#### -Directions-

This exam includes 25 multiple-choice questions and three open-response questions that might be used as tie breakers. For questions 1 through 25 (the multiple-choice questions), mark your answer choice in the appropriate location on the sheet provided. After completing questions 1 through 25, answer each tie breaker question in sequential order (i.e. complete Question #1 first, then Question #2, and then Question #3 last). Be sure that your name is printed on each of the tie break questions. When time is called, you will be asked to turn in your multiple-choice question answer sheet and your written responses to the tie breaker questions.

- 1. What percentage of observed measurements in a dataset fall above the third quartile?
  - a. 75%
  - b. 25%
  - c. 24%
  - d. Cannot Be Determined
- 2. The histogram to the right represents the number of reported deaths due to overdose for each state in the US for 2017. Using the histogram, determine the relationship between the mean and the median.
  - a. Mean = Median
  - b. Mean  $\approx$  Median
  - c. Mean < Median
  - d. Mean > Median



- 3. Refer to the histogram above. How many states reported more than 500 deaths in 2017? Please select the best answer of those provided below.
  - a. 20
  - b. 37
  - c. 50

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d. Cannot Be Determined

NAME: \_\_\_\_\_

For Questions 4–6, refer to the setting and data provided below.

While pregnant or nursing, the Environmental Protection Agency (EPA) recommends that women avoid consumption of certain fish, thought to contain high levels of mercury.

Two different varieties of Mackerel (Spanish and King) are listed as fish to limit or avoid altogether due to their high levels of mercury. While similar in appearance, it is believed that King Mackerel contain much higher levels of mercury than Spanish Mackerel. Due to this belief, intake of King Mackerel should be avoided entirely, rather than limited.

An experiment was conducted to see if the average mercury content (ppm) for King Mackerel is higher than that of Spanish Mackerel. To determine this five randomly selected King Mackerel and five randomly selected Spanish Mackerel were sampled and their mercury content (ppm) was recorded. Mercury contents are listed below for both groups of fish. In addition, a variable labeled 'Differences' is provided, which is the mercury content of the King Mackerel observation minus the mercury content of the Spanish Mackerel observation.

King Mackerel	0.709	0.947	0.555	0.712	0.675
Spanish Mackerel	0.453	0.212	0.758	0.405	0.355
Differences	0.256	0.735	-0.203	0.307	0.320

- 4. Which of the following statistical procedures would be most appropriate to test the claim that average mercury content (ppm) for King Mackerel is higher than that of Spanish Mackerel? Assume that any necessary normality requirements hold.
  - a. One-tailed two-sample paired/dependent t-test of means
  - b. One-tailed two-sample independent t-test of means
  - c. Two-tailed two-sample paired/dependent z-test of means
  - d. Two-tailed two-sample independent z-test of means
- 5. Referring to the setting and data provided above, what is the appropriate p-value for testing the claim that the average mercury content (ppm) for King Mackerel is higher than that of Spanish Mackerel? Round to 3 decimal places.
  - a. 0.065
  - b. 0.018
  - c. 0.002
  - d. 0.036

- 6. Refer to Questions 4-5. Using a 0.10 significance level, which of the following is the most appropriate conclusion for the hypothesis test given the results?
  - a. Reject the null hypothesis; there is sufficient evidence to suggest that the average mercury content for King Mackerel is higher than that of Spanish Mackerel.
  - b. Reject the null hypothesis; there is not sufficient evidence to suggest that the average mercury content for King Mackerel is higher than that of Spanish Mackerel.
  - c. Fail to reject the null hypothesis; there is sufficient evidence to suggest that the average mercury content for King Mackerel is higher than that of Spanish Mackerel.
  - d. Accept the null hypothesis; there is sufficient evidence to suggest that the average mercury content for King Mackerel is the same as that of Spanish Mackerel.
- 7. A survey on lifestyle habits revealed that an individual's preferred time to shower (i.e. morning or evening) might differ by age. The survey was administered to 461 individuals, including both older individuals (aged 25 to 50 years old) and younger individuals (aged 24 years old or younger). Survey results indicated that 73% of older individuals reported a preference for morning showers while only 59% of younger individuals reported a preference for morning showers. Among 422 older individuals, 307 reported a preference for morning showers.

