

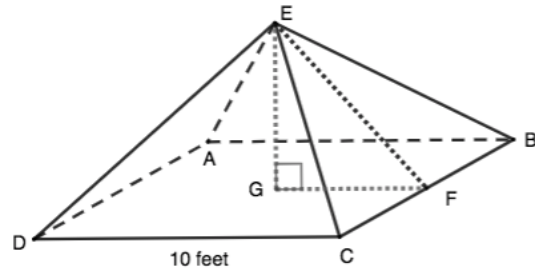
# ACTM 2019 Regional Geometry Exam

NAME \_\_\_\_\_

This exam includes 25 multiple-choice questions and three open-response questions that might be used as tie breakers. For questions 1 through 25 (the multiple-choice questions), mark your answer choice in the appropriate location on the sheet provided. After completing questions 1 through 25, answer each tie breaker question in sequential order (i.e., complete Question #1 first, then Question #2, and then Question #3 last). Be sure that your name is printed on each of the tie breaker questions. When time is called, you will be asked to turn in your multiple choice answer sheet and the tie-breaker pages.

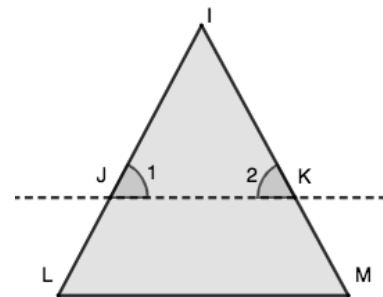
1. A tent is in the shape of a square pyramid as shown below. The center pole,  $\overline{EG}$ , is 5 feet long. A zipper is installed from the top of the tent, point  $E$ , to the midpoint of a side, point  $F$ . What is the length of the zipper?

- (a) 5 feet
- (b)  $5\sqrt{2}$  feet
- (c)  $5\sqrt{3}$  feet
- (d) 10 feet



2. Adam is cutting triangular tiles for his kitchen. He's cutting  $\triangle ILM$  along  $\overleftrightarrow{JK}$ , which is parallel to  $\overline{LM}$ . In order to increase the size of his piece, he moves  $\overleftrightarrow{JK}$  toward  $\overline{LM}$  while keeping them parallel. Which is the most accurate description of the effect on the measures of  $\angle 1$  and  $\angle 2$ ?

- (a)  $m\angle 1$  and  $m\angle 2$  increase
- (b)  $m\angle 1$  and  $m\angle 2$  stay the same
- (c)  $m\angle 1$  and  $m\angle 2$  decrease
- (d)  $m\angle 1$  increases and  $m\angle 2$  decreases

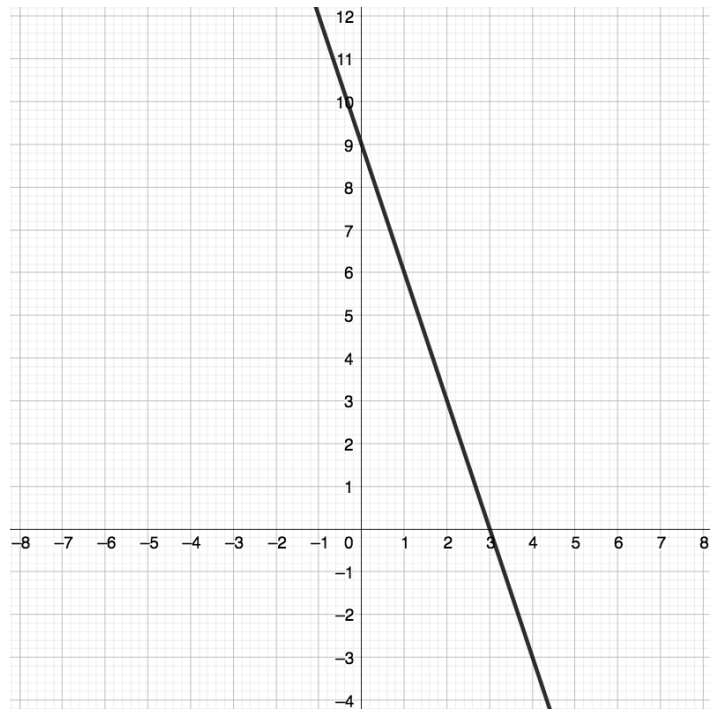


3. The point  $(2, -5)$  is an endpoint to a diameter of circle  $C$ . The equation  $x^2 + y^2 - 4x + 2y = 11$  represents circle  $C$ . What is the  $y$ -coordinate of the other endpoint of this diameter?

