ACTM 2019 State Geometry Exam

NAME_____

This exam includes 25 multiple-choice questions and three open-response questions that might be used as tie breakers. For questions 1 through 25 (the multiple-choice questions), mark your answer choice in the appropriate location on the sheet provided. After completing questions 1 through 25, answer each tie breaker question in sequential order (i.e., complete Question #1 first, then Question #2, and then Question #3 last). Be sure that your name is printed on each of the tie breaker questions. When time is called, you will be asked to turn in your multiple choice answer sheet and the tie-breaker pages. Note that figures are not necessarily drawn to scale.

1. Which of the following *must* be a parallelogram?

   (a) A trapezoid

   (b) A kite

   (c) A rhombus

   (d) The union of two triangles that share a common side.

2. A guy line is attached to a radio antenna at a point $\frac{2}{3}$ of the distance from the ground to the top of the antenna. It forms an angle of $60°$ with the ground and is anchored 50 feet from the base of the tower. What is the height of the tower, in feet? Round to the nearest tenth of a foot if necessary.

   (a) 65 ft

   (b) 75 ft

   (c) 86.6 ft

   (d) 129.9 ft

3. A grain silo is in the shape of a right circular cylinder topped by a right circular cone. The interior radius of the silo is 11 feet, the cylindrical portion is 24 feet tall, and the conical top is 9 feet tall. What is the total interior volume of the silo?

   (a) 786.5π ft^3

   (b) 3146π ft^3

   (c) 3267π ft^3

   (d) 3630π ft^3

4. What is the area of a square inscribed in a circle of circumference 8π cm?

   (a) 32 cm^2

   (b) $4\sqrt{2}$ cm^2

   (c) 32π cm^2

   (d) 8 cm^2

5. Object X' is the image of object X resulting from two rotations (possible around different points). Object X' could *possibly* be be obtained from object X as the result of what single transformation?

   (a) A reflection

   (b) A translation

   (c) A glide reflection

   (d) None of the above are possible

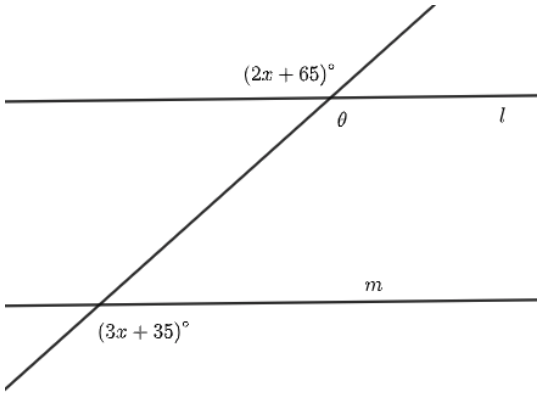
6. Lines l and m are parallel in the figure below. What is the measure of angle θ? (The given angle measures are $(2x + 65)^\circ$ along line l and $(3x + 35)^\circ$ along line m.)

   (a) 125°

   (b) 97°

   (c) 30°

   (d) 16°



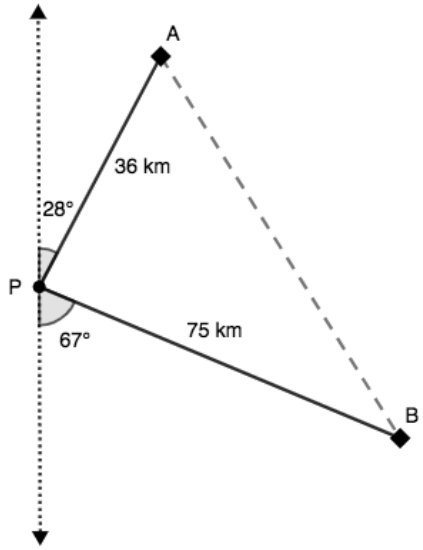
7. Ship A left Port P on a bearing of N28°E and is currently 36 km from port. Ship B left Port P on a bearing of S67°E and is currently 75 km from port. How far apart, rounded to the nearest $\frac{1}{10}$ km, are the two ships currently? (Note: *for the purpose of this problem, consider the surface of the body of water to be a flat plane, ignoring the curvature of the earth*)

