\big(g(x)\big)$ if $f(x) = x^2 - 1$ and $g(x) = x + 3$?
   A. $x^3 + 3x^2 - x - 3$
   B. $x^2 + 6x + 8$
   C. $x^2 + x + 2$
   D. $x^2 + 8$
   E. None of the above

25. A linear equation which has a slope perpendicular to $m = -\frac{3}{2}$ and x-intercept of 8 is:
   A. $2x - 3y = 16$
   B. $2x + 3y = -16$
   C. $3x + 2y = 24$
   D. $3x - 2y = -24$
   E. None of the above

Tie Breaker #1

Name: _____

School: _____

Determine a function that fits the criteria such that a quadratic has complex root $1 - 3i$ and passing through the point $(1, 2)$. Write the answer in standard or vertex form.

Tie Breaker #2

Name: _____

School: _____

Find the coefficient of $s^3 t^4$ in the expansion of $(s - 2t)^7$.

Tie Breaker #3

Name: _____

School: _____

Based on recent data, a logistic model for the population of Dallas, t years after 1900, is as follows:
$$P(t) = \frac{1{,}300{,}000}{1 + 22e^{-0.05054t}}$$
According to this model, in which year did the population reach 1 million? Justify your answer.

ANSWER KEY

1) E	11) C	21) E
2) A	12) A	22) B
3) B	13) D	23) C
4) D	14) C	24) B
5) C	15) B	25) A
6) C	16) D	
7) A	17) A	
8) C	18) E	
9) C	19) E	
10) B	20) C	

Tie Breaker Answers

1. One possible answer. Others are possible.

$$y = \frac{2}{9}x^2 - \frac{4}{9}x + \frac{20}{9} = \frac{2}{9}(x-1)^2 + 2$$

2. Several methods exist, but the solution should be this term:

$$s^7 - 14s^6t + 84s^5t^2 - 280s^4t^3 + \boxed{560s^3t^4} - 672s^2t^5 + 448st^6 - 128t^7$$

   The desired coefficient is 560.

3. Solve to find $t = 84.98$, which is about 1985.