- (a) -9
- (b) -1
- (c) 3
- (d) 6

4. The point  $H = (-5, -7)$  is reflected over the  $y$ -axis, then rotated  $180^\circ$  clockwise about the origin. What are the coordinates of the image of  $H$  after both transformations?
- (a)  $(5, -7)$
  - (b)  $(-5, 7)$
  - (c)  $(7, 5)$
  - (d)  $(7, -5)$
5. A pair of parallel lines is cut by a transversal. Which of the following is *not* true?
- (a) Consecutive interior angles are congruent
  - (b) Corresponding angles are congruent
  - (c) Alternate interior angles are congruent
  - (d) Vertical angles are congruent
6. An adult (full-size) basketball has a diameter 2 inches greater than the diameter of a youth basketball. What is the difference in circumference between an adult and youth basketball?
- (a)  $2\pi$  inches
  - (b)  $\pi + 2$  inches
  - (c)  $\pi + 4$  inches
  - (d)  $4\pi$  inches

7. Which equation represents a line that is perpendicular to the line shown in the graph below?

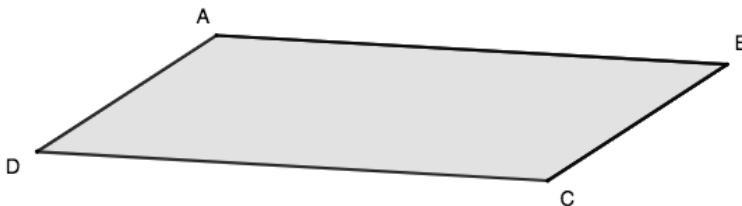


- (a)  $y = -3x - \frac{1}{9}$
- (b)  $y = -3x + \frac{1}{9}$
- (c)  $y = \frac{1}{3}x + 9$
- (d)  $y = -\frac{1}{9}x + 9$

8.  $\overline{AB}$  has endpoints at  $A = (-5, 24)$  and  $B = (10, -3)$ . Point  $N$  lies on the line segment so that  $AN = 3BN$ . What are the coordinates of point  $N$ ?

- (a) (1.25, 17.25)
- (b) (5, 6)
- (c) (2.5, 10.5)
- (d) (6.25, 3.75)

9. Parallelogram  $ABCD$  is shown below, and  $m\angle A = 4 \cdot (m\angle B)$ . What is  $m\angle C$ ?



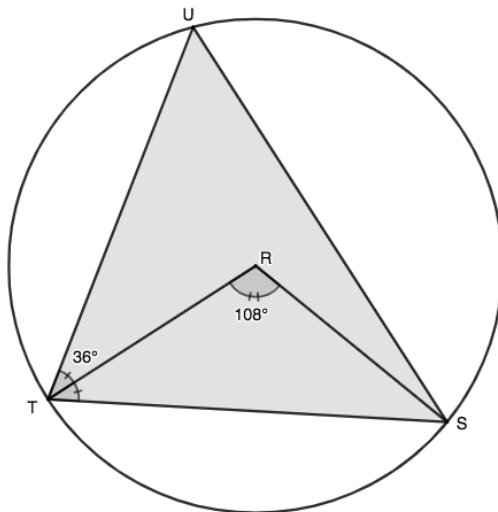
- (a)  $36^\circ$
  - (b)  $45^\circ$
  - (c)  $135^\circ$
  - (d)  $144^\circ$
10. The following information is known about the quadrilateral  $ABCD$ :
- $\overline{BC}$  is parallel to  $\overline{AD}$
  - $\overline{AB}$  is not congruent to  $\overline{CD}$
  - $\angle CDA$  is a right angle

Which of the following must be true about quadrilateral  $ABCD$ ?

