ACTM State Pre-Calculus/Trig Competition 2018

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Angles are given in radians unless otherwise stated.

1. Divide $3 - 2i$ by $2 + 3i$.
   a. $-i$
   b. $\frac{12-13i}{13}$
   c. $\frac{12+13i}{13}$
   d. None of the above

2. Which are polar coordinates of the point with rectangular coordinates $(-1, 1)$?
   a. $\left(\sqrt{2}, \frac{3\pi}{4}\right)$
   b. $\left(\sqrt{2}, \frac{\pi}{4}\right)$
   c. $\left(2, -\frac{3\pi}{4}\right)$
   d. $\left(2, -\frac{\pi}{4}\right)$

3. Find the inverse function of $f(x) = a^{2x} + 1$. Assume $a > 0$.
   a. $f^{-1}(x) = a^{x/2} - 1$
   b. $f^{-1}(x) = \log_a \sqrt{x+1}$
   c. $f^{-1}(x) = \log_a (x-1)$
   d. $f^{-1}(x) = \log_a \sqrt{x-1}$

4. Let vector $\vec{u} = 2i + j$ and vector $\vec{v} = i - 3j$. Find $\|\vec{u} + \vec{v}\|$.
   a. 1
   b. 5
   c. $\sqrt{5}$
   d. $\sqrt{13}$

5. Find the vertex of the parabola $y = 2x^2 - 8x + 14$
   a. $(2, 6)$
   b. $(-2, 6)$
   c. $(2, -6)$
   d. $(-2, -6)$

6. What is the solution set to the inequality $|9x - 1| > 5$?
   a. $\left\{x : x < -\frac{4}{9}\right\}$
   b. $\left\{x : x > \frac{2}{3}\right\}$
   c. $\left\{x : -\frac{4}{9} < x < \frac{2}{3}\right\}$
   d. $\left\{x : x < -\frac{4}{9} \text{ or } x > \frac{2}{3}\right\}$

7. Suppose f is an odd function. What is an equivalent expression for $(f \circ f)(-x)$?
   a. $f(f(x))$
   b. $-f(f(x))$
   c. $-f(f(-x))$
   d. None of these

8. How long does it take your money to double if you invested in an account that earns 9% annual interest compounded continuously? Round to the nearest month.
   a. 264
   b. 92
   c. 22
   d. 8

9. Simplify the expression:

$$\frac{n!\,(n+2)!}{\left((n+1)!\right)^2}$$

   a. $\frac{n+2}{n+1}$
   b. $\frac{n(n+2)}{(n+1)^2}$
   c. $\frac{n}{n+1}$
   d. $\frac{n+2}{n(n+1)}$

10. Find the sum: $-17 - 9 - 1 + 7 + \cdots + 79$
   a. 310
   b. 324
   c. 403
   d. 420

11. What is the smallest angle, to the nearest degree, in a triangle with sides of length 20, 21, & 29?
   a. 35°
   b. 36°
   c. 44°
   d. 47°

12. Find the exact value of $\log_\pi a \times \log_a(\cos^{-1}(-1))$. Assume $a > 0$.
   a. a
   b. 0
   c. 1
   d. Undefined

13. What is the coefficient of x^7y^8 when $(x+y)^{15}$ expanded?
   a. 15
   b. 6435
   c. 32432400
   d. 259459200

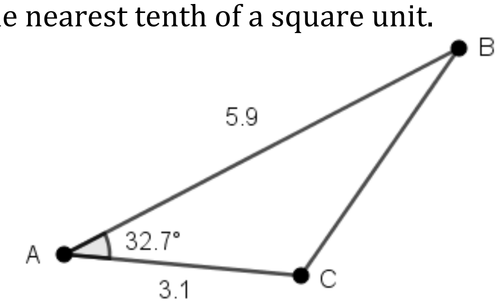
14. What is the domain of $f(x) = \sin^{-1}(3x+1)$?
    a. $[-1,1]$
    b. $\left[-\frac{2}{3}, 0\right]$
    c. All real numbers
    d. None of these

15. Determine the extraneous solution of the equation $\log_6(x-2) = 1 - \log_6(x-1)$.
    a. $\{4\}$
    b. $\{-1, 4\}$
    c. $\{-1\}$
    d. None of these