What is the 99% confidence interval to estimate  $p_{\text{Older}} - p_{\text{Younger}}$ , the difference between the population proportions of individuals with a preference for morning showers among older individuals and younger individuals? Is there evidence to suggest that the proportion of individuals with a preference for morning showers differs between older and younger individuals?

Please select the best answer of those provided below.

- a. (-0.073, 0.348); There does not appear to be a significant difference in the proportions of older and younger individuals that report a preference for morning showers
- b. (-0.022, 0.298); There does not appear to be a significant difference in the proportions of older and younger individuals that report a preference for morning showers
- c. (-0.073, 0.348); There does appear to be a significant difference in the proportions of older and younger individuals that report a preference for morning showers
- d. (-0.022, 0.298); There does appear to be a significant difference in the proportions of older and younger individuals that report a preference for morning showers

# ARKANSAS COUNCIL OF TEACHERS OF MATHEMATICS 2018 STATISTICS STATE EXAM

- 8. The Global Assessment of Functioning (GAF) is a numeric scale used by mental health clinicians to rate the social, occupational, and psychological functioning of an individual. Scores range from 100 (extremely high functioning) to 1 (severely impaired functioning). The distribution of observed GAF scores for several individuals is negatively skewed with a mean of 72 and a standard deviation of 15. If the set of all observed GAF scores are converted to a z-score scale, which of the following describes the shape, center, and spread of the new distribution of these z-scores?
  - a. Normally distributed with a mean of 0 and a standard deviation of 1
  - b. Normally distributed with a mean of 72 and a standard deviation of 15
  - c. Negatively skewed with a mean of 0 and a standard deviation of 1
  - d. Cannot Be Determined
- 9. The battery for a heart pacemaker is believed to have an average life of 300 days. A researcher decides to perform a one-tailed, one-sample t-test to determine if the average life of the battery for a heart pacemaker is actually more than 300 days. Consider that incorrectly claiming the battery has a longer life than it truly does would put patients that rely on heart pacemakers at great risk. Given this consideration and assuming that you are a quality control manufacturer for the battery company, would it be more preferable to make a Type I Error or a Type II Error in this setting? Please select the best answer of those provided below.
  - a. A Type I Error would be preferable
  - b. A Type II Error would be preferable
  - c. Neither error would be possible in a one-tailed, one-sample t-test
  - d. Neither error would be possible when rejecting the null hypothesis
- 10. There is nothing worse than a favorite shirt that is too tight or a favorite pair of pants that is too short after washing. What influences the shrinkage of our clothing? A simple random sample of clothing items was to be utilized in the study. The fiber of each clothing item would be recorded (cotton, linen, or wool). Each clothing item was then dried in the same manner. Once the clothing was determined to be dry, the percentage of length shrinkage was recorded (with higher values indicating greater length shrinkage of the clothing item). Which of the following statistical procedures would be most appropriate to determine if length shrinkage is related to type of fiber for clothing items? Please select the best answer of those provided below, assuming that any necessary statistical requirements hold.
  - a. One-Way Analysis of Variance (ANOVA)
  - b. Chi-Square test of independence
  - c. Linear Correlation test
  - d. Two-sample independent t-tests

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NAME:

For Questions 11-13, refer to the table, which summarizes conditional probabilities for camping types given primary camping location of US campers. Note that 38% of all US campers report camping primarily in National Parks while the other 62% of US campers report camping primarily elsewhere (e.g., State Parks, Private Campgrounds, etc.).

	Type of Camping				
Primary Camping Location	Tent	RV	Yurt	No Shelter (or Bivy)	
National Parks	0.803	FIND ME	0.005	0.030	
Elsewhere	0.615	0.280	0.100	0.005	

- 11. What is the probability of one randomly selected US camper using an RV for camping given they report camping primarily in National Parks (i.e., report the value for 'FIND ME')? Round to 3 decimal places.
  - a. 0.720
  - b. 0.062
  - c. 0.162
  - d. 0.174
- 12. What is the probability of one randomly selected US camper using an RV for camping? Round to 3 decimal places.
  - a. 0.235
  - b. 0.045
  - c. 0.442
  - d. 0.342
- 13. What is the probability of one randomly selected US camper reporting that they camp primarily in National Parks given they use a Tent for camping? Round to 3 decimal places.
  - a. 0.445
  - b. 0.686
  - c. 0.305
  - d. 0.566