- (a)  $ABCD$  is a rhombus
- (b)  $ABCD$  is a rectangle
- (c)  $ABCD$  is a parallelogram
- (d)  $ABCD$  is a trapezoid

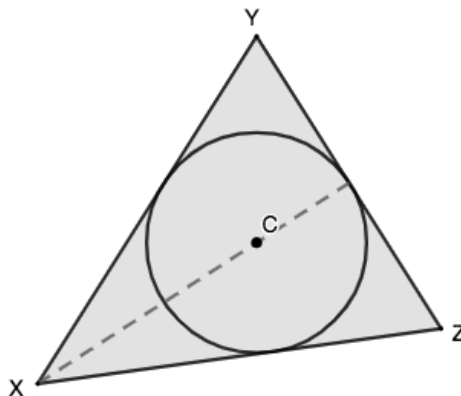
11. Triangle  $\triangle STU$  is inscribed in a circle with center  $R$ .  $\overleftrightarrow{RT}$  bisects  $\angle STU$ . As shown in the figure below,  $m\angle SRT = 108^\circ$  and  $m\angle RTU = 36^\circ$ . Determine the measure of  $\angle RSU$ .

- (a)  $18^\circ$
- (b)  $20^\circ$
- (c)  $36^\circ$
- (d)  $54^\circ$



12. A circle with center  $C$  is inscribed in  $\triangle XYZ$ . Which of the following *must* be true?

- (a)  $\overleftrightarrow{XC}$  bisects angle  $\angle ZXY$
- (b)  $\overleftrightarrow{XC}$  and  $\overline{YZ}$  are perpendicular
- (c)  $\overleftrightarrow{XC}$  intersects  $\overline{YZ}$  at its midpoint
- (d) Both (b) and (c) must be true

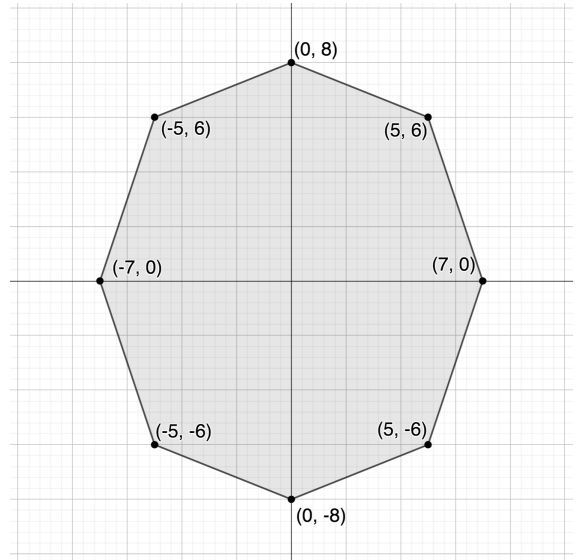


13.  $\triangle ABC$  is dilated from some center  $D$  by a scale factor of 3 to form  $\triangle A'B'C'$ . What is the value of

$$\frac{\text{Area of } A'B'C'}{\text{Area of } ABC}$$

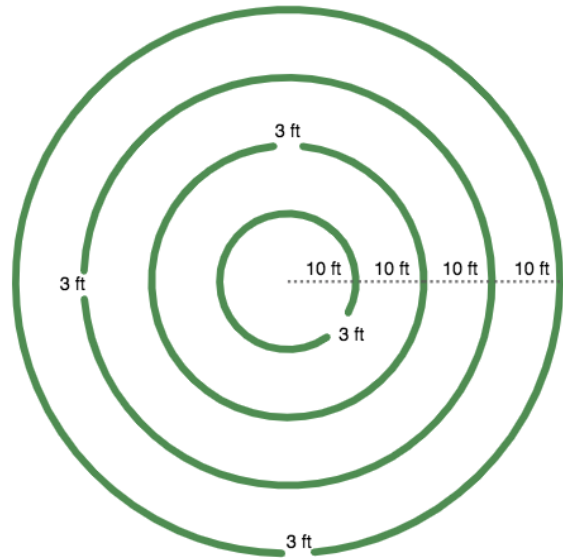
- (a) 1
- (b) 3
- (c) 9
- (d) 27

14. Rotation about the origin by angle  $\theta$  maps the octagon in the figure below onto itself. If  $0 < \theta \leq 2\pi$ , how many different values of  $\theta$  are possible?



