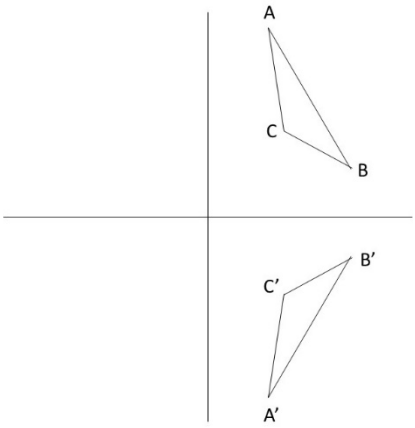
ACTM State Geometry Competition 2018

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie-breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie-breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

NOTE: Figures are not necessarily drawn to scale.

1. Given Triangle ABC. Triangle A'B'C' is the result of a transformation on Triangle ABC. Which of the following individual transformations will return A'B'C' back to the original position of ABC without the use of any additional transformations?



A. Translation

B. Rotation

C. Dilation

D. Reflection

E. None of these.

2. Find the angle between the hour and minute hands of a clock at 8:24 PM.

A. 108°

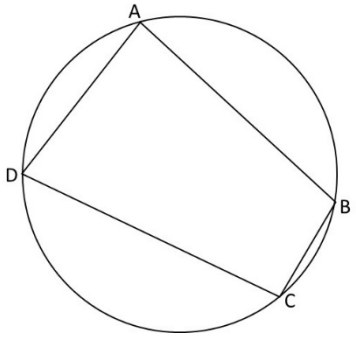
B. 96°

C. 90°

D. 102°

E. None of these.

3. Quadrilateral ABCD is inscribed in the given circle. The $m\angle B = 120°$, the $m\widehat{AB}$ is 2x°, the $m\widehat{BC}$ is x°, and the $m\angle A = (2x+5)°$. Find the $m\angle C$.



A. 85°

B. 95°

C. 40°

D. 60°

E. None of these.

4. Which of the following statements is not true about the complementary angles A and B?

A. $\sin A = \sin B$

B. $\sin A = \cos B$

C. $\tan A = \dfrac{1}{\tan B}$

D. $\dfrac{1}{\cos A} = \dfrac{1}{\sin B}$

E. All are correct.

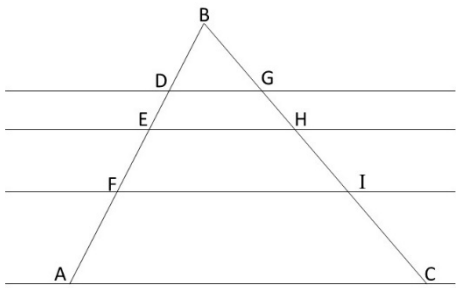
5. Given the lines DG, EH, FI and AC are all parallel. Which of the following statements is not true?

A. $\dfrac{BE}{DF} = \dfrac{BH}{GI}$

B. $\dfrac{DE}{FA} = \dfrac{GH}{IC}$

C. $\dfrac{CI}{IH} = \dfrac{AF}{FE}$

D. $\dfrac{AB}{DE} = \dfrac{CB}{GH}$



E. None of these.

6. Given a rhombus that is inscribed in a circle. Which of the following statements is not true about the rhombus and the circle?

A. The diagonals of the rhombus are congruent to each other.

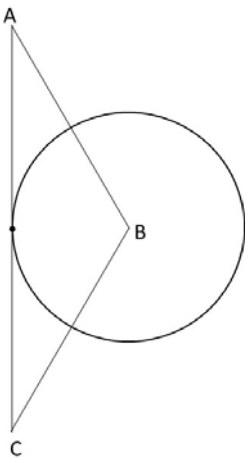
B. The four angles of the rhombus are congruent to each other.

C. The opposite angles of the rhombus are supplementary to each other.

D. The sides of the rhombus have the same measurement as the diameter of the circle.

E. All of these are true.

7. Given isosceles triangle ABC with vertex B at the center of Circle B and side AC tangent to Circle B. The $m\angle ABC$ =120° and AC = 10 *cm*. Find the area of the sector of Circle B that lies on the interior of Δ ABC.



