

Arkansas Council of Teachers of Mathematics
2017 State Exam for Algebra I

For questions 1- 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2). Be sure that your name is printed on each of the tiebreaker pages.

- A dog eats 7 cans of food in 3 days. At this rate, how many cans of food does the dog eat in $3 + d$ days?
 - $7 + d$
 - $\frac{7}{3} + \frac{d}{3}$
 - $\frac{7}{3} + \frac{7}{3d}$
 - $7 + \frac{d}{3}$
 - $7 + \frac{7d}{3}$
- The formula for the surface area (A) of a rectangular solid is given by $A = 2lw + 2lh + 2wh$, where l represents length; w , width; and h , height. By doubling each of the dimensions (l , w , and h), the surface area will be multiplied by what factor?
 - 4
 - 6
 - 8
 - 12
 - 64
- If a , b , and c are positive integers such that $a^b = x$ and $c^b = y$, then $xy = ?$
 - ac^b
 - ac^{2b}
 - $(ac)^b$
 - $(ac)^{2b}$
 - $(ac)^{b^2}$
- $\frac{a}{\sqrt{2}} + \frac{b}{\sqrt{3}} = ?$
 - $\frac{a\sqrt{3}+b\sqrt{2}}{\sqrt{5}}$
 - $\frac{a\sqrt{3}+b\sqrt{2}}{\sqrt{6}}$
 - $\frac{a+b}{\sqrt{2}+\sqrt{3}}$
 - $\frac{a+b}{\sqrt{5}}$
 - $\frac{ab}{\sqrt{6}}$
- Given that $f(x) = 4x^2 + 3x - 24$ and $g(x) = 2x^2 + 12x - 6$, what is the product of the solutions for the function formed by $f(x) - g(x) = 0$?
 - 9
 - 4.5
 - 4
 - 9
 - Each of the answers is incorrect.
- Jalea has a camera that automatically takes pictures of hummingbirds visiting her hummingbird feeder. The camera takes 4 pictures on the first day and 10 pictures every day after that. How many pictures has her camera taken after 4 weeks?
 - 34
 - 44
 - 274
 - 284
 - Each of the answers is incorrect.
- Perform the indicated operation and simplify the expression: $\frac{7x-14}{6x+18} - \frac{4x-8}{3x-6}$
 - $\frac{3x-2}{6(x+3)}$
 - $\frac{-(x+38)}{6(x+3)}$
 - $\frac{3x-26}{6x-18}$
 - $\frac{-x+10}{6(x+3)}$
 - Each of the answers is incorrect.

8. The domain of $f(x) = \frac{1}{\sqrt[3]{x^2}} - 15$ is

- a. $(-\infty, \infty)$
- b. $(-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$
- c. $(-\infty, 0) \cup (0, \infty)$
- d. $(-\infty, -15) \cup (-15, 15) \cup (15, \infty)$
- e. $(0, \infty)$

9. Which of the following describes a true relationship between the functions

$$f(x) = (x - 3)^2 + 112 \quad \text{and} \quad g(x) = \frac{1}{2}x + 111?$$

- a. $f(x) = g(x)$ for exactly 2 values of x .
- b. $f(x) = g(x)$ for exactly 1 value of x .
- c. $f(x) < g(x)$ for all x .
- d. $f(x) > g(x)$ for all x .
- e. $f(x)$ is the inverse of $g(x)$.

10. A formula to estimate the monthly payment, p dollars, on a short-term loan is

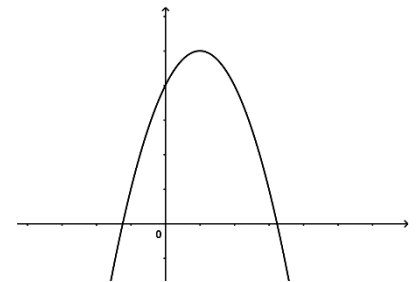
$$p = \frac{\frac{1}{2}ary + a}{12y}$$

where a dollars is the amount of the loan, r is the annual interest rate expressed as a decimal, and y years is the length of the loan. When a is multiplied by 2, what is the effect on p ?

- a. p is divided by 6
- b. p is divided by 2
- c. p does not change
- d. p is multiplied by 2
- e. p is multiplied by 4

11. The equation $y = ax^2 + bx + c$ is graphed in the standard (x, y) coordinate plane below for real values of a , b , and c . When $y = 0$, which of the following best describes the solutions for x ?

