ACTM State Algebra II Competition 2018

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures are not necessarily drawn to scale. Angles are given in radians unless otherwise stated.

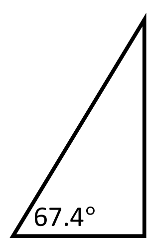
1. Jill and Peter are stacking cans of mixed fruit in a display at the end of the grocery aisle. The top layer of the stack of cans has 1 can, the second layer has 4 cans, the third layer has 9 cans, and so on. How many cans are in the stack if it has 12 layers?
   a. 600 cans
   b. 814 cans
   c. 650 cans
   d. 720 cans

2. Simplify the expression:

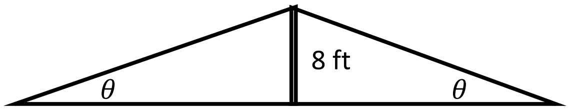
$$\frac{\frac{3x}{4x-1}}{1+\frac{3x}{x-1}}$$

   a. $\frac{x(3x-1)}{(4x-1)(x-1)}$
   b. $\frac{3x(x-1)}{(4x-1)^2}$
   c. $\frac{12x^2-3x-1}{(4x-1)}$
   d. $\frac{x}{4x-1}$

3. The sail of boat is in the shape of a right triangle. The bottom edge of the sail, called the foot, is 60 inches long. The vertical edge of the sail that attaches to the mast, called the luff, is 144 inches long. The angle at the bottom of the sail measures 67.4°. Find the value of sine and cosine for this angle.
   a. $\sin 67.4° = \frac{5}{13}; \cos 67.4° = \frac{12}{13}$
   b. $\sin 67.4° = \frac{12}{5}; \cos 67.4° = \frac{5}{13}$
   c. $\sin 67.4° = \frac{12}{13}; \cos 67.4° = \frac{5}{13}$
   d. $\sin 67.4° = \frac{12}{13}; \cos 67.4° = \frac{12}{5}$



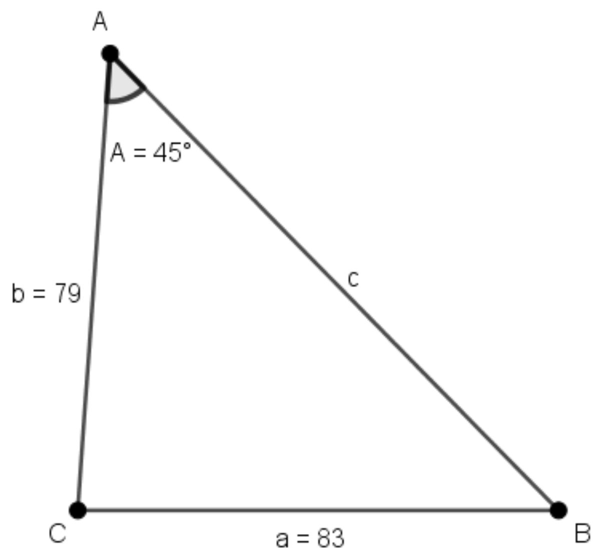
4. The support for a roof is shaped like two right triangles each with one leg 8 feet long and the hypotenuse 16 feet long. The sine of the angle θ will be $\frac{1}{2}$. What will be θ?



   a. $\frac{\pi}{3}$
   b. $\frac{\pi}{6}$
   c. $\frac{\pi}{4}$
   d. $\frac{\pi}{8}$

5. Find $\tan\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$:
   a. $\frac{5}{12}$
   b. $\frac{12}{5}$
   c. $\frac{12}{13}$
   d. $\frac{13}{5}$

6. What volume of a 35% salt solution must be added to a 10% salt solution to get 250 milliliters of a 20% solution?
   a. 62.5 mL
   b. 100 mL
   c. 300 mL
   d. 125 mL



7. Use the Law of Sines to solve ΔABC.
   a. $B \approx 47°59'$; $C \approx 74°1'$; $c \approx 114.59$
   b. $B \approx 92°42'$; $C \approx 42°18'$; $c \approx 117.25$
   c. $B \approx 42°18'$; $C \approx 92°42'$; $c \approx 117.25$
   d. $B \approx 45°$; $C \approx 90°$; $c \approx 114.59$

8. Simplify the expression:

$$\frac{x^2+49}{x^2-49}+\frac{x}{7-x}+\frac{7}{x+7}$$

   a. $\frac{2x^2+98}{x^2-49}$
   b. 0
   c. $\frac{14x}{x^2-49}$
   d. $\frac{x^2-x+56}{x^2-49}$