A. $\frac{100\pi}{9} cm^2$

B. $\frac{25\pi}{3} cm^2$

C. $\frac{25\pi}{9} cm^2$

D. $\frac{100\pi}{3} cm^2$

E. None of these.

8. A student makes the following claim: "If 2 adjacent angles are a linear pair, then they are supplementary. Conversely, if 2 adjacent angles are not a linear pair, then they are not supplementary." Which of the following is the best description of the error?

A. There is no error because both statements are correct.

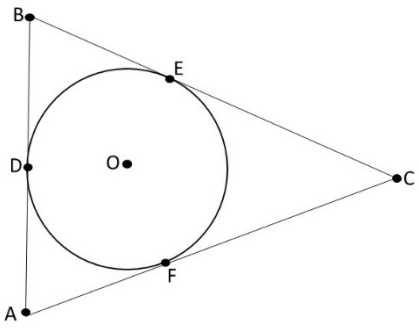
B. Neither statement is correct.

C. The second statement is stated correctly, but it is not true.

D. The second statement is not the converse of the first statement.

E. The second statement is actually the contrapositive and therefore correct.

9. Given that each side of ΔABC is tangent to Circle O and the radius of Circle O is 5 cm. Which of the following statements is true?



A. $m\angle ABF = m\angle CBF$

B. BD = AD

C. BO = OF

D. $m\angle ABC = m\angle BAC$

E. None of these.

10. 2 park rangers observe a fire from their respective fire towers that are 5 miles apart. The angle between the line of sight from tower 1 to the other tower and the line of sight from tower 1 to the fire is 48.4°. The angle between the line of sight from tower 2 to the other tower and the line of sight from tower 2 to the fire is 32.6°. How much further is the distance from the second ranger to the fire than the distance from the first ranger to the fire?

A. 2.7 miles

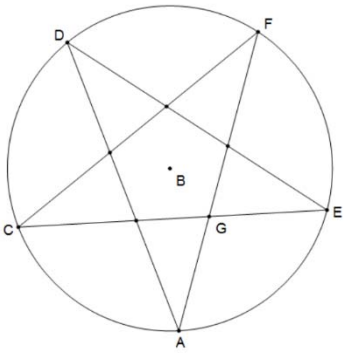
B. 1.1 miles

C. 3.8 miles

D. 2.3 miles

E. None of these.

11. Given ACDFE is a regular star pentagram where all of the corresponding sides and corresponding angles are congruent. Given that AG is 6 cm long. Find the perimeter of the pentagon in the center of the pentagram.



A. 30 cm

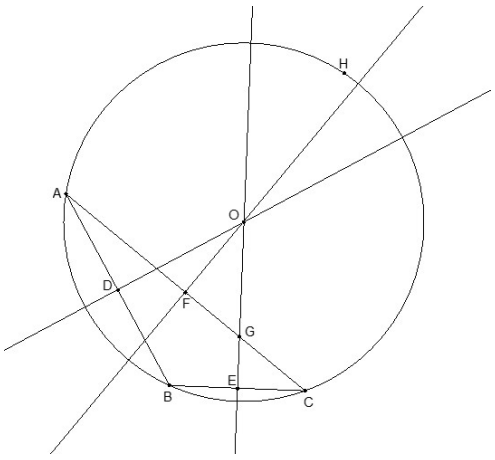
B. 3.7 cm

C. 18.5 cm

D. 20 cm

E. None of these.

12. Given Circle O with inscribed triangle ABC. $\overset{\frown}{AHC}$ =260°. BC = 10 cm. EG = 3.5 cm. Find the $m\angle A$ to the nearest tenth of a degree.