- a. 2 distinct positive real solutions
- b. 2 distinct negative real solutions
- c. 1 positive real solution and 1 negative real solution
- d. 2 real solutions that are not distinct
- e. 2 distinct solutions that are not real



12. Every minute, 2.2 gallons of water flow from a shower. A family of 5 people each shower for an average of $\frac{1}{6}$ of an hour every morning. How many gallons of water does the family use for showering each morning?

- a. 1.83 gallons
- b. 11 gallons
- c. 18.3 gallons
- d. 22 gallons
- e. 110 gallons

13. Which of the following best describes the domain for the radical expression $\sqrt[n]{x^n}$, where n is an odd positive integer?
- All real numbers
 - All non-negative real numbers
 - All positive integers
 - All positive real numbers
 - Each of the answers is incorrect.

14. Which of the following is the solution statement for the inequality shown below?

$$-1 < \frac{1}{5} - \frac{3}{5}x < 2$$

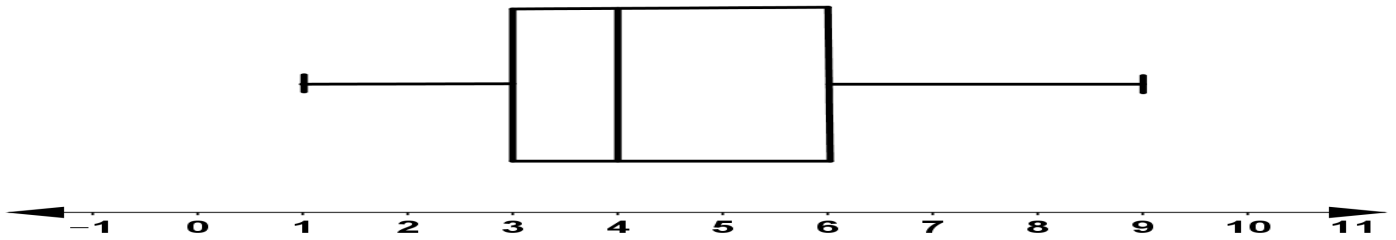
- $-5 < x < 10$
 - $-3 < x$
 - $-3 < x < 2$
 - $-2 < x < 3$
 - $x < -3$ or $x > -2$
15. Which statement about the graph of the equation $f(x) = -\left(\frac{1}{a}\right)^x$ is true for $0 < a < 1$?
- $f(x)$ is an increasing function with a range of $(0, \infty)$.
 - $f(x)$ is a decreasing function with a range of $(0, \infty)$.
 - $f(x)$ is an increasing function with a range of $(-\infty, 0)$.
 - $f(x)$ is a decreasing function with a range of $(-\infty, 0)$.
 - None of the statements are true.
16. Before driving to her grandmother's house, Paula buys 10 gallons of gasoline for \$23.00. She drives 300 miles over the course of 3 hours. At the end of each hour, she records how many miles she has driven. In a graph of her distance over time, what would be the independent variable?
- Dollars
 - Gallons
 - Hours
 - Miles
 - Each of the answers is incorrect.

17. Sarah is comparing the graphs of $f(x) = (a + 1)^x$ and $g(x) = (2a + 1)^x$ for $a > 0$.

Consider the following statements:

- The y -intercept of $f(x)$ is $(0, a + 1)$ and the y -intercept of $g(x)$ is $(0, 2a + 1)$.
 - For $x > 0$, the graph of $g(x)$ is steeper than the graph of $f(x)$.
 - The horizontal asymptote for the graphs of both $f(x)$ and $g(x)$ is $y = 1$.
- Only statements (i) and (ii) are true.
 - Only statement (ii) is true.
 - Only statements (ii) and (iii) are true.
 - All three statements are true.
 - None of the statements are true.

