Arkansas Council of Teachers of Mathematics – Algebra I State Exam
April 23, 2016

For questions 1-25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items in sequence order (do #1 first, followed by #2, then #3). Be sure that your name is printed on each of the tiebreaker pages.

1. The graphs of the equations $y = 2^x$ and $y = -2x + a$ intersect in Quadrant I for which values of a?

   a. $0 < a < 1$ b. $a < 1$ c. $a \geq 1$ d. $a > 1$

2. Which ordered pair is NOT in the solution set of $y > -\frac{1}{2}x + 5$ and $y \leq 3x - 2$?

   a. (3, 5) b. (4, 3) c. (3, 4) d (4, 4)

3. A car depreciates at a rate of 4.5% annually. Greg purchased a car for $12,500. Which equation can be used to determine the value of the car, V, after 5 years?

   a. $V = 12{,}500(0.55)^5$
   b. $V = 12{,}500(0.955)^5$
   c. $V = 12{,}000(1.045)^5$
   d. $V = 12{,}500(1.45)^5$

4. A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which function(s) shown below can be used to determine the height, f(n), of the sunflower in n weeks?

   I. $f(n) = 2n + 3$
   II. $f(n) = 2n + 3(n - 1)$
   III. $f(n) = f(n - 1) + 2$ where $f(0) = 3$

   a. I and II b. II only c. III only d. I and III

5. Find the solution set to $\frac{2}{3}x + 13.5 > \frac{1}{6}x - 2.4$

   a. $(-\infty, -31.8)$
   b. $(-31.8, \infty)$
   c. $(-\infty, 9.25)$
   d. $(9.25, \infty)$

6. The weights of 11 people in John's class are averaged to be 94 pounds. John takes out one person's weight and the average becomes 97 pounds. What weight was removed from the data set?

   a. 33 pounds b. 64 pounds c. 97 pounds d. 100 pounds

7. At most gas stations, gasoline prices include $\frac{9}{10}$ of a cent in addition to an amount stated in dollars and cents. However, customers can only pay for their purchases using dollars and whole cents. What is the maximum difference between the actual price of a gasoline purchase and the price the customer pays?

   a. $0.001 b. $0.005 c. $ 0.009 d. There is no maximum difference

8. Joe has a rectangular patio that measures 10 feet by 12 feet. He wants to increase the area by 50% and plans to increase each dimension by equal lengths, *x*. Which equation could be used to determine *x*?

   a. $(10 + x)(12 + x) = 120$
   b. $(10 + x)(12 + x) = 180$
   c. $(15 + x)(18 + x) = 180$
   d. $(15)(18) = 120 + x^2$

9. The equation $y = 0.45x + 2.99$ determines the total cost that a restaurant charges for its $2.99 regular hamburger plus toppings at $0.45 each. Which of the following is a true statement?

   a. The y-intercept represents the cost of a regular hamburger with no toppings.
   b. The y-intercept represents the additional cost per each topping.
   c. The slope represents the number of hamburger toppings added.
   d. The slope represents the total cost a hamburger with *x* toppings.

10. Which quadrant will be completely shaded in the graph of the inequality $4x + 3 < 9$?

    a. Quadrant I b. Quadrant II c. Quadrant III d. Quadrant IV

11. Delaware has an area of 2,000 square miles. Which is true if Delaware is included in the data set?

a. The standard deviation increases.
b. The range decreases.
c. The interquartile range decreases.
d. The mean increases.

State	**Area** (thousands of square miles)
Connecticut	6
Georgia	59
Maryland	12
Massachusetts	11
New Hampshire	9
New York	54
North Carolina	54
Pennsylvania	46

12. In a beauty contest, the scores awarded by eight judges were:

    5.9 6.7 6.8 6.5 6.7 8.2 6.1 6.3

Only six scores are to be used. Which two scores may be omitted that will leave the value of the median the same?

a. 6.5 and 6.7
b. 5.9 and 6.3
c. 5.9 and 8.2
d. The median cannot be duplicated using six scores.

13. Given the function $f(x) = 2x^2 + 5, x \geq 0$ find $f^{-1}(x)$.

a. $\sqrt{\frac{x+5}{2}}$ b. $\sqrt{\frac{x-5}{2}}$ c. $\pm\sqrt{\frac{-x+5}{2}}$ d. $\pm\sqrt{\frac{-x-5}{2}}$

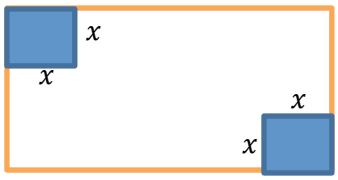
14. An arithmetic sequence has a first term of 10 and a sixth term of 40. What is the 20th term of this sequence?

a. 105 b. 110 c. 124 d. 130

15. Which situation below correctly models the exponential equation $y = 1500(.975)^x$.

a. A population of 1,500 geese losing its population by 2.5% each year.
b. A population of 1,500 geese growing at a rate of 97.5% each year.
c. A population of 1,500 geese losing 25% of its population a year.
d. A population of 1,500 geese adding 97.5 new geese to its population each year.