A. 22.5°

B. 15.0°

C. 30.0°

D. 29.6°

E. None of these.

13. Find the center of the circle given by the equation $36x^2 + 36y^2 = -36x + 48y + 119$.

A. $\left(-\frac{1}{2}, \frac{2}{3}\right)$

B. $\left(-1, \frac{4}{3}\right)$

C. $(-18, 24)$

D. $\left(-\frac{1}{4}, \frac{4}{9}\right)$

E. None of these.

14. A river is flowing directly south at 10 km/hr. A dog on the west bank tries to swim directly east across the river, which is 100 m wide. It takes the dog 20 seconds to get across the river and lands at a point downriver from a point directly east of where it started. What is the angle between the line directly across the river and the line of sight of where the dog landed on the opposite bank?

A. 63°

B. 19°

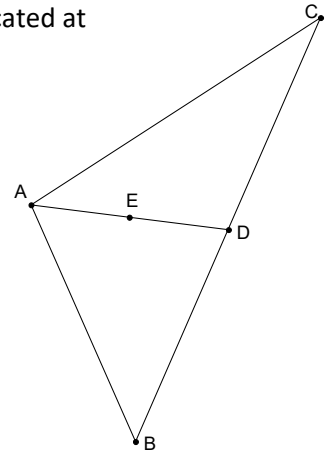
C. 82°

D. 29°

E. None of these.

15. Given Triangle ABC where D is the midpoint of BC and E is the midpoint of AD.

If Point A is located at $(3, 4)$, Point B is at $(4.5, -4.5)$, and Point E is located at $\left(\frac{11}{2}, \frac{5}{2}\right)$. Find the location of Point C.



A. $\left(\frac{23}{2}, \frac{13}{2}\right)$

B. $\left(\frac{35}{8}, -\frac{5}{8}\right)$

C. $(8, 1)$

D. $(-6.25, -1.75)$

E. None of these.

16. The area of a trapezoid is known to be 120 units^2. One base has endpoints at (5, 10) and (16, 10). The other base has an endpoint at (2, 2). Find the other endpoint.

A. (20, 2)

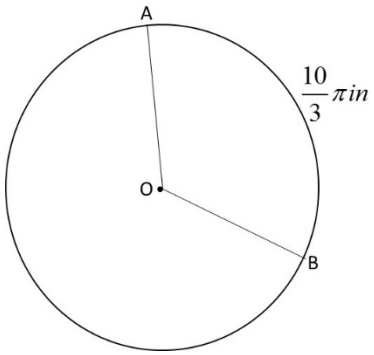
B. (2, 21)

C. (19, 2)

D. (22,2)

E. None of these.

17. Given circle O with arc AB as given. If the radius of the circle is 4 in, then find the central angle, AOB.



A. 300°

B. 120°

C. 135°

D. 150°

E. None of these.

18. Which of the following steps is NOT required when constructing 2 parallel line segments?

A. Construct a line segment

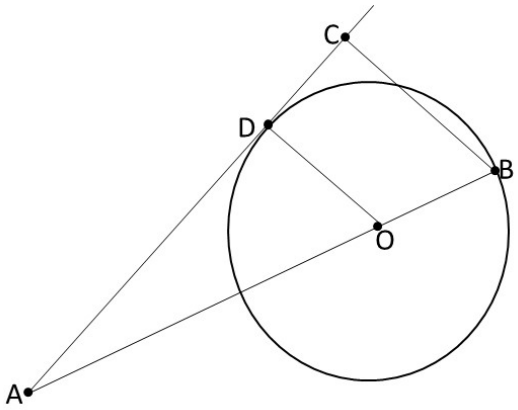
B. Construct a point not on the line segment

C. Construct 2 arcs

D. Construct a segment through the point on the transversal and the point located at the intersection of 2 arcs.

E. All of these are required.

19. Given that AB = 16 m, AO = 10 m, AD = $\frac{1}{2}$ AB and AC is tangent to Circle 0. Points B and D lie on Circle O. Find the length of segment BC.



A. 7.7 m

B. 10 m

C. 9.6 m

D. 3.6 m

E. None of these.

20. The surface area of a cylinder is 96 π . If the height of the cylinder is twice its radius, then find the volume of the cylinder.

