

ACTM State Algebra I Competition 2018

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures are not necessarily drawn to scale.

- Solve the equation: $16 = 2^{7x-5}$
 - 0.1143
 - .3010
 - $\frac{13}{7}$
 - $\frac{9}{7}$
- Which of the following is a factor of the polynomial $-12x^2 - 36x + 48$?
 - $x + 1$
 - $-12x + 4$
 - $x - 1$
 - $12x + 4$
- Rachael invests \$1,000 in an account that has an interest rate of 5%, compounded annually. How much money will be in her account after 30 months?
 - \$1,153.75
 - \$1,102.50
 - \$1,500.00
 - \$1,157.63
- If $3x - y = 12$, what is the value of $\frac{2^{3x}}{2^y}$?
 - 2^{12}
 - 4^4
 - 8^2
 - The value cannot be determined from the information given.
- The following function models the height, $h(t)$, of a ball t seconds after it is kicked into the air
$$h(t) = -16t^2 + 64t$$
What does each zero of this function represent?
 - The minimum height of the ball
 - The maximum height of the ball
 - The times at which the ball is on the ground
 - The time the ball reaches its maximum height
- If the value of z decreases by 2, how does the value of the expression $y(16 + z)$ change?
 - Decreases by $2y$
 - Decreases by $32y$
 - Increases by $14y$
 - Increases by $18y$
- If $a < 0$, and $b > |a|$, which of the following must be true?
 - $|a - b| < |b - a|$
 - $|b - a| < |a - b|$
 - $|a - b| = |b - a|$
 - Not enough information is given

ACTM State Algebra I Competition 2018

8. Sally and George both start walking to school from the playground at the same time. George is walking 3 times faster than Sally. After 4 minutes, he walks 1,000 feet, realizes he has forgotten his jacket, and turns to walk back to the playground at the same pace. How many minutes after turning back will it take for George and Sally to cross paths?

- a. 2 minutes
- b. 3 minutes
- c. 4 minutes
- d. 6 minutes

9. If $x < 2$, simplify the following:

$$|x - 2| - 4| - 6|$$

- a. $x + 22$
- b. $x - 22$
- c. $-x + 22$
- d. $-x - 22$

10. The expression $\left(\frac{(x^3)^{-2}}{x^2 \cdot x^3}\right)^3$ simplifies to the form x^m , for all nonzero values of x .

Which of the following represents the value of m when the expression is fully simplified?

- a. -23
- b. -33
- c. $-\frac{18}{5}$
- d. 2

11. A teacher examined the scores from a recent test. The sum of all the scores was 1800. The median of the scores was 75. The mean of the scores was 72. How many students took this test?

- a. 75
- b. 72
- c. 25
- d. 24

12. The graphs of the two equations $y = ax^2 + bx + c$ and $g = Ax^2 + Bx + C$, such that a and A have different signs and that the quantities $b^2 - 4ac$ and $B^2 - 4AC$ are both negative,

- a. Intersect at two points
- b. Intersect at one point
- c. Do not intersect
- d. None of the above

13. The perimeter of a triangle is 59 cm. Side a is 5 cm shorter than side b . Side c is twice as long as side a . Find the length of side c .

- a. 37
- b. 18.5
- c. 13.5
- d. 27

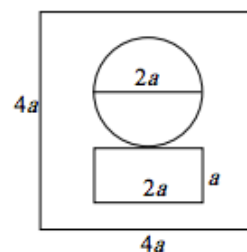
14. If $a^2 - b^2 = 648$ and $a - b = 24$, what is the value of $a + b$?

- a. 21
- b. 25
- c. 26
- d. 27

ACTM State Algebra I Competition 2018

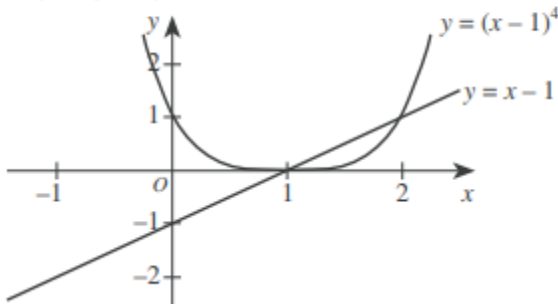
15. Assuming $x \neq 0$, how can the expression $(3x)^2 + 6x^0 + (5x)^0$ be simplified?
- $9x^2 + 7$
 - $3x^2 + 11$
 - $3x^2 + 7$
 - $9x^2 + 11$
16. Given $C = \frac{5}{9}(F - 32)$, at what temperature are Celsius and Fahrenheit the same?
- 17.8
 - 40
 - 57.6
 - 72
17. When a parabola represented by the equation $y = 2x^2 + 8x + 5$ is translated 3 units to the left and 2 units up, the new parabola has its vertex at
- $(-5, -1)$
 - $(-1, -5)$
 - $(-5, 15)$
 - $(-2, -3)$
18. The graphs of the two linear equations $ax + by = c$ and $bx - ay = c$, where a , b , and c are all not equal to zero
- Are parallel
 - Intersect at two points
 - Are perpendicular
 - None of the above
19. A woman has two rectangular gardens. The larger garden is five times as wide and three times as long as the smaller one. If the area of the smaller one is x , what is the difference in size of the two gardens?
- $15x$
 - $8x$
 - $7x$
 - $14x$
20. A square sheet of metal with sides of length $4a$ has a circle of diameter $2a$ and a rectangle with length $2a$ and width a removed from it. What is the area of the remaining metal?