Use the figure below to answer question 16.



$-\frac{2}{3}x + 5$

$3x^2 - \frac{1}{2}$

16. What is the area of the unshaded part of the figure?

a. $-2x^3 + \frac{40}{3}x^2 - \frac{5}{2}$
b. $-2x^3 - \frac{40}{3}x^2 - \frac{5}{2}$
c. $-2x^3 + 13x^2 - \frac{1}{3}x - \frac{5}{2}$
d. $-2x^3 + 13x^2 + \frac{1}{3}x - \frac{5}{2}$

17. One type of redwood tree has an average height of 65 feet when it is 20 years old. If the tree is more than 20 years old, the average height, h, can be modeled by the function $h = 1.95(a - 20) + 65$, where a is the age of the tree in years. Which statement about this situation is true?

a. Each additional year of age over 20 years adds 1.95 ft. to the average height of this type of redwood tree.
b. Every additional 1.95 ft. of length over 20 ft. adds 45 years to the age of this type of redwood tree.
c. For this type of redwood tree, the average height increases by 1.95 ft. per year throughout its lifetime.
d. For this type of redwood tree, the average height increases by 65 ft. for every 20 years of growth.

18. The perimeter of a rectangle is 42 centimeters. The length of the rectangle can be represented by $(x + 4)$, and its width can be represented by $(2x - 7)$. What are the dimensions of this rectangle in centimeters?
    a. Length = 10 and width = 11
    b. Length = 8 and width = 13
    c. Length = 6 and width = 15
    d. Length = 12 and width = 9

19. Which of the following is not equivalent to $(3)^2 \left(\frac{12}{25}\right)^{-\frac{3x}{2}}$?

    a. $3^2 \left(\frac{5}{(12)^{\frac{1}{2}}}\right)^{3x}$ b. $\frac{9(125)^x}{(2\sqrt{3})^{3x}}$ c. $9\left(\frac{12\sqrt{12}}{25}\right)^{-x}$ d. $3^2 \left(\frac{2\sqrt{3}}{5}\right)^{-3x}$

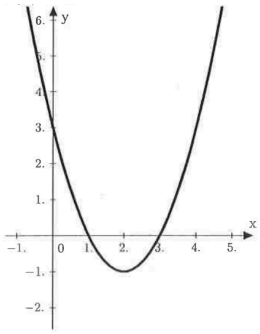
20. The intercepts of a given upward facing quadratic function are (3,0) and (5,0). What is the axis of symmetry of the quadratic function?

    a. $x = 15$ b. $x = -15$ c. $x = -4$ d. $x = 4$

21. Given the function $h(t) = t^2 + 10t + 24$, what is the average rate of change from [5, 10]?

    a. 25 b. -25 c. 125 d. -125

22. Let f be a function whose graph is the parabola sketched to the right. In which interval(s) is $f(x) > 0$?

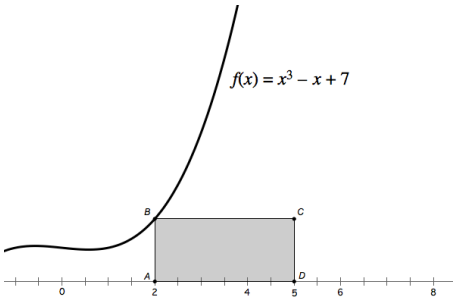


    a. $(1, 3)$ b. $[1, 3]$ c. $(-\infty, 1) \cup (3, \infty)$ d. $(-\infty, 1] \cup [3, \infty)$

23. At 3:00 PM, 250 bacterial cells are being grown in a lab. If the number of cells continued to double every half hour, at what time will there be 32,000 cells?

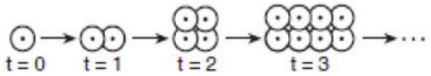
    a. 6:00 PM b. 6:30 PM c. 10:00 PM d. 11:00 PM

24. What is the area of the rectangle shown in the figure to the right? (Note: The graph is not drawn to scale.)



    a. 15 units^2 b. 20 units^2 c. 39 units^2 d. 65 units^2

25. The accompanying diagram represents the biological process of cell division.



If this process continues, which expression best represents the number of cells at any time, t?

    a. $t + 2$ b. $2t$ c. t^2 d. 2^t

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Open Response #1

The formula $s = \sqrt{30df}$ is used to find how fast a car was traveling before an accident, where s = speed (in mph), d = the length (in feet) of the skid mark left by the car, and f = the coefficient of friction (road condition). Officer Barton must decide if the driver in a car accident was speeding in a 35 mph speed zone. The car's skid mark length was measured to be 76 feet. The coefficient of friction is 0.80.

a. Write the equation that models the speed of the car before the accident.

b. Solve the equation in part (a). Round your answer to the nearest tenth.

c. Was the driver speeding in the 35 mph speed zone? Show your work and/or explain your answer.

d. If the skid mark had been 4 times longer, how many times as fast would the driver have been going as compared to the speed in part (b)? Show your work and/or explain your answer.