A. 256 π

B. 128 π

C. 96 π

D. 72 π

E. None of these.

21. A beverage company wants to change their existing cans. They want to keep the volume about the same while at the same time reducing the surface area. Which of the following is the best option to consider? (Assume that the can is a cylinder.)

A. Reduce the diameter of the can

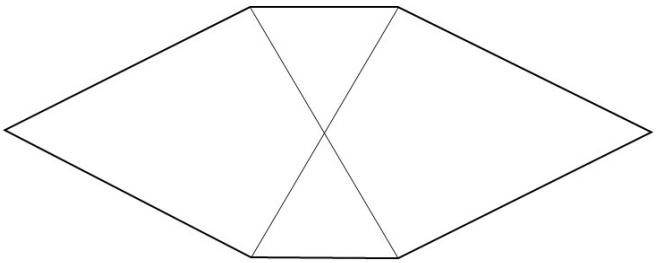
B. Reduce the diameter and increase the height of the can

C. Increase the diameter and decrease the height of the can

D. Reduce both the diameter and the height of the can

E. None of these.

22. A hexagon is constructed with 2 congruent equilateral triangles and 2 congruent kites. The bases of the equilateral triangles are parallel to each other. The lengths of the sides of the triangles are $4\sqrt{3}$ in. The longer diagonal of the kite is 10 in long. What is the area of the hexagon?



A. $\left(120+24\sqrt{3}\right)in^2$

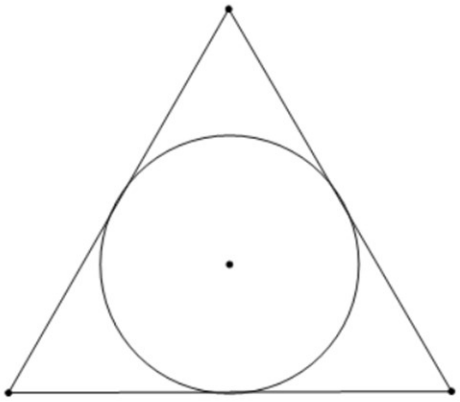
B. $72\sqrt{3}\,in^2$

C. $184\sqrt{3}\,in^2$

D. $\left(240+24\sqrt{3}\right)in^2$

E. None of these.

23. Given a circle inscribed in an equilateral triangle. What is the percentage of the area of the triangle that lies outside of the circle?



A. 65.5%

B. 60.4%

C. 33.3%

D. 39.6%

E. None of these.

24. A plane flies on a bearing of 110° at a speed of 480 mph. The wind is blowing at 30 mph at a bearing of 50°. What is the resulting ground speed of the plane?

A. 480.9 mph

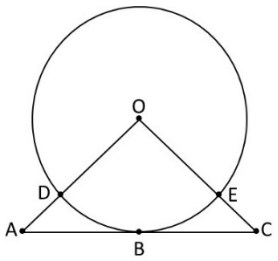
B. 480.0 mph

C. 495.7 mph

D. 465.7 mph

E. None of these.

25. Given that ΔAOC is an isosceles triangle, $\overline{AC}$ is tangent to Circle O at B, the radius of Circle O is 8 cm and EC = 4.8 cm. Find the approximate area of ΔAOC that is not inside of Circle O.



A. 80 cm^2

B. 38.8 cm^2

C. 51.3 cm^2

D. 22.7 cm^2

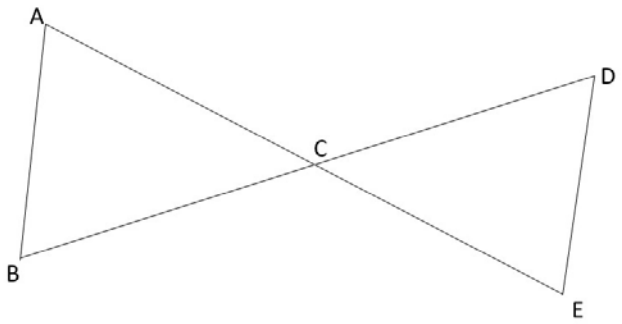
E. None of these.

