

Arkansas Council of Teachers of Mathematics
2017 State Exam for Algebra II

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, then #3). Be sure that your name is printed on each of the tiebreaker pages.

- 1.) How many digits are in 2017^{2017} ?
- a.) **6666**
 - b.) 2017
 - c.) 9227
 - d.) 8
 - e.) Each of the other answers is incorrect.
- 2.) Simplify: $\log_3\left(\frac{1}{\sqrt[4]{729}}\right) =$
- a.) $-4/5$
 - b.) $4/5$
 - c.) **$-3/2$**
 - d.) $3/2$
 - e.) Each of the other answers is incorrect.
- 3.) Bob sells widgets. When the price is \$15 per widget, Bob sells 100 widgets. When the price is \$12 per widget, Bob sells 130 widgets. Assuming the relationship between the number of widgets sold and the price per widget is linear, find, R, the revenue produced by Bob's widgets sales, giving R as a function of w, the number of widgets sold.
- a.) $R = -10w + 250$
 - b.) $R = \frac{w-250}{10}$
 - c.) $R = w(-10p + 250)$
 - d.) **$R = \frac{250w - w^2}{10}$**
 - e.) Each of the other answers is incorrect.
- 4.) Find the sum of all the roots of the following equation.
- $$x^2 + x + 2 = 0$$
- a.) **-1**
 - b.) 1
 - c.) 3
 - d.) There are no roots.
 - e.) Each of the other answers is incorrect.

- 5.) If A is the area of a circle of radius, r , and C is the circumference, find a formula for the circumference as a function of the area.
- $C = 2\sqrt{A}/\pi$
 - $C = 2\pi\sqrt{A}$
 - $C = 2\sqrt{A/\pi}$
 - $C = 2\sqrt{A\pi}$**
 - Each of the other answers is incorrect.
- 6.) If $\log_a 2 = b$, $\log_a 3 = c$, and $\log_a 5 = d$, what is $\log_a 7.5$?
- cd/b
 - $c + d - b$**
 - $(c + d)/b$
 - $cd - b$
 - Each of the other answers is incorrect.
- 7.) A point, a , is a *fixed point* for a function, f , if $f(a) = a$. Find all fixed points of the function $f(x) = x^2 - x$.
- $x = 0, x = 2$**
 - $x = 0, x = 1$
 - $x = 1, x = 2$
 - $x = 0, x = 1, x = 2$
 - Each of the other answers is incorrect.
- 8.) A ball is dropped from the top of a building. If the ball has a height of $s(t) = -16t^2 + s_0$ at t seconds where s_0 is the initial height of the ball, how tall is the building if it takes 2.5 seconds for the ball to hit the ground?
- 75 feet
 - 40 feet
 - 200 feet
 - 100 feet**
 - Each of the other answers is incorrect.
- 9.) Consider $f(x) = 2.4 - 3.7 \sin(2.8x - 3.14)$. What is the range of f ?
- $[-3.7, 3.7]$
 - $[-1.3, 6.1]$**
 - $[-6.1, 1.3]$
 - $[-2.4, 2.4]$
 - Each of the other answers is incorrect.