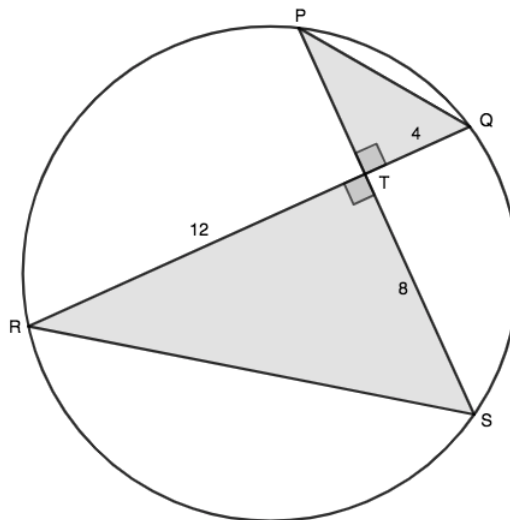
- (a) One
- (b) Two
- (c) Four
- (d) Eight

15. The Pun Family Maize Maze is designed as a series of concentric circles this year. Four concentric circles of corn are planted as shown in the figure, each 10 feet apart. A 3 foot opening is left in each circle, as shown below. How many linear feet of corn will need to be planted?



- (a)  $80\pi - 12$
- (b)  $100\pi - 12$
- (c)  $200\pi - 12$
- (d)  $3000\pi - 12$

16. Chords  $\overline{PS}$  and  $\overline{QR}$  are perpendicular and create triangles  $\triangle PQT$  and  $\triangle RST$ . The segments have lengths as shown in the figure below. What is the value of  $\sin(m\angle PQT)$ ?



- (a)  $\frac{3}{\sqrt{13}}$   
 (b)  $\frac{3}{2\sqrt{13}}$   
 (c)  $\frac{2}{\sqrt{13}}$   
 (d)  $\frac{1}{\sqrt{13}}$
17. A circle has center  $(-1, 5)$ , and the point  $(4, -4)$  is on the circle. Which of the following is an equation of the circle?
- (a)  $(x - 1)^2 + (y + 5)^2 = 106$   
 (b)  $(x + 1)^2 + (y - 5)^2 = 106$   
 (c)  $(x + 1)^2 + (y - 5)^2 = \sqrt{106}$   
 (d)  $(x - 1)^2 + (y + 5)^2 = \sqrt{106}$
18. Which of the following transformations *will not* carry the set of points on a non-square rectangle onto itself?
- (a) A reflection over the segment joining the midpoints of opposite sides  
 (b) A rotation of  $180^\circ$  clockwise about the intersection of the diagonals  
 (c) A rotation of  $360^\circ$  counterclockwise about one vertex  
 (d) A reflection over a diagonal
19. Which of the following is *not* true for all values of  $x$  for which the functions are defined?
- (a)  $\sin^2 x + \cos^2 x = \tan^2 x$   
 (b)  $1 + \cot^2 x = \csc^2 x$   
 (c)  $1 + \tan^2 x = \sec^2 x$   
 (d)  $1 = \sin^2 x + \cos^2 x$

20. Figure A shows a right triangular prism. Its base has length 5 and height 8. The prism is 16.3 units tall. Figure B shows an oblique triangular prism. Its base has length 8 and height 5. The prism is 16.3 units tall. How do the volumes of the prisms compare?