Tie Breaker 1

Name:_____

Given that $\overline{AC} \cong \overline{BC}$ and $\angle A \cong \angle D$.

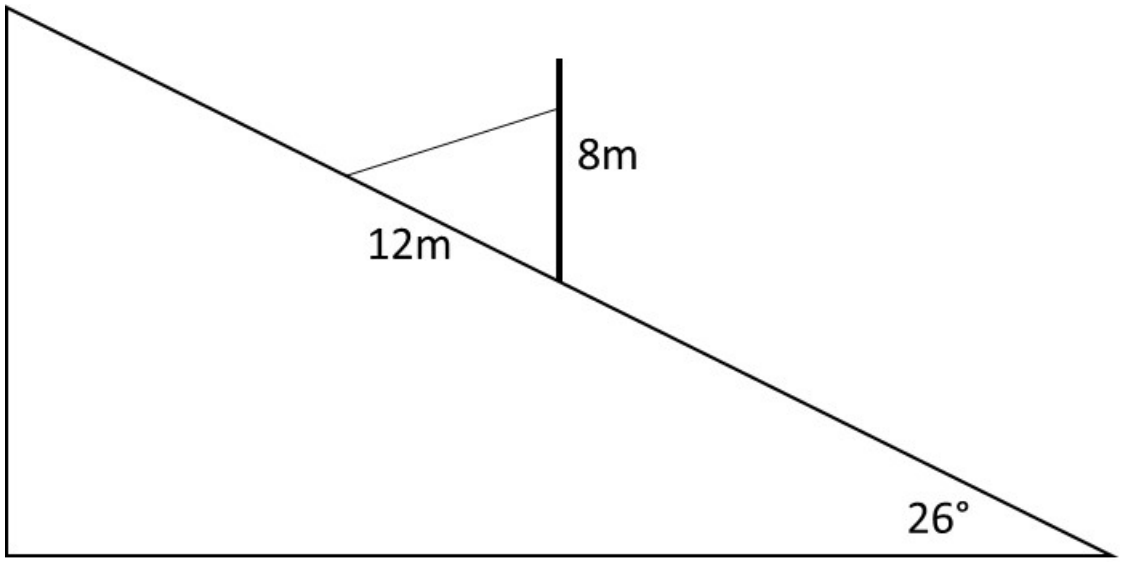
Prove that $\overline{AB} \parallel \overline{DE}$



Tie Breaker #2

Name:_____

An 8m tall pole stands on the side of a mountain. The pole has a wire attached 2m from the top of the pole and is anchored in the ground 12m from the base of the pole. Find the angle formed between the wire and the side of the mountain.