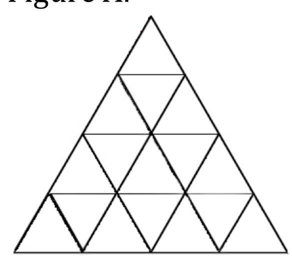
9. The expression $cos^2x + tan^2x \cdot cos^2x$ is equivalent to:
   a. 1
   b. tan^2x
   c. sin^2x
   d. sec^2x

10. Write the simplest polynomial function with the integral coefficients if two of its zeros are -2 and $4 + 3i$.
    a. $f(x) = x^4 - 8x^3 + 21x^2 + 32x - 100$
    b. $f(x) = x^3 - 10x^2 + 41x - 50$
    c. $f(x) = x^3 - 6x^2 + 9x + 50$
    d. $f(x) = x^3 + 2x^2 + 12ix - 7x + 24i - 14$

11. The sets of numbers to which $\sqrt{36}$ belongs are:
    a. Natural numbers, whole numbers, rational numbers
    b. Natural numbers, whole numbers, real numbers
    c. Natural numbers, whole numbers, integers, real numbers
    d. Natural numbers, whole numbers, integers, real numbers, rational numbers

12. Find the total number of triangles in Figure A.
    a. 25
    b. 16
    c. 19
    d. 27

Figure A:



13. The equation of a line that passes through $(6, -5)$ and is perpendicular to the line whose equation is $3x - \frac{1}{5}y = 3$ is
    a. $x + 15y = -69$
    b. $-15x + y = -3$
    c. $-\frac{1}{15}x + 5y = -\frac{1}{3}$
    d. $x + 15y = 81$

14. Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines whose equations are $2x + y = -12$, $2x - y = -8$, $2x - y - 4 = 0$, and $4x + 2y = 24$.
    a. (5, 2), (-4, -4), (2, 8), (1, 12)
    b. (2, -12), (2, -8), (4, 24), (4, 0)
    c. (-5, -2), (4, 4), (-2, -8), (1, 10)
    d. (-5, -2), (4, -4), (-2, 8), (1, -10)

15. Find the maximum and minimum values of the function $f(x, y) = -2x + y$ for the region defined by the following inequalities: $2x - 4 \leq y, -2x - 4 \leq y, 2 \geq y$:
    a. Maximum value = 2; minimum value = 0
    b. Maximum value = 8; minimum value = -4
    c. Maximum value = 2; minimum value = -4
    d. Maximum value = 3; minimum value = 0

16. Simplify the expression:

$$\frac{(-2t^3)^2(t^{-2})^{-1}}{(t^2)^{-3}}$$

    a. $-2t^2$
    b. $4t^{12}$
    c. $4t^{14}$
    d. $-2t^{14}$

17. Determine the dimension of the matrix product $A_{3\times 2} \cdot B_{3\times 2}$.
    a. 3×2
    b. 3×3
    c. 2×3
    d. Not defined

18. Find k such that $-\frac{3}{2}$ is a root of the equation $2x^2 + kx - 12 = 0$
    a. -3
    b. 15
    c. -5
    d. 2

19. Solve the equation $y^3 - 16y^{\frac{3}{2}} = -64$.
    a. 4
    b. $8, -8$
    c. 64
    d. 8

20. Identify the vertex and axis of symmetry $f(x) = \frac{1}{3}x^2 - 4x + 15$.
    a. Vertex $(-6, 3)$; axis of symmetry $x = -6$
    b. Vertex $(-6, 3)$; axis of symmetry $x = 3$
    c. Vertex $(6, 3)$; axis of symmetry $x = 6$
    d. Vertex $(6, 3)$; axis of symmetry $x = 3$

21. Write the equation of the ellipse whose major axis is 12 units long and parallel to the y-axis. The minor axis is 8 units long and the center is at $(-2, 3)$.
    a. $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{36} = 1$
    b. $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{36} = 1$
    c. $\frac{(x-2)^2}{12} + \frac{(y-3)^2}{8} = 1$
    d. $\frac{(x-2)^2}{144} + \frac{(y+3)^2}{64} = 1$

22. Find the 2018th digit to the right of the decimal in the expansion of $\frac{1}{13}$.
    a. 2
    b. 3
    c. 6
    d. 7

23. If $n = 3^x + 3^x + 3^x$, find an expression for n^2.
    a. 9^{x+1}
    b. 9^{3x}
    c. 27^{2x}
    d. 27^{3x}

24. If $\log 2 = a$ and $\log 3 = b$ then $\log_5 12$ equals:
    a. $\frac{2a+b}{1-a}$
    b. $\frac{a+2b}{1-a}$
    c. $\frac{a+b}{1+a}$
    d. $\frac{2a+b}{1+a}$

25. Find the sum of the arithmetic series $3 + 7 + 11 + \cdots + 1283$.
    a. 206,403
    b. 207,046
    c. 412,806
    d. 414,092