- 14. The statement that "A 95% confidence interval obtained from a simple random sample of 1000 people has a greater chance of containing the true population parameter than a 95% confidence interval obtained from a simple random sample of 500 people" is: Please select the best answer of those provided below.
  - a. Always True
  - b. Never True
  - c. Sometimes True
  - d. Not Enough Information
- 15. The first quartile of a distribution is 20.3, the median is 26.5, the range is 24.0, and the maximum value is 41. What is the interquartile range for the distribution? Please select the best answer of those provided below.
  - a. 20.7
  - b. 6.2
  - c. 24.0
  - d. Cannot Be Determined
- 16. For a particular hypothesis test, the significance level is 0.05 and the statistical power is 0.85. What is the probability of making a Type II Error in this setting? Round to 3 decimal places.
  - a. 0.050
  - b. 0.150
  - c. 0.143
  - d. 0.950
- 17. Assume that 20 large simple random samples of the same size were taken from the same population and corresponding 95% confidence intervals for the population mean were constructed for each. How many of the 20 constructed confidence intervals would be expected to contain the true value of the population mean? Please select the best answer of those provided below.
  - a. 20
  - b. 19
  - c. 0
  - d. None of the above

- NAME:
- 18. A student brings cupcakes to school for her birthday. Her mother helped her make 24 cupcakes but she would like to share these with her homeroom class that has 27 students (not including herself). Assume that the student with the birthday will not take one of the cupcakes. Further, assume that the other 27 students will independently refuse a cupcake or not be present on the day of her birthday 8% of the time. What is the probability that on the day of her birthday these 24 cupcakes will be enough to share for all of the students present in her homeroom class that would like a cupcake? Round to 3 decimal places.
  - a. 0.202
  - b. 0.368
  - c. 0.166
  - d. 0.632
- 19. The 115<sup>th</sup> Congress has an all-time high of 22 female senators. Note that there are 100 total senators. Four senators will be randomly selected to form a recreational 3 vs. 3 soccer team for an upcoming tournament. What is the probability that there will be *more* women than men selected for the four-member team? Round to 3 decimal places.
  - a. 0.032
  - b. 0.121
  - c. 0.209
  - d. 0.036
- 20. Willow oak trees are typically planted as ornamental trees in cities but, due to poor planning, causes sidewalks to crack. The trunk diameter for these trees are normally distributed with a mean of 1.3 meters and a standard deviation of 0.3 meters. City planners have estimated that 1.5 meter diameter allotments are appropriate for planted trees. Given that the city planted 60 willow oak trees, how many would be expected to exceed the 1.5 meter diameter allotment? Round to the nearest whole number.
  - a. 1
  - b. 0
  - c. 15
  - d. 45

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- 21. Assume that a multiple choice quiz consists of ten independent questions. Five of the questions are similar to AP examination questions and have five answer choices, with one correct option. Three of the questions have four answer choices, with one correct option. The last two questions have True/False answer choices, with one correct option. Using this information, what is the probability of guessing on each of the 10 questions and getting all of the 10 questions wrong? Round to 3 decimal places.
  - a. 0.999
  - b. 0.000
  - c. 0.266
  - d. 0.035
- 22. A professor is late to teach a class! Typically, students follow a 'guideline' rule stating that they should wait at least 15 minutes for a late professor before leaving the classroom. Assume that 'late' professor arrival times are uniformly distributed between 2 and 30 minutes after the official class start time. What is the probability that a randomly selected 'late' professor will arrive before his students leave the classroom (i.e., what is the probability that the professor will arrive between 2 and 15 minutes after the official class start time)? Round to 3 decimal places.
  - a. 0.500
  - b. 0.352
  - c. 0.067
  - d. 0.464
- 23. A researcher is attempting to predict the GPA of students ('GPA') using the number of social media accounts that they access often ('Social Media Accounts'). A small sample is collected and data is provided.

Social Media Accounts	12	2	1	0	4	4	9
GPA	3.2	3.9	4.0	3.0	3.4	2.6	2.8

Referring to the setting and data provided above, what proportion of the variability in student GPA can be accounted for by the number of social media accounts that they access often? Round to 3 decimal places.