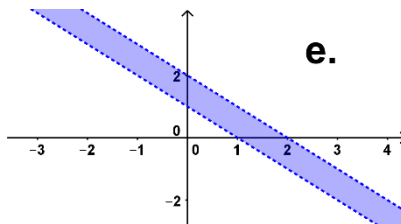
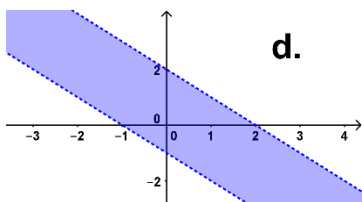
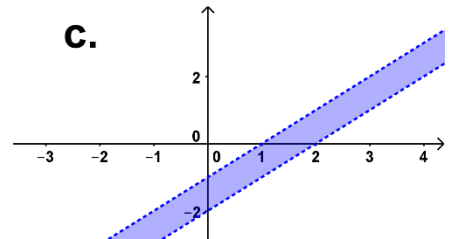
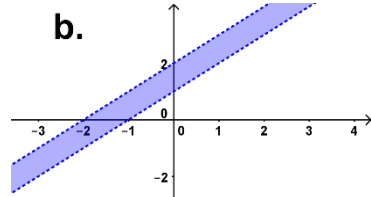
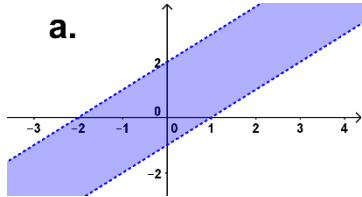
18. A movie theater recorded the number of tickets sold daily for a popular movie during the month of June. The box-and-whisker plot shown below represents the data for the number of tickets sold, in hundreds.



Which conclusion can be made using this plot?

- a. The second quartile is 600.
- b. The mean of the attendance is 400.
- c. The range of the attendance is 300
- d. Twenty-five percent of the attendance is between 300 and 400.
- e. None of the above statements are true.

19. Which of the following is the graph of the region $1 < x + y < 2$ in the standard (x, y) coordinate plane?



20. The size of the computer monitor is given by the length of the diagonal of the screen. The ratio of the height to the width of wide-screen monitors is 9:16. Find expressions for the width and height of a wide-screen monitor in terms of the length of its diagonal.

- a. $W = \frac{16}{\sqrt{337}}d$, $H = \frac{9}{\sqrt{337}}d$
- b. $W = \frac{9}{\sqrt{337}}d$, $H = \frac{16}{\sqrt{337}}d$
- c. $W = \frac{9}{25}d$, $H = \frac{16}{25}d$
- d. $W = \frac{16}{25}d$, $H = \frac{9}{25}d$
- e. Each of the answers is incorrect.

21. A ball is thrown into the air from a height of 1500 feet with an upward velocity of 48 feet/second. Its height h in feet above the ground after t seconds is given by the function $h(t) = -16t^2 + 48t + 1500$. How long does it take the ball to reach the ground? Round to the nearest tenth of a second, if necessary.
- a. 1.5 secs b. 5.6 secs c. 11.3 secs d. 1500 secs e. Each of the answers is incorrect.
22. Using the principles of translation/transformation, describe how the function $f(x) = \frac{3}{2}(x - 11.7)^2 + 11.6$, can be obtained from $g(x) = x^2$.
- a. Shift it horizontally 11.6 units to the left. Shrink it vertically by a factor of $\frac{3}{2}$. Shift it vertically 11.7 units upward.
- b. Shift it horizontally 11.6 units to the right. Stretch it vertically by a factor of $\frac{3}{2}$. Shift it vertically 11.7 units upward.
- c. Shift it horizontally 11.7 units to the right. Shrink it vertically by a factor of $\frac{3}{2}$. Shift it vertically 11.6 units upward.
- d. Shift it horizontally 11.7 units to the right. Stretch it vertically by a factor of $\frac{3}{2}$. Shift it vertically 11.6 units upward.
- e. Each of the answers is incorrect.
23. If $xy = 1$ and x is greater than 0, which of the following statements is true?
- a. When x is greater than 1, y is negative.
- b. When x is greater than 1, y is greater than 1.
- c. When x is less than 1, y is less than 1.
- d. As x increases, y increases.
- e. As x increases, y decreases.

24. The number of branches on a large tree after the year 2000 is represented by the following table:

Time (Years)	Branches
0	16
2	23
4	33
6	48
8	69
10	99

Which model for $B(t)$, the number of branches t years after the year 2000, best fits the data?

- a. $B(t) = 7.4t + 8.1$
- b. $B(t) = 8.1t + 7.4$
- c. $B(t) = 16t^{1.2}$
- d. $B(t) = 1.2(16)^t$
- e. $B(t) = 16(1.2)^t$
25. If x^2 is added to x , the sum is 42. What is the sum of the solutions?
- a. -13 b. -1 c. 1 d. 13 e. Each of the answers is incorrect.

Tie Breaker Questions

Name: _____

Reminder: Attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2). Be sure that your name is printed on each of the tiebreaker pages.