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Open Response #2

Ms. Daniels' sewing class students are making stuffed teddy bears for a fundraiser. They are making boy and girl bears. Each week they purchase enough materials to create 40 bears. Let x represent the number of girl bears and y represent the number of boy bears.

a. Write an inequality that shows the possible number of boy and girl bears they can make each week.

b. It cost $3 to create each bear. The class will charge $15 for girl bears and $12 for boy bears. They want to earn at least $540 a week. Write an inequality to describe this situation.

c. Give three possible solutions for the numbers of boy and girl bears that can be made to earn a profit over $540.

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Open Response #3

The $150 million Deep Space 1 spacecraft launched on October 22, 1998 used an ion engine to travel from Earth to the Comet Borrelly. It arrived on September 22, 2001. By ejecting a constant stream of xenon atoms into space, at speeds of thousands of kilometers per second, the new ion engine could run continuously for months. This allowed the spacecraft to accelerate to speeds that eventually could exceed the fastest rocket-powered spacecraft.

The Deep Space 1 ion engine produced a constant acceleration, starting from a speed of 44,000 km/hr, reaching a speed of 56,060 km/hr as it passed the comet 36 months later. The sequence for the first 7 months of operation is given by:

n	1	2	3	4	5	6	7
V_n	44,000	44,335	44,670	45,005	45,340	45,675	46,010

a. What is the general formula for V_n?

b. How much time had passed when the spacecraft reached a speed of 50,000? Round to the nearest whole number.

c. Suppose the Deep Space I ion engine could be left on for 30 years. What would be the speed of the spacecraft at that time?

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Answer Key:

1. D
2. B
3. B
4. D
5. B
6. B
7. C
8. B
9. A
10. C
11. A
12. C
13. B
14. C
15. A
16. D
17. A
18. D
19. C
20. D
21. A
22. C
23. B
24. C
25. D

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e. Write the equation that models the speed of the car before the accident.

$$\boldsymbol{S = \sqrt{30 \cdot 76 \cdot 0.80}}$$

f. Solve the equation in part (a). Round your answer to the nearest tenth.

$$\boldsymbol{S = 42.7}$$

g. Was the driver speeding in the 35 mph speed zone? Show your work and/or explain your answer.

Yes. The driver's speed was 42.7 mph (as shown in part b), which is greater than the speed limit (35 mph).

h. If the skid mark had been 4 times longer, how many times as fast would the driver have been going as compared to the speed in part (b)? Show your work and/or explain your answer.

Hypothetical skid mark $\boldsymbol{76 \times 4 = 304}$ **feet**

Hypothetical Speed $\boldsymbol{S = \sqrt{30 \cdot 304 \cdot 0.08} = 7296}$

$$\boldsymbol{\frac{hypothetical\ speed}{actual\ speed} = \frac{7296}{42.7} = 170.87}$$

The driver would have been going 170.87 times faster

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d. Write an inequality that shows the possible number of boy and girl bears they can make each week.

$$x + y \leq 40$$

or

$$y \leq 40 - x$$

e. The class will charge \$15 for girl bears and \$12 for boy bears. They want to earn at least \$540 a week. Write an inequality to describe this situation.

$$15x + 12y \geq 540$$

or

$$y \geq 45 - \frac{5}{4}x$$

f. Give three possible solutions for the numbers of boy and girl bears that can be made to earn at least \$540.

Any of the combinations given in the table are acceptable answers. (x=girls, y=boys)

X=20 Y=20	X=21 Y=19	X=22 Y=18	X=23 Y=17	
X=24 Y=15 or 16	X=25 Y=14 or 15	X=26 Y=13 or 14	Y=27 Y=12 or 13	
X=28 Y=10,11 or 12	X=29 Y=9,10 or 11	X=30 Y=8,9 or 10	X=31 Y=7,8 or 9	
X=32 Y=5,6,7 or 8	X=33 Y=4,5,6 or 7	X=34 Y=3,4,5 or 6	X=35 Y=2,3,4 or 5	
X=36 Y=0,1,2,3 or 4	X=37 Y=0,1,2 or 3	X=38 Y=0,1 or 2	X=39 Y=0 or 1	X=40 Y=0

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V_n	44,000	44,335	44,670	45,005	45,340	45,675	46,010

d. What is the general formula for V_n?

$$\boldsymbol{V_n = 44000 + 335(n - 1)}$$

e. How much time had passed when the spacecraft reached a speed of 50,000? Round to the nearest whole number.

$$\boldsymbol{50000 = 44000 + 335(n - 1)}$$
$$\boldsymbol{n = 19}$$

Answer: 19 months, or 1 year and 7 months

f. Suppose the Deep Space I ion engine could be left on for 30 years. What would be the speed of the spacecraft at that time?

$$\boldsymbol{V_{360} = 44000 + 335(360 - 1)}$$

$$\boldsymbol{V_{360} = 164265\ km/hr}$$