- a. 0.143
- b. 0.377
- c. 0.404
- d. -0.377

24. 'Hill races' are races that involve steep climbs. These races are well known in Scotland and are held throughout the year. It is hypothesized that the total climb (meters) for these hill races is related to the fastest recorded race time (minutes) on each course.

A simple random sample of 90 Scottish hill races was used to construct the simple linear regression equation  $\hat{y} = 2.890 + 0.083x$  where x represents the climb of the race in meters and y represents the fastest recorded race time in minutes.

One race has climb of 1500 meters and a residual of -18.931. What was the observed fastest recorded race time in minutes for this particular race? Round to 3 decimal places.

- a. 1481.069
- b. 127.390
- c. 108.459
- d. 146.321
- 25. The scatterplot to the right reveals that one data point lies very far from the rest of the data. What is the effect of including this data point when fitting a simple linear regression model?

Please select the best answer of those provided below.

- a. Including the point would increase the intercept term and increase the slope term
- b. Including the point would decrease the intercept term and decrease the slope term
- c. Including the point would have little to no influence on the intercept term or the slope term but including the point would change the value of R<sup>2</sup> substantially
- d. Including the point would have little to no influence on the intercept term or the slope term and including the point would not change the value of R<sup>2</sup> substantially



NAME:

NAME:

#### -Tie Breaker Question 1-

A cohort study of several currently pregnant women was conducted to examine certain health outcomes for pregnant women (e.g., the occurrence of gestational diabetes, fasting blood glucose levels during the third trimester, blood pressure levels during the third trimester) and how these outcomes were related to exposures such as smoking, age, and pre-pregnancy weight.

Correct descriptions of variables in a study is a benefit to researchers by providing them with information that can guide their data analysis and presentation choices. Describe the following data, which was recorded during the study, as Categorical or Quantitative.

<u>Note</u>: If you describe the data as Categorical you must also indicate whether it is nominal, binary/dichotomous, or ordinal. If you describe the data as Quantitative you must also indicate if it is discrete or continuous.

- State of Residence
- Marital Status (Single, Married)
- Pre-Pregnancy Weight (kg)
- Pre-Pregnancy BMI Classification (Underweight, Normal Weight, Overweight, Class I Obesity, Class II Obesity, and Class III Obesity)
- Smoking Status (Currently Smoking, Previously Smoked, Never Smoked)
- Gravidity (i.e. the number of times a female is or has been pregnant)
- Parity (i.e. the number of times a female has given birth at a viable gestational age)
- Gestational Diabetes (Yes/No)
- Fasting Blood Glucose Level (mg/dL)

NAME:

#### -Tie Breaker Question 2-

Several studies have reported that eating Soy has been linked to lowered low-density lipoprotein (LDL) cholesterol levels. Eighty total participants volunteered and agreed to participate in a study designed to determine if the amount of Soy consumption in the diet is related to LDL cholesterol levels. The participants would be randomly assigned to four groups of Soy consumption for a year long period (No Soy Meals per Week, 1 Soy Meal per Week, 4 Soy Meals per Week, and 7 Soy Meals per Week). Twenty of the participants were randomly assigned to each of the four groups. At the end of the year, LDL cholesterol levels for the participants were recorded.

Partial output, applying the methods of one-way analysis of variance (ANOVA), is provided below. Using both the study information above and the partial output below, provide all of the missing parts of the one-way ANOVA table as the answer to this question.

Source	Degrees of Freedom (DF)	Sum of Squares (SS)	Mean Square (MS)	F
Between/ Treatment	??	??	155.35	??
Within/ Error	??	??	17.10	

Round to 3 decimal places.

NAME:

#### -Tie Breaker Question 3-

The table below represents data from a survey and provides the self-reported level of religiosity as well as the sex for a moderately large sample of individuals. The goal of the study was to determine if there was a statistically significant association between self-reported level of religiosity and the sex of an individual.

	Religiosity				
Sex	Very Moderately Slightly Not at				
Female	170	340	174	95	
Male	98	266	161	123	

Do the data indicate an association between self-reported level of religiosity and the sex of an individual? Conduct an appropriate hypothesis test to answer this question using a 0.05 significance level. Provide the hypotheses, test statistic(s), p-value(s), and a formal conclusion.