1. The freezing points of water in degrees Fahrenheit and Celsius are 32 and 0, respectively. The boiling points of water in degrees Fahrenheit and Celsius are 212 and 100, respectively.
 - a. Find the linear equation for converting degrees Celsius to degrees Fahrenheit. Use F to represent degrees Fahrenheit and use C to represent degrees Celsius. Show all work, even if you know the formula.
 - b. Find the linear equation for converting degrees Fahrenheit to degrees Celsius. Use F to represent degrees Fahrenheit and use C to represent degrees Celsius.
 - c. Check given values to see if the equations in a. and b. are correct.
 - d. A meteorologist in the Upper Peninsula of Michigan predicts an overnight low of -10°F . What would a Canadian meteorologist predict for the same location in Celsius?
 - e. Find the temperature at which the Celsius and Fahrenheit scales coincide.

Tie Breaker Questions

Name: _____

2. If \$2.00 had been placed in a bank account in year 0 and forgotten until now, how much would be in the account at the end of 2017 if the money earned

a. 2% simple interest?

b. 2% interest compounded annually? Do not leave answers in Scientific Notation. Place commas appropriately.

c. 2% interest compounded continuously?

- d. A formula for the future value of an annuity S of n payments of R dollars each at the end of each consecutive

interest period, with interest compounded at a rate r per period m is $S = R \left[\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}} \right]$. If \$2 deposits

had been placed in a bank account annually since the year 0, how much would be in the account at the end of 2017 if the money earned 2% interest compounded annually? Do not leave answers in Scientific Notation. Place commas appropriately. For annual payments, $m = 1$.

- e. The formula in part d can be used to determine the amount of money necessary to periodically invest at a given interest rate to reach a particular goal. Derive the formula for a Sinking Fund Payment by solving for R in the future value annuity formula above.
- f. Using the formula from part e, calculate the dollar amount that should have been deposited each year since year 0 to reach the savings goal found in part b by the end of this year if the account earns 2% interest compounded annually. Round to the nearest cent.

Answer Key

1. E	6. C	11. C	16. C	21. C
2. A	7. B	12. E	17. B	22. D
3. C	8. C	13. A	18. D	23. E
4. B	9. A	14. C	19. E	24. E
5. A	10. D	15. D	20. A	25. B

Solution to Tie Breaker 1.

The freezing points of water in degrees Fahrenheit and Celsius are 32 and 0, respectively. The boiling points of water in degrees Fahrenheit and Celsius are 212 and 100, respectively.

- a. Find the linear equation for converting degrees Celsius to degrees Fahrenheit. Use F to represent degrees Fahrenheit and use C to represent degrees Celsius. Show all work, even if you know the formula.

Since the Fahrenheit temperature will *depend* on the Celsius temperature, we will set our points as (C, F) where the Celsius temperature is the *independent* variable.

$$\begin{array}{ll}
 \text{(C, F)} & y - y_1 = m(x - x_1) \\
 (0, 32) & m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5} \quad y - 32 = m(x - 0) \\
 (100, 212) & y = \frac{9}{5}x + 32 \\
 & F = \frac{9}{5}C + 32
 \end{array}$$

- b. Find the linear equation for converting degrees Fahrenheit to degrees Celsius. Use F to represent degrees Fahrenheit and use C to represent degrees Celsius.

$$F = \frac{9}{5}C + 32 \Rightarrow F - 32 = \frac{9}{5}C \Rightarrow \frac{5}{9}(F - 32) = \frac{9}{5}C * \frac{5}{9} \Rightarrow \frac{5}{9}(F - 32) = C$$

- c. Check given values to see if the equations in a. and b. are correct.

degrees Celsius to degrees Fahrenheit

$$F(C) = \frac{9}{5}C + 32$$

$$F(0) = \frac{9}{5}(0) + 32 = 32$$

$$F(100) = \frac{9}{5}(100) + 32 = 212$$

degrees Fahrenheit to degrees Celsius

$$C(F) = \frac{5}{9}(F - 32)$$

$$C(32) = \frac{5}{9}(32 - 32) = 0$$

$$C(212) = \frac{5}{9}(212 - 32) = 100$$

- d. A meteorologist in the Upper Peninsula of Michigan predicts an overnight low of -10°F . What would a Canadian meteorologist predict for the same location in Celsius?