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Tiebreaker Question 1

Name ____________________

School ____________________

Solve the system of equations

$$\begin{cases} \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{5}{8} \\ \dfrac{3}{x} + \dfrac{2}{y} = -\dfrac{5}{8} \end{cases}$$

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Tiebreaker Question 2

Name ____________________

School ____________________

A refractometer uses the basic principles of Snell's Law. If n represents the index of refraction, I represents the angle of incidence, and r represents the angle of refraction, then the equation below states their relationship as light passes from a vacuum into another medium.

$$n = \frac{\sin I}{\sin r}, \text{ where } 0 < I \leq 90°, 0 < r \leq 90°$$

A beam of light moves from a vacuum to glass. If the angle of refraction is 30° and the index of refraction is $\sqrt{2}$, what is the angle of incidence?

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Tiebreaker Question 3

Name ____________________

School ____________________

Vonda and Bill are standing 100 feet apart and in a straight line with television tower. The angle of elevation from Bill to the tower is 30° and the angle of elevation from Vonda is 20°. Find the height of the television tower to the nearest foot.

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2018 State Algebra II Exam Key

1. C
2. B
3. C
4. B
5. A
6. B
7. C
8. B
9. A
10. C
11. D
12. D
13. A
14. C
15. B
16. C
17. D
18. C
19. A
20. C
21. B
22. D
23. A
24. A
25. A

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Tie Breaker #1 Solution:

Let $m = \frac{1}{x}$ and $n = \frac{1}{y}$. Then we have $m - n = \frac{5}{8}$ and $3m + 2n = -\frac{5}{8}$. Solving by substitution we have the solution (8, -2).

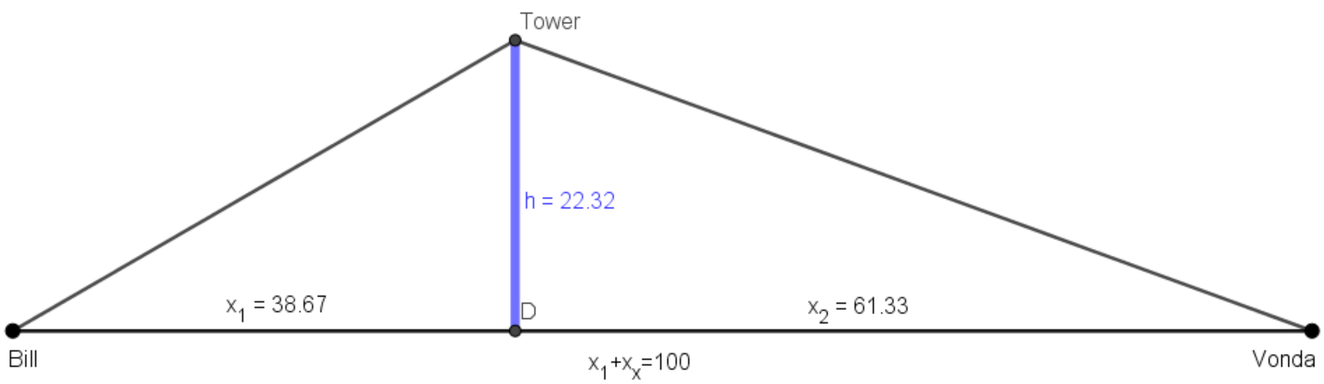
Tie Breaker #2 Solution:

By Snell's Law we have $n = \frac{\sin I}{\sin r}$

$$\sqrt{2} = \frac{\sin I}{\sin 30°}$$
$$\sqrt{2} = \frac{\sin I}{\frac{1}{2}}$$
$$\frac{\sqrt{2}}{2} = \sin I$$
$$45° = I$$

Thus, the angle of incidence is 45°.

Tie Breaker #3 Solution:



$$\tan 30° = \frac{h}{x_1} \qquad \tan 20° = \frac{h}{x_2} = \frac{h}{100 - x_1}$$
$$h = x_1 \tan 30° \qquad h = \tan 20° \,(100 - x_1)$$

$$x_1 \cdot \tan 30° = \tan 20° \cdot (100 - x_1)$$
$$x_1 \tan 30\,° = 100 \tan 20° - x_1 \tan 20°$$
$$x_1 \tan 30\,° + x_1 \tan 20° = 100 \tan 20°$$
$$x_1(\tan 30° + \tan 20°) = 100 \tan 20°$$
$$x_1 = \frac{100 \tan 20°}{\tan 30° + \tan 20\,°} \approx 38.67ft$$

$$h = \frac{100 \tan 20°}{\tan 30° + \tan 20\,°} \cdot \tan 30° = 22.32ft$$

Thus, the antenna is about 22.3 feet tall.