# **ACTM STATISTICS**

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### Multiple Choice Key

- 1. d
- 2. d
- 3. d
- 4. b
- 5. b
- 6. a
- 7. a
- 8. c
- 9. b
- 10. a
- 11. c
- 12. a
- 13. a
- 14. b
- 15. d
- 16. b
- 17. b
- 18. b
- 19. a
- 20. c
- 21. d
- 22. d
- 23. a
- 24. c
- 25. d

#### **ACTM STATISTICS**

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#### -Tie Breaker Question 1-

ABBREVIATED Question: Describe the following data as Categorical or Quantitative. If you describe the data as Categorical you must also indicate whether it is nominal, binary/dichotomous, or ordinal. If you describe the data as Quantitative you must also indicate if it is discrete or continuous.

#### **Solution:**

- State of Residence - Categorical; Nominal
- Marital Status (Single, Married) - Categorical; Binary/Dichotomous
- Pre-Pregnancy Weight (kg) - Quantitative; Continuous
- Pre-Pregnancy BMI Classification (Underweight, Normal Weight, Overweight, Class I Obesity, Class II Obesity, and Class III Obesity)
   Categorical; Ordinal
- Smoking Status (Currently Smoking, Previously Smoked, Never Smoked) - Categorical; Nominal
- Gravidity (i.e. the number of times a female is or has been pregnant) - Quantitative; Discrete
- Parity (i.e. the number of times a female has given birth at a viable gestational age) - Quantitative; Discrete
- Gestational Diabetes (Yes/No)
   Categorical; Binary/Dichotomous
- Fasting Blood Glucose Level (mg/dL) - Quantitative; Continuous

#### Rubric: 0 pts to 9 pts Possible

1 point for each completely correct description above  $\frac{1}{2}$  point for each partially correct description above

e.g., Pre-Pregnancy Weight (kg) described as Quantitative but Discrete (rather than Continuous) 0 points for each incorrect description above

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#### -Tie Breaker Question 2-

ABBREVIATED Question Information: Eighty total participants in a study designed to determine if the amount of Soy consumption in the diet is related to LDL cholesterol levels are randomly assigned to four groups of Soy consumption for a year long period (No Soy Meals per Week, 1 Soy Meal per Week, 4 Soy Meals per Week, and 7 Soy Meals per Week).

Partial output, applying the methods of one-way analysis of variance (ANOVA), is provided below. Using both the study information above and the partial output below, provide all of the missing parts of the one-way ANOVA table as the answer to this question.

Round to 3 decimal places.

Source	Degrees of Freedom (DF)	Sum of Squares (SS)	Mean Square (MS)	F
Between/ Treatment	4 - 1 = 3	<u>155.35*3 = 466.050</u>	155.35	<u>155.35/17.10 = 9.085</u>
Within/ Error	<u>80 - 4 = 76</u>	<u>17.10*76 = 1299.600</u>	17.10	

### **Rubric:** 0 pts to 5 pts Possible

1 point for each completely correct answer above (correct student provided answers are underlined above) 0 points for each incorrect answer above

### **ACTM STATISTICS**

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## -Tie Breaker Question 3-

	Religiosity			
Sex	Very	Moderately	Slightly	Not at All
Female	170	340	174	95
Male	98	266	161	123

Do the data indicate an association between self-reported level of religiosity and the sex of an individual? Conduct an appropriate hypothesis test to answer this question using a 0.05 significance level. Provide the hypotheses, test statistic(s), p-value(s), and a formal conclusion.

#### **Solution:**

• Hypotheses

 $\begin{cases}
H_0: \text{Religiosity and Sex of an Individual are Independent (there is no association)} \\
H_1: \text{Religiosity and Sex of an Individual are Dependent (there is association)}
\end{cases}$ 

- Test Statistic  $\chi^2 = 20.628, df = 3$
- **P-Value**  $p - value \approx 0.0001$
- Formal Conclusion at  $\alpha = 0.05$  (in terms of  $H_0$ ) Reject the null hypothesis at the 5% significance level. There is sufficient evidence to support the claim that self-reported religiosity and the sex of an individual are associated/dependent.

#### Rubric: 0 pts to 4 pts Possible

1 point for each completely correct answer and 0 points for each incorrect answer of the following tie breaker components: (1) hypotheses, (2) test statistic, (3) p-value, and (4) formal conclusion