- 10.) Identify any vertical asymptotes for the function $f(x) = \frac{2x^2-3x+1}{x^2-1}$
- a.) $x = -1$
 - b.) $x = 1$
 - c.) $x = -1$ and $x = 1$
 - d.) f has no vertical asymptotes.
 - e.) Each of the other answers is incorrect.
- 11.) If $\sin A = 4/7$ and A is not in quadrant one, then $\cos A = \underline{\hspace{2cm}}$.
- a.) $\frac{\sqrt{33}}{7}$
 - b.) $-\frac{\sqrt{33}}{7}$
 - c.) $\frac{3}{7}$
 - d.) $-\frac{3}{7}$
 - e.) Each of the other answers is incorrect.
- 12.) In a box are four red balls, five green balls, and two plaid balls. If two balls are pulled out of the box, what is the probability that the two balls are different colors?
- a.) **38/55**
 - b.) 87/121
 - c.) 19/55
 - d.) 38/121
 - e.) Each of the other answers is incorrect.
- 13.) Replace all the balls from #12 back in the box. Two balls are pulled from the box. What is the probability that neither ball is green?
- a.) 2/11
 - b.) 1/5
 - c.) **3/11**
 - d.) 12/55
 - e.) Each of the other answers is incorrect.
- 14.) The price of a jacket was \$150. The price was reduced 15% and then Mary bought the jacket. If the sales tax is 8.3%, how much did Mary pay for the jacket?
- a.) \$186.82
 - b.) \$158.18
 - c.) \$116.92
 - d.) **\$138.08**
 - e.) Each of the other answers is incorrect.

- 15.) A refrigerator's price was reduced 15%. What was the original price if the sale price was \$731? (round to the nearest dollar)
- a.) \$4873
 - b.) \$746
 - c.) \$841
 - d.) \$860**
 - e.) Each of the other answers is incorrect.
- 16.) Suppose $f(x) = 2x - 3$ and $g(x) = x^2 + 1$. Find $f \circ g(x)$.
- a.) $2x^3 - 3x^2 + 2x - 3$
 - b.) $2x^2 - 1$**
 - c.) $4x^2 - 12x + 10$
 - d.) $x^2 + 2x - 2$
 - e.) Each of the other answers is incorrect.
- 17.) Simplify $\left(\frac{3^{-1}x^4y^{-3}z^5}{6^{-2}x^2y^{-4}z^{-2}}\right)^{-1}$
- a.) $12^{-1}x^{-2}y^{-1}z^{-7}$**
 - b.) $2x^2yz^7$
 - c.) $12x^2y^{-1}z^3$
 - d.) $2^{-1}x^{-2}yz^{-7}$
 - e.) Each of the other answers is incorrect.
- 18.) A standard six-sided die is rolled four times. What is the probability we get at least one three?
- a.) $625/1296$
 - b.) $1/6$
 - c.) $671/1296$**
 - d.) $5/6$
 - e.) Each of the other answers is incorrect.
- 19.) If $f(x) = \sqrt{2x + 5}$ and $g(x) = \sqrt{9 - x}$, find the domain of $g \circ f(x)$.
- a.) $[-5/2, 38]$**
 - b.) $[-5/2, 9]$
 - c.) $(-\infty, 9]$
 - d.) All real numbers
 - e.) Each of the other answers is incorrect.

- 20.) If $A = \begin{pmatrix} 4 & 1 \\ 3 & 1 \end{pmatrix}$, find A^{-1} .
- a.) $A^{-1} = \begin{pmatrix} 4 & 1 \\ 3 & 1 \end{pmatrix}$
 - b.) $A^{-1} = \begin{pmatrix} 1/4 & 1 \\ 1/3 & 1 \end{pmatrix}$
 - c.) $A^{-1} = \begin{pmatrix} 1 & -1 \\ -3 & 4 \end{pmatrix}$
 - d.) $A^{-1} = \begin{pmatrix} -4 & 3 \\ 1 & -1 \end{pmatrix}$
 - e.) Each of the other answers is incorrect.
- 21.) You put \$5000 into an account paying 1.75% interest compounded quarterly. After three years you move all of the money into an account paying 2% interest compounded continuously. How much money would you have after the money has been in the second account for three years?
- a.) \$5596.85
 - b.) \$5594.72**
 - c.) \$5592.82
 - d.) \$5595.36
 - e.) Each of the other answers is incorrect.
- 22.) \$5000 is invested in an account that pays 2.5% interest compounded monthly. Rounding to the nearest hundredth, find the number of years it would take for the account to grow to \$7000.
- a.) 13.46 years
 - b.) 13.47 years**
 - c.) 11.20 years
 - d.) 11.25 years
 - e.) Each of the other answers is incorrect.
- 23.) Solve for x : $4^{2x-1} = 8^{3x+1}$
- a.) $x = -1$**
 - b.) $x = -2$
 - c.) $x = 3/4$
 - d.) $x = 4/3$
 - e.) Each of the other answers is incorrect.