- $4a - 4\pi a^2 - 2a^2$
- $14a^2 - \pi a^2$
- $4a^2 - 4\pi a^2 - 2a^2$
- $14a - 4\pi a^2$



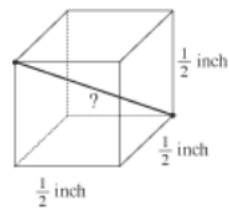
ACTM State Algebra I Competition 2018

21. The graphs of the equations $y = x - 1$ and $y = (x - 1)^4$ are shown in the standard (x, y) coordinate plane below. What real values of x , if any, satisfy the inequality $(x - 1)^4 < (x - 1)$?



- a. No real values
 b. $x < 1$ and $x > 2$
 c. $1 < x < 2$
 d. All real numbers
22. For every positive 2-digit number, x , with tens digit t and ones digit u , let y be the 2-digit number formed by reversing the digits of x . Which of the following expressions is equivalent to $x - y$?
- a. $9(t - u)$
 b. $9t - u$
 c. $ut - tu$
 d. 0
23. A cube has edges that measure $\frac{1}{2}$ inches in length. What is the length, in inches, of a diagonal that runs from one corner of the cube to the opposite corner?

- a. $\frac{1}{4}$
 b. $\frac{\sqrt{3}}{2}$
 c. $\frac{\sqrt{2}}{2}$
 d. $\frac{1}{8}$



24. What is the sum of the first 30 positive integers?
- a. 465
 b. 550
 c. 450
 d. 565

25. If $\frac{A}{30} + \frac{B}{105} = \frac{7A + 2B}{x}$ and A , B , and x are integers greater than 1, then what must x equal?

- a. 3,150
 b. 450
 c. 225
 d. 210

ACTM State Algebra I Competition 2018

Tiebreaker Question 1

Name _____

School _____

A machine can make 27,000 watches in one week. The Quality Assurance manager checks 1 out of every 9 watches for flaws. If more than 0.5% of the checked watches are flawed, then the batch must be thrown out.

- a) For the first batch of checked watches, 0.4% are flawed. How many watches are flawed?
- b) For the second batch, the Quality Assurance manager finds that 18 watches are flawed. Does this batch get thrown out? Explain your answer.

ACTM State Algebra I Competition 2018

Tiebreaker Question 2

Name _____

School _____

At the beginning of a party, there are twice as many women as men. By the end of the party, 17 additional men have arrived and 16 women have left. There are now the same number of men and women. How many men and women were originally at the party?

ACTM State Algebra I Competition 2018

Tiebreaker Question 3

Name _____

School _____

The University of Central Arkansas is hosting a dinner to raise money for scholarships. It expects that 225 people will attend the dinner when tickets are sold for \$45 each. The ticket price will never be less than \$45. For each \$1 increase in ticket price, 3 fewer people will attend.

- a) If x represents the number of \$1 price increases, what is the domain of this situation?

- b) There is a cost of \$10 per person for each meal, and a \$1,000 flat rate to rent the dining hall. Find a function $C(x)$ to represent the total cost of the dinner.

- c) Revenue is given by, $R(x) = (\text{Number who attend the dinner}) \cdot (\text{Price of one ticket})$. Find the revenue function.

- d) Profit is given by, $P(x) = R(x) - C(x)$.
How much should be charged per ticket in order to maximize the profit at the dinner?

- e) What is the maximum profit from the dinner?

ACTM State Algebra I Competition 2018

SOLUTIONS

Multiple Choice Key

1. D
2. C
3. B
4. A
5. C
6. A
7. C
8. A
9. D
10. B
11. C
12. C
13. D
14. D
15. A
16. B
17. A
18. C
19. D
20. B
21. C
22. A
23. B
24. A
25. D

ACTM Algebra I Tie Breaker Solutions

1. Watch Problem

- a. Start by dividing $\frac{27,000}{9} = 3,000$ watches that are checked by QA.
Take .04% of that to get $(3,000)(.004) = \mathbf{12 \text{ flawed watches}}$.
- b. $(3,000)(.005) = 15$, so the batch gets thrown out if any more than 15 watches are flawed. Since 18 are flawed in the second batch, the batch gets thrown out. **YES.**

2. Start with same amount of men and women. $W_i = 2M_i$. Then 17 men arrive and 16 women leave, so $M_f = M_i + 17$ and $W_f = W_i - 16$. The party ends with the same amount of men and women, so $M_f = W_f$, which gives $M_i + 17 = W_i - 16$. From here, we know that $W_i = 2M_i$, so substituting gives, $M_i + 17 = 2M_i - 16$. Thus, initially there are **33 men and 66 women**.

3. UCA dinner

- a. The minimum number of people is 0, so solving $225 - 3x = 0$ gives a **domain of [0, 75]**.
- b. Remember x represents the number of price bumps.
 $C(\text{people}) = 10(\text{people}) + 1000$.
Which means $C(x) = 10(225 - 3x) + 1000$
Or $C(x) = -30x + 3250$
- c. Revenue:
 $R(x) = (225 - 3x)(45 + x) = -3x^2 + 90x + 10,125$
- d. Profit function: $P(x) = -3x^2 + 120x + 6875$
Maximum @ $x = 20$.
Then ticket price should be $45 + 60 = \mathbf{\$65}$.
- e. Maximum profit: $P(20) = \mathbf{\$8075}$