(a) The volume of the prism in Figure A must be greater

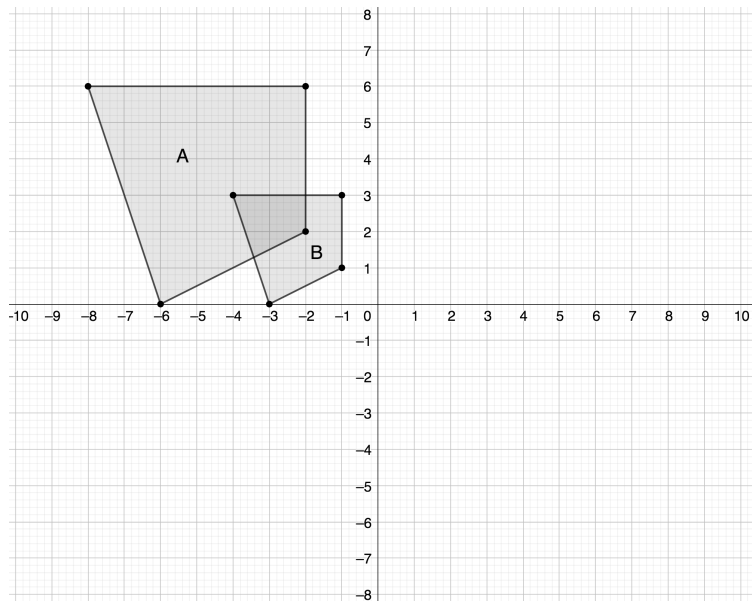
(b) The volume of the prism in Figure B must be greater

(c) The volume of the prisms in the two figures must be the same

(d) There is not enough information to determine which volume is greater

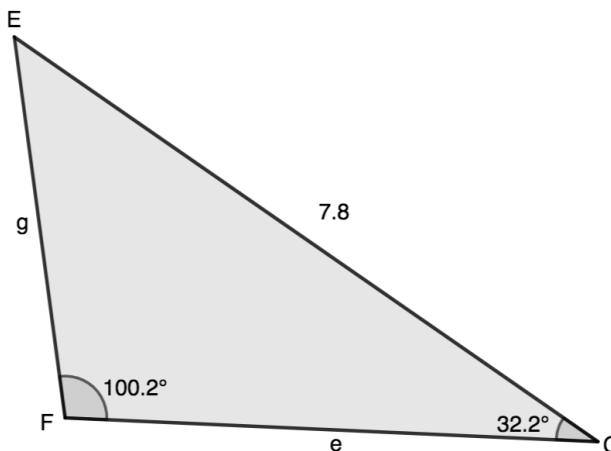
21. The diagram shows two quadrilaterals graphed on a coordinate plane. Which transformation on quadrilateral A can be used to verify that it is similar to quadrilateral B?

- (a) A dilation by a scale factor of  $\frac{1}{2}$  centered at the origin
- (b) A dilation by a scale factor of  $\frac{1}{2}$  centered at  $(-6, 0)$
- (c) A dilation by a scale factor of  $\frac{3}{4}$  centered at the origin
- (d) A dilation by a scale factor of 2 centered at the origin



22. Triangle  $\triangle EFG$  has measurements shown below. What is the length of  $g$ , rounded to the nearest  $\frac{1}{10}$ ?

- (a) 14.0  
 (b) 5.8  
 (c) 4.2  
 (d) 2.6



23. A sector of a circle has an area of  $\frac{104\pi}{9}$  square inches and a central angle with measure  $65^\circ$ . What is the radius of the circle, in inches?

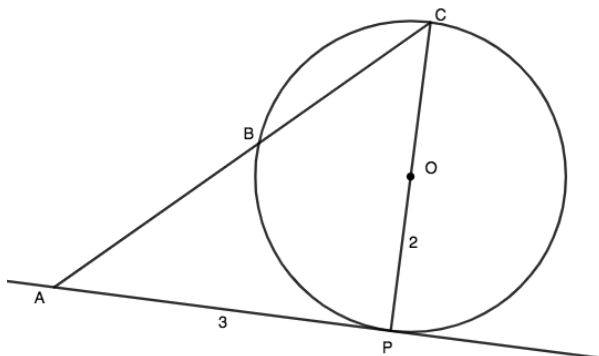
- (a) 5.7  
 (b) 8  
 (c) 64  
 (d) 104

24. In a right triangle  $\triangle ABC$ ,  $m\angle B = 90^\circ$ , and  $\tan(m\angle A) = \frac{4}{5}$ . What is the value of  $\sin(m\angle A)$ ?