Geometry Test

Tie Breaker #3

Name:_____

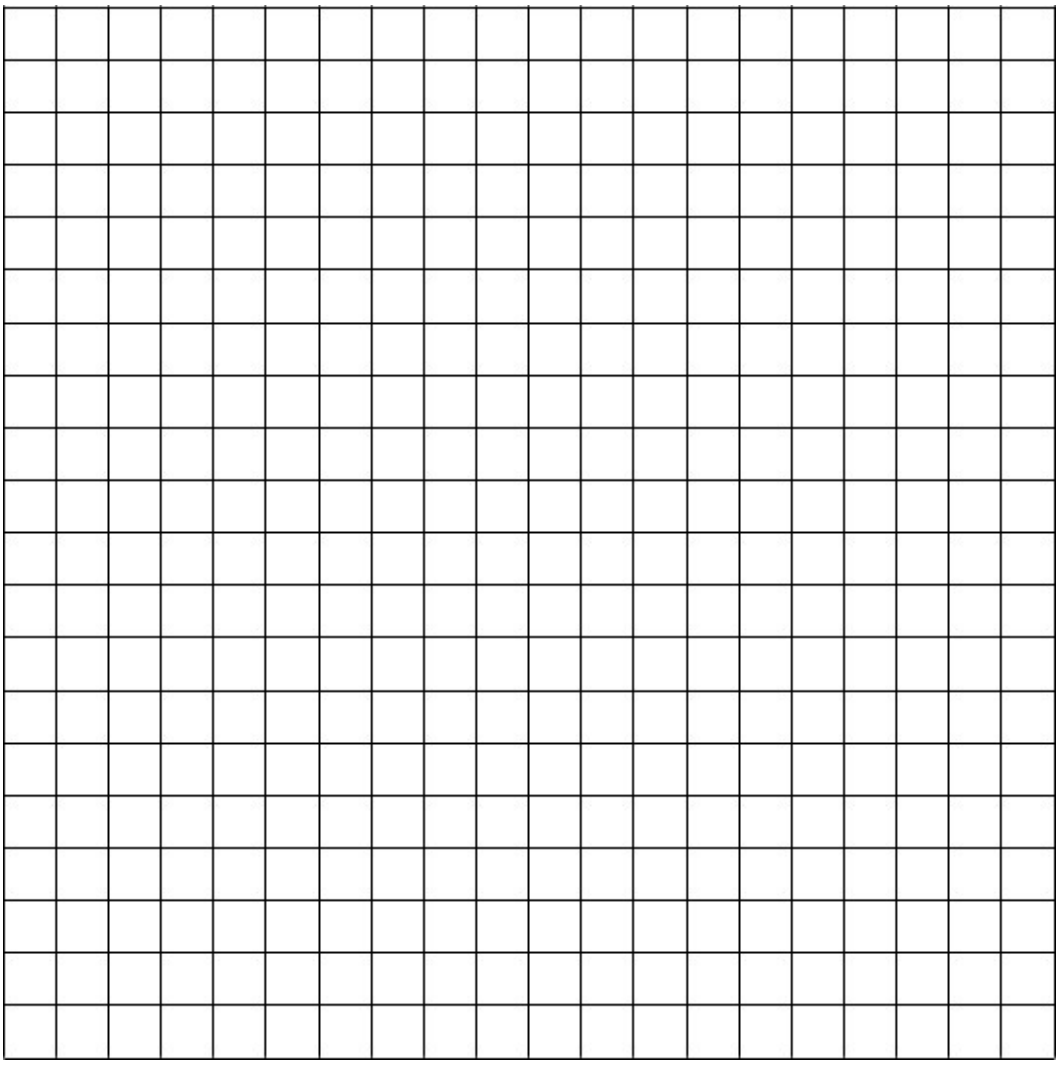
Given trapezoid ABCD with vertices at (2, – 2), (3, 2), (5, 2) and (8, – 2). Find the location of the vertices A'B'C'D' where A', B', C' and D' are the locations of the original vertices after a reflection across the line $y = -2x + 4$.

A'=_____

B'=_____

C'=_____

D'=_____



2018 – State Mathematics Contest

Geometry Test

Solutions:

1. D

2. A

3. B

4. A

5. E

6. D

7. C

8. D

9. A

10. B

11. C

12. B

13. A

14. D

15. A

16. E

17. D

18. E

19. C

20. B

21. B

22. A

23. D

24. C

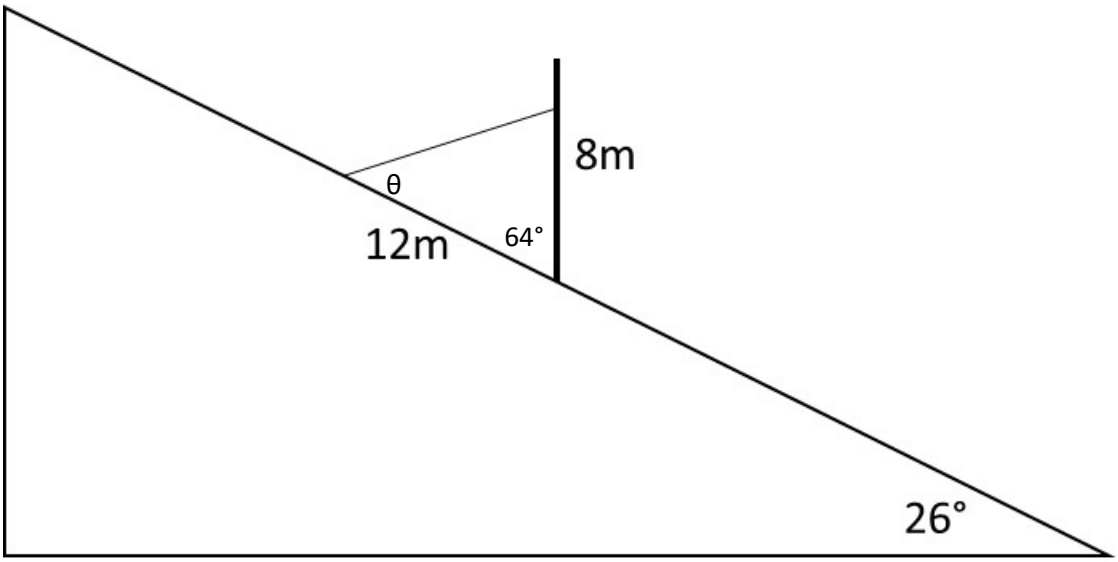
25. D

Tie Breaker 1

One possible proof in 2-column format is:

Statement	Reason
$\overline{AC} \cong \overline{BC}$	Given
$\angle A \cong \angle B$	Isosceles Triangle Theorem
$\angle A \cong \angle D$	Given
$\angle B \cong \angle D$	Transitive Property
$\overline{AB} \parallel \overline{DE}$	If 2 lines in a plane are cut by a transversal such that the alternate interior angles are congruent, then the lines are parallel.

Tie Breaker #2



The amount of pole between the ground and the wire is 6m. The angle between the pole and the ground is 64°. Use the law of cosines to find the missing side of the triangle.

$$c^2 = 12^2 + 6^2 - 2(12)(6)\cos(64)$$

That gives c = 10.8m. Now use the law of sines to find the missing angle. $\frac{sin\theta}{6m} = \frac{sin64}{10.8m}$

This gives θ as 30°. They may give the uphill angle as being 150°.

Tie Breaker #3

A' is at (18/5, – 6/5)

B' is at (– 1/5, 2/5)

C' is at (– 7/5, – 6/5)

D' is at (0, – 6)

Find the equation of each line that passes through each vertex and that is perpendicular to y = – 2x +4.

The line through A and A' is y = ½ x – 3. The line through B and B' is y= ½ x + ½. The line through C and C' is y = ½ x – ½ . The line through D and D' is y = ½ x – 6. Then find where the points of intersection are for each pair of lines. The point of intersection will be the midpoint for the segment connecting A and A', B and B', C and C', and D and D'. The midpoint of the segment AA' is (14/5, – 16/10). The midpoint of the segment BB' is (7/5, 6/5). The midpoint for the segment CC' is (9/5, 2/5). The midpoint for the segment DD' is (4, – 4).

Using the midpoints and the endpoints, the other endpoint can be found by:

A': $\frac{x+2}{2}=\frac{14}{5}\Rightarrow x+2=\frac{28}{5}\Rightarrow x=\frac{18}{5}$ and $\frac{y-2}{2}=-\frac{8}{5}\Rightarrow y-2=-\frac{16}{5}\Rightarrow y=-\frac{6}{5}$

B': $\frac{x+3}{2}=\frac{7}{5}\Rightarrow x+3=\frac{14}{5}\Rightarrow x=-\frac{1}{5}$ and $\frac{y+2}{2}=\frac{6}{5}\Rightarrow y+2=\frac{12}{5}\Rightarrow y=\frac{2}{5}$

C': $\frac{x+5}{2}=\frac{9}{5}\Rightarrow x+5=\frac{18}{5}\Rightarrow x=-\frac{7}{5}$ and $\frac{y+2}{2}=\frac{2}{5}\Rightarrow y+2=\frac{4}{5}\Rightarrow y=-\frac{6}{5}$

D': $\frac{x+8}{2}=4\Rightarrow x+8=8\Rightarrow x=0$ and $\frac{y-2}{2}=-4\Rightarrow y-2=-8\Rightarrow y=-6$