16. Find the solution set of the equation $x^3 - x^2 - 7x + 15 = 0$.
    a. $\{-3\}$
    b. $\{3, -3\}$
    c. $\{3, 2+i, 2-i\}$
    d. $\{-3, 2+i, 2-i\}$

17. Find the inverse function of $f(x) = \frac{2x+1}{2x-1}$.
    a. $f^{-1}(x) = \frac{2x-1}{2x+1}$
    b. $f^{-1}(x) = \frac{x+1}{x-1}$
    c. $f^{-1}(x) = \frac{x+1}{2x-1}$
    d. $f^{-1}(x) = \frac{x+1}{2(x-1)}$

18. Find the area of ΔABC with the dimensions given. Round to the nearest tenth of a square unit.
    a. $A = 4.9\ units^2$
    b. $A = 7.7\ units^2$
    c. $A = 9.1\ units^2$
    d. $A = 15.4\ units^2$



19. Find the exact value of $\cos\left(\frac{\sin^{-1}\left(-\frac{1}{5}\right)}{2}\right)$:
    a. $-\frac{\sqrt{10(5+2\sqrt{6})}}{10}$
    b. $\frac{\sqrt{10(5+\sqrt{26})}}{10}$
    c. $-\frac{\sqrt{10(5+\sqrt{26})}}{10}$
    d. $\frac{\sqrt{10(5+2\sqrt{6})}}{10}$

20. Let $u > 0$, express $\cot(\csc^{-1} u)$ in terms of u.
    a. $u - 1$
    b. $\sqrt{u^2 - 1}$
    c. $u + 1$
    d. $\sqrt{u^2 + 1}$

21. Find the length of side a in a triangle with sides $b = 8$, $c = 9$, and angle $A = 28°$, where side a is opposite of angle A. Round to the nearest tenth of a unit.
    a. 4.2
    b. 16.5
    c. 17.9
    d. 272.1

22. Simplify $\frac{(1-i)^{5020}}{2^{2510}}$.
    a. $1 - i$
    b. $-1 + i$
    c. $-i$
    d. -1

23. What is the domain of the inverse function for $f(x) = \log_{\pi/2}(\csc^{-1} x)$?
    a. $(-\infty, 1]$
    b. $[-1,1]$
    c. $[-1,0) \cup (0,1]$
    d. $[-1, \infty)$

24. If $\tan\theta = t$, express $\sin 2\theta$ in terms of t.
    a. $\frac{2t}{1+t^2}$
    b. $\frac{2t}{1-t^2}$
    c. $2t$
    d. None of these

25. Consider the functions: $f(x) = x + 1$, $g(x) = 2 - x^2$. Evaluate $(f \circ g \circ f)(1)$
    a. 1
    b. -4
    c. -1
    d. 2

ACTM State Pre-Calculus/Trig Competition 2018

Tiebreaker Question 1

Name ___________________

School ___________________

Solve the triangle $b = 7, c = 8, B = 17°$, where side a is opposite of angle A, side b is opposite of angle B, and side c is opposite of angle C. (Round your answers to one decimal place.)

ACTM State Pre-Calculus/Trig Competition 2018

Tiebreaker Question 2

Name ____________________

School ____________________

A boat leaves point A and travels 780 miles to another point B on a bearing of N43°E. the boat later leaves point B and travels to point C, 630 miles away on a bearing of S71°E. find the distance between the points A and C to the nearest mile.

ACTM State Pre-Calculus/Trig Competition 2018

Tiebreaker Question 3

Name ___________________

School ___________________

Given that the terminal side of angle θ lies in the 2nd quadrant, and that $\sin(\theta) = \frac{4}{5}$, evaluate the sum:

$$\sum_{n=1}^{\infty} \sin^n(2\theta)$$

ACTM State Pre-Calculus/Trig Competition 2018

Multiple Choice Answers

1)	A
2)	A
3)	D
4)	D
5)	A
6)	D
7)	B
8)	B
9)	A
10)	C
11)	C
12)	C
13)	B
14)	B
15)	C
16)	D
17)	D
18)	A
19)	D
20)	B
21)	A
22)	D
23)	A
24)	A
25)	C