- (a)  $\frac{3}{5}$   
 (b)  $\frac{5}{\sqrt{41}}$   
 (c)  $\frac{3}{4}$   
 (d)  $\frac{4}{\sqrt{41}}$

25. In the circle below with center  $O$  and radius 2,  $\overleftrightarrow{AP}$  is tangent to the circle at  $P$ , and  $\overline{AP}$  has length 3.  $\overline{CP}$  is a diameter of the circle. What is the length of  $\overline{BC}$ ?

- (a) 1.25  
 (b) 2  
 (c) 3.2  
 (d) 5

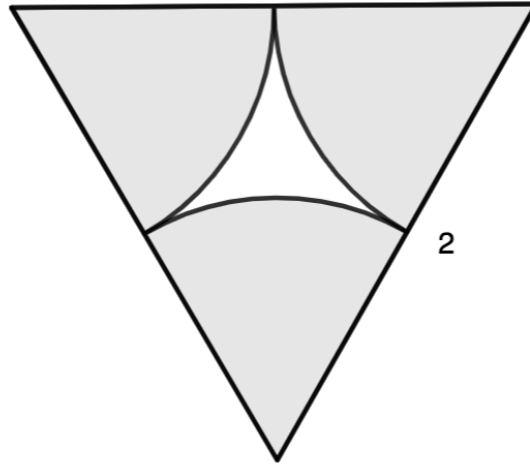




# Tiebreaker Question #1

NAME \_\_\_\_\_

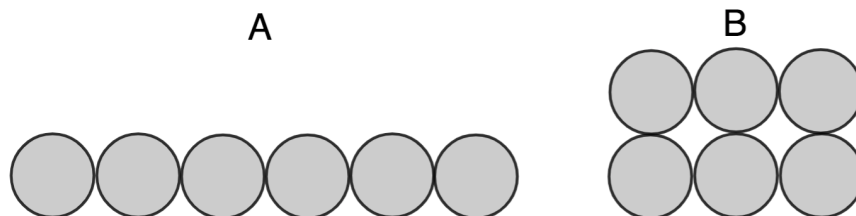
The figure below consists of 3 equal arcs in an equilateral triangle. Each side of the triangle measures 2 units. What is the area of the unshaded center section?



## Tiebreaker Question #2

NAME \_\_\_\_\_

A standard soda can is 4.83 inches high and 2.60 inches in diameter. A package of six soda cans can be boxed for shipment in one of two rectangular arrays for shipment: A single row of 6 cans side-by-side (Option A), or two rows of 3 cans each (Option B).



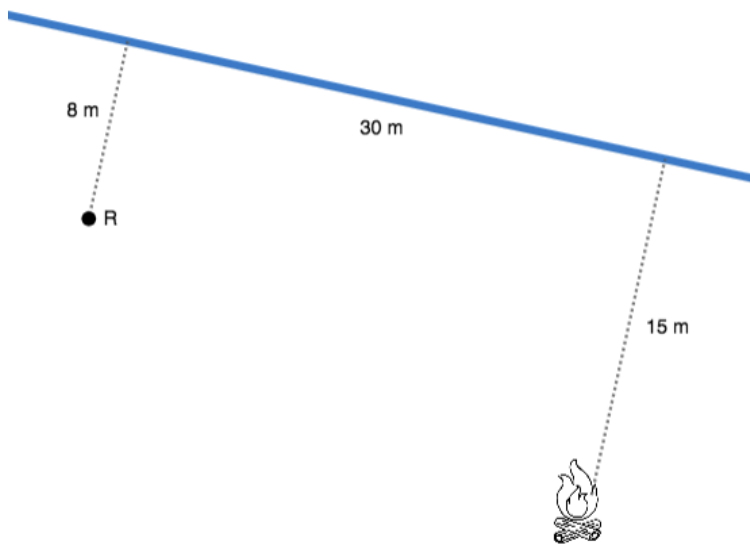
Cardboard will be used to box the cans on all sides (including top and bottom) with no overlap. Respond to both of the following questions, explaining or justifying your answer:

- Which box will require less cardboard to construct?
- How do the volumes of the two boxes compare?