Since degrees Celsius depends on the degrees Fahrenheit in this situation,

$$C(F) = \frac{5}{9}(F - 32)$$

$$C(-10) = \frac{5}{9}(-10 - 32) = -23.3$$

A temperature of -10°F is equivalent to -23°C .

- e. Find the temperature at which the Celsius and Fahrenheit scales coincide.

Use either equation.

$$C(F) = \frac{5}{9}(F - 32)$$

or

$$F(C) = \frac{9}{5}C + 32$$

Let $F = C$

Let $C = F$.

$$C(C) = \frac{5}{9}(C - 32)$$

$$F(F) = \frac{9}{5}F + 32$$

$$C = \frac{5}{9}C - \frac{5}{9} * 32$$

$$F = \frac{9}{5}F + 32$$

$$C - \frac{5}{9}C = -\frac{160}{9}$$

$$-\frac{9}{5}F + F = 32$$

$$\frac{4}{9}C = -\frac{160}{9}$$

$$-\frac{4}{5}F = 32$$

$$C = -\frac{160}{9} * \frac{9}{4}$$

$$F = 32 * \left(-\frac{5}{4}\right)$$

$$C = -40$$

$$F = -40$$

-40°F is equivalent to -40°C

Solution to Tie Breaker 2.

If \$2.00 had been placed in a bank account in year 0 and forgotten until now, how much would be in the account at the end of 2017 if the money earned

$$A = P(1 + rt)$$

- a. 2% simple interest?

$$A = 2(1 + (0.02)(2017))$$

$$A = 82.68$$

For an account earning 2% simple interest, the accumulated amount after 2017 years would be \$82.68.

- b. 2% interest compounded annually? Do not leave answers in Scientific Notation. Place commas appropriately.

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

$$A = 2 \left(1 + \frac{0.02}{1} \right)^{(1)(2017)}$$

$$A = 4.44197837E17$$

For an account earning 2% interest compounded annually, the accumulated amount after 2017 years would be approximately \$444,197,837,000,000,000.

- c. 2% interest compounded continuously?

$$A = Pe^{rt}$$

$$A = 2e^{(0.02)(2017)}$$

$$A = 6.61407927E17$$

For an account earning 2% interest compounded continuously, the accumulated amount after 2017 years would be approximately \$661,407,927,000,000,000.

- d. A formula for the future value of an annuity S of n payments of R dollars each at the end of each consecutive interest period, with interest compounded at a rate r per period m is

$$S = R \left[\frac{\left(1 + \frac{r}{m} \right)^n - 1}{\frac{r}{m}} \right]. \text{ If } \$2 \text{ deposits had been placed in a bank account annually since the year 0,}$$

how much would be in the account at the end of 2017 if the money earned 2% interest compounded annually? Do not leave answers in Scientific Notation. Place commas appropriately. For annual payments, $m = 1$.

$$S = R \left[\frac{\left(1 + \frac{r}{m} \right)^n - 1}{\frac{r}{m}} \right] = 2 \left[\frac{\left(1 + \frac{0.02}{1} \right)^{2017} - 1}{\frac{0.02}{1}} \right] = 2.220989185E19$$

For a future value of annuity account earning 2% interest compounded annually, where \$2 is deposited each year, the accumulated amount after 2017 years would be approximately \$22,209,891,850,000,000,000.

- e. The formula in part d can be used to determine the amount of money necessary to periodically invest at a given interest rate to reach a particular goal. Derive the formula for a Sinking Fund Payment by solving for R in the future value annuity formula above.

$$S = R \left[\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}} \right] \Rightarrow S * \frac{\frac{r}{m}}{\left(1 + \frac{r}{m}\right)^n - 1} = R \left[\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}} \right] * \frac{\frac{r}{m}}{\left(1 + \frac{r}{m}\right)^n - 1} \Rightarrow$$

$$R = S * \frac{\frac{r}{m}}{\left(1 + \frac{r}{m}\right)^n - 1}$$

- g. Using the formula from part e, calculate the dollar amount that should have been deposited each year since year 0 to reach the savings goal found in part b by the end of this year if the account earns 2% interest compounded annually. Round to the nearest cent.

$$R = \frac{4.44197837 * 10^{17} \left(\frac{0.02}{1}\right)}{\left(1 + \frac{0.02}{1}\right)^{2017} - 1}$$

$$R = 0.04$$

If four cents had been placed in a sinking fund every year since year 0, the accumulated amount after 2017 years would be approximately \$444,197,837,000,000,000.