Tie Breaker Answers

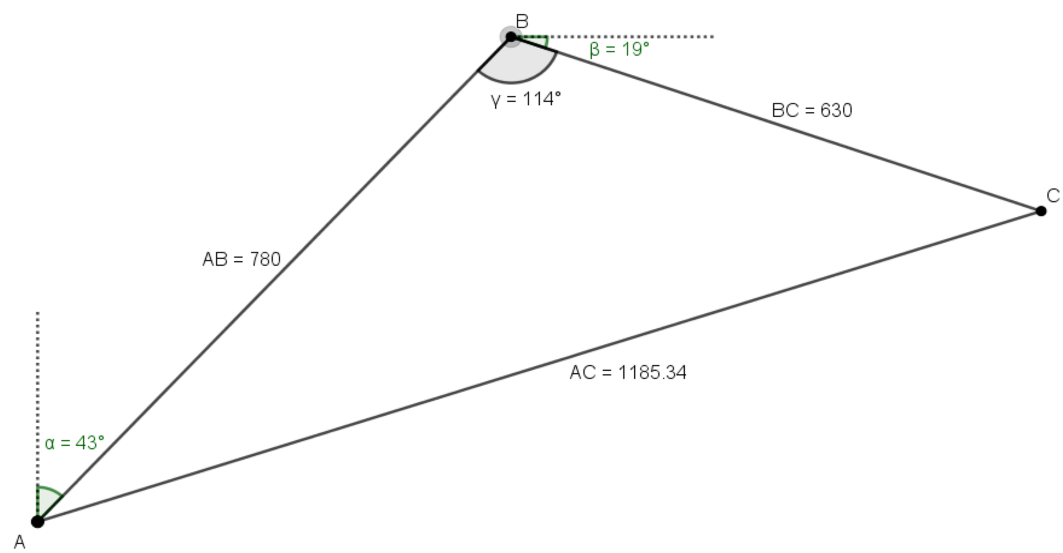
Tiebreaker 1: Solve problem using Law of Sines. The triangle described has a side-side-angle arrangement, which indicates that there are two different possible resulting triangles.

$$\frac{\sin C}{8} = \frac{\sin 17°}{7} \to C = \sin^{-1}\left(\frac{8 \cdot \sin 17°}{7}\right) = \begin{cases} C_1 \approx 19.5° \\ C_2 \approx 180 - 19.5 \approx 160.5° \end{cases}$$

Case 1: Use $C_1 = 19.5°$
$A_1 \approx 180 - 17 - 19.5 \approx 143.5°$
$$a_1 \approx \frac{7 \sin 143.5°}{\sin 17°} \approx 14.2$$

Case 2: Use $C_2 \approx 160.5°$
$A_2 \approx 180 - 17 - 160.5 \approx 2.5°$
$$a_2 \approx \frac{7 \sin 2.5°}{\sin 17°} \approx 1.1$$

Tiebreaker 2:
One possible method is to use the law of cosines.
Angle B is $43° + (90° - 19°) = 114°$.



$$AC = \sqrt{780^2 + 630^2 - 2 \cdot 780 \cdot 630 \cdot \cos 114°} \approx 1185 \text{ miles}$$

Tiebreaker 3:
Given that the angle is in Q2, $sin(t) = \frac{4}{5}$ and $cos(t) = \frac{-3}{5}$.
Note the identity $sin(2t) = 2\sin(t)\cos(t)$,

$$\sum_{n=1}^{\infty} \sin^n(2\theta) = \sum_{n=1}^{\infty} (\sin(2\theta))^n = \sum_{n=1}^{\infty} (2\sin(\theta)\cos(\theta))^n$$
$$= \sum_{n=1}^{\infty} \left(2 \cdot \frac{4}{5} \cdot \frac{-3}{5}\right)^n = \sum_{n=1}^{\infty} \left(\frac{-24}{25}\right)^n$$

Since $-1 < -\frac{24}{25} < 1$, this is a geometric series and the result converges.

$$\sum_{n=1}^{\infty} \left(\frac{-24}{25}\right)^n = \frac{1}{1 - \left(-\frac{24}{25}\right)} = \frac{25}{49}$$