### Tiebreaker Question #3

NAME \_\_\_\_\_

Rebekah is enjoying a campfire on the beach at her local lakeshore. She builds the campfire 15 m from the shoreline. She is 30 m downshore from her campfire and 8 m away from the shoreline when she notices her fire is growing out of control. Luckily she has a bucket with her. What's the minimum distance she can travel from her current location, to the shoreline to get water, then to the campfire to put it out?



# ACTM 2019 Regional Geometry Exam

## ANSWER KEY

### MULTIPLE CHOICE

1. b
2. b
3. c
4. b
5. a
6. a
7. c
8. d
9. d
10. d
11. a
12. a
13. c
14. b
15. c
16. a
17. b
18. d
19. a
20. c
21. a
22. c
23. b
24. d
25. c

### TIEBREAKER 1

Height of Triangle =  $h = \sqrt{2^2 - 1^2} = \sqrt{3}$  units

Area of Triangle =  $A_{\Delta} = \frac{1}{2} \cdot 2 \cdot \sqrt{3} = \sqrt{3}$  units<sup>2</sup>

Note that the three sectors each have angle of  $60^\circ$ , so they create a semicircle with radius 1 when taken together.

Area of shaded region =  $A_s = \frac{1}{2} \cdot \pi \cdot 1^2 = \frac{\pi}{2}$  units<sup>2</sup>

**Thus, the desired area is  $\sqrt{3} - \frac{\pi}{2} \approx 0.1612545$  units<sup>2</sup>**

### TIEBREAKER 2

Note that one can reason through the problem even without the given measurements. If the cans have diameter  $d$  and height  $h$ , Option A has surface area:

$$S_A = 2 \cdot 6d \cdot h + 2 \cdot d \cdot h + 2 \cdot 6d \cdot d = 14dh + 12d^2$$

and Option B has surface area

$$S_B = 2 \cdot 3d \cdot h + 2 \cdot 2d \cdot h + 2 \cdot 3d \cdot 2d = 10dh + 12d^2$$

Since  $d$  and  $h$  are both positive,  $S_A - S_B = 4dh > 0$ .

(If numbers are used, note  $S_A \approx 256.9$  in<sup>2</sup> and  $S_B \approx 206.7$  in<sup>2</sup>.)

**Thus, Option B requires less cardboard.**

Since both boxes have the same height, one need only consider the area of the base to compare the volumes. Since  $6d \cdot d = 3d \cdot 2d$ , the base areas are the same.

(If numbers are used, note  $V_A = V_B = 391.8$  in<sup>3</sup>.)  $V=159.9$  in<sup>3</sup>

**Thus, the volumes will be the same.**

*Full credit should be given for correct responses with correct reasoning, whether numbers are used or not. Correct answers with incorrect or missing reasoning should not be given full credit.*

### TIEBREAKER 3

The minimum distance can be determined by reflecting one point (say, the campfire) over the line determined by the bank. The point where the line between  $R$  and the reflection of the campfire crosses the shoreline is the point where Rebekah should visit the water. The length of that line segment is the minimum distance.

Assigning a coordinate system makes that easier - for example, letting the shoreline be the horizontal axis,  $R = (0, -8)$ , and the reflection of the campfire be  $(30, 15)$ . Using that coordinate system, **the distance between the points is**

$$D = \sqrt{(30 - 0)^2 + (15 + 8)^2} = \sqrt{1429} \approx 37.8 \text{ m}$$

*Note that other coordinate systems are possible, and credit should be given for either exact or approximate (rounded) answers.*