24.) Consider the following system of equations.

$$\begin{cases} x + 2y - 3z = 4 \\ 2x - 4y - 3z = -5 \\ 3x - 2y - 6z = -1 \end{cases}$$

Choose the statement below which is true.

- a.) The system has no solution.
 - b.) The system has exactly one solution.
 - c.) The system has exactly three solutions.
 - d.) The system has infinitely many solutions.**
 - e.) Each of the other answers is incorrect.
- 25.) What is the coefficient of the x^5 term in the expansion of $(2 - 3x)^7$.
- a.) -5103
 - b.) 40824
 - c.) 5103
 - d.) 20412
 - e.) Each of the other answers is incorrect.**

Tie Breaker Questions

Name: _____

Reminder: Attempt the tiebreaker questions in sequential order (Do #1 first, followed by #2, then #3). Be sure that your name is printed on each of the tiebreaker pages.

Tiebreaker #1

Find the exact value of $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$.

Tiebreaker #2

Using algebraic methods only, find any points of intersection between the two hyperbolas given below.

$$xy = 1$$
$$x^2 - y^2 = 4$$

Tiebreaker #3

Find a polynomial function, $f(x)$, that satisfies all of the following conditions.

- x -intercepts $(-2, 0)$, $(-1, 0)$, and $(1, 0)$
- y -intercept $(0, -2)$
- for all $x > 0$, $f(x) \leq 0$

Tie Breaker Questions

Name: _____

Answer Key

- | | | | | |
|------|-------|-------|-------|-------|
| 1. a | 6. b | 11. b | 16. b | 21. b |
| 2. c | 7. a | 12. a | 17. a | 22. b |
| 3. d | 8. d | 13. c | 18. c | 23. a |
| 4. a | 9. b | 14. d | 19. a | 24. d |
| 5. d | 10. a | 15. d | 20. c | 25. e |

Tiebreaker #1

Find the exact value of $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$.

Answer = $\sqrt{2} - 1$

Solution - If $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}} = x$, then $x = 1/(2 + x)$ and $x^2 + 2x = 1$. Quadratic formula gives $x = \sqrt{2} - 1$ and $x = -\sqrt{2} - 1$.

Since the expression is obviously positive, the latter result is discarded.

Tiebreaker #2

Using algebraic means only, find any points of intersection between the two hyperbolas given below.

$$xy = 1$$

$$x^2 - y^2 = 4$$

Solution - From the first equation, $y = 1/x$. Substituting into the second gives $x^2 - 1/x^2 = 4$. Multiplying through by x^2 gives $x^4 - 4x^2 - 1 = 0$. Let $t = x^2$. Then $t^2 - 4t - 1 = 0$. Quadratic formula gives $x^2 = t = 2 \pm \sqrt{5}$. Since t cannot be negative, we have $x^2 = 2 + \sqrt{5}$. Thus $x = \pm\sqrt{2 + \sqrt{5}}$. So the points of intersection are

$$\left(\sqrt{2 + \sqrt{5}}, \frac{1}{\sqrt{2 + \sqrt{5}}}\right) \text{ and } \left(-\sqrt{2 + \sqrt{5}}, \frac{-1}{\sqrt{2 + \sqrt{5}}}\right).$$

Tiebreaker #3

Find a polynomial function, $f(x)$, that satisfies all of the following conditions.

- x-intercepts $(-2, 0)$, $(-1, 0)$, and $(1, 0)$
- y-intercept $(0, -2)$
- for all $x > 0$, $f(x) \leq 0$

Solution - Many solutions are possible. One possibility is

$$f(x) = -(x + 2)(x + 1)(x - 1)^2$$