   (a) 69.4 km

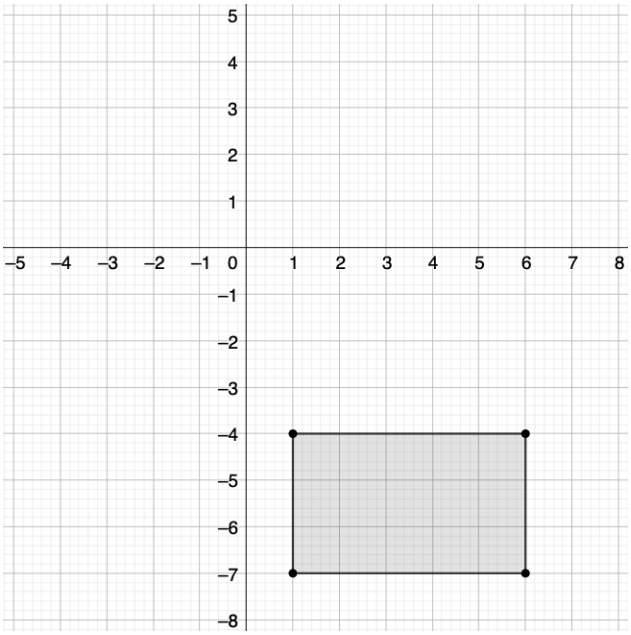
   (b) 80.3 km

   (c) 83.2 km

   (d) 86.0 km



8. The rectangle shown below has vertices at $(1, -4)$, $(6, -4)$, $(6, -7)$, and $(1, -7)$. Suppose the rectangle is dilated by a factor of $\frac{1}{4}$ centered at the origin. Which of these points will be on the interior of the image of the rectangle under the dilation?



(a) $(10, -20)$

(b) $(1, -4)$

(c) $(1, -2)$

(d) $(1, \frac{-3}{2})$

9. Line l has slope equal to the radius of the circle described by the equation $x^2+12x+y^2-5y = 0$ and passes through the center of the circle. Which of the following points is on line l?

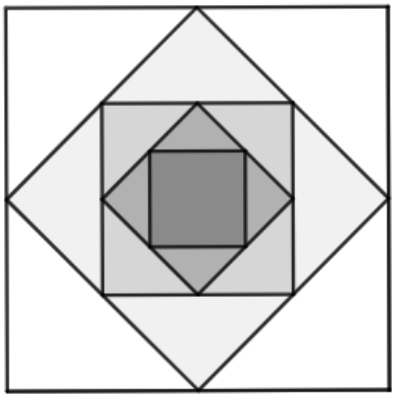
(a) (-3,-17)

(b) (-3,22)

(c) (2,52)

(d) $(6, -\frac{5}{2})$

10. In the sequence of 5 squares shown below, each square is symmetrically inscribed within its predecessor. That is, its vertices of the midpoints of the next-largest square. Find the ratio of the side lengths of the largest square to the smallest.



(a) 2:1

(b) $\sqrt{2} : 1$

(c) $2\sqrt{2} : 1$

(d) 4:1

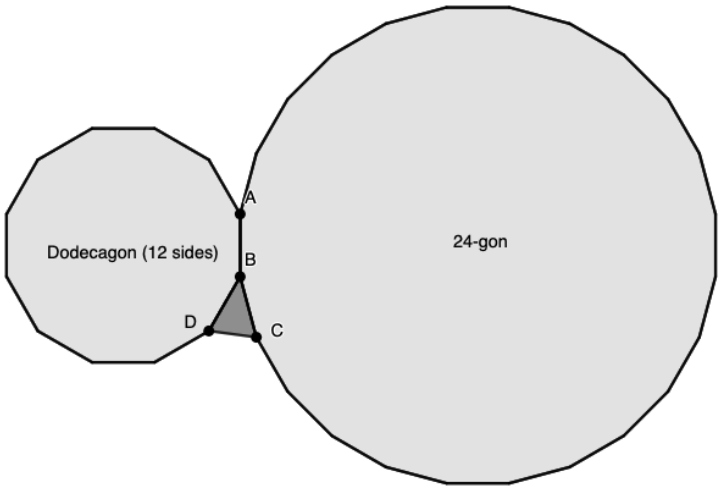
11. The regular dodecagon and 24-gon shown in the figure below are regular, and the labeled points are vertices of the polygons as indicated in the figure. What is the measure of $\angle BDC$?

(a) $67.5°$

(b) $60°$

(c) $45°$

(d) $30°$



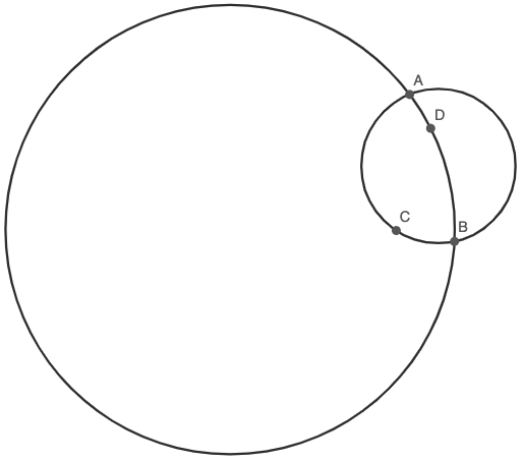
12. Two circles intersect at A and B. $m\widehat{ACB} = 170°$ on the smaller circle and $m\widehat{ADB} = 40°$ on the larger circle, and the radius of the larger circle is 12, what is the radius of the smaller circle (if necessary, round your answer to the nearest tenth of unit)?

(a) 0.2

(b) 2.8

(c) 4.2

(d) 6.0



13. Each edge of a solid cube of silver measures 4 centimeters. A metallurgist melts the cube and uses all of the molten silver to make two smaller identical solid cubes. What is the length of the edge of one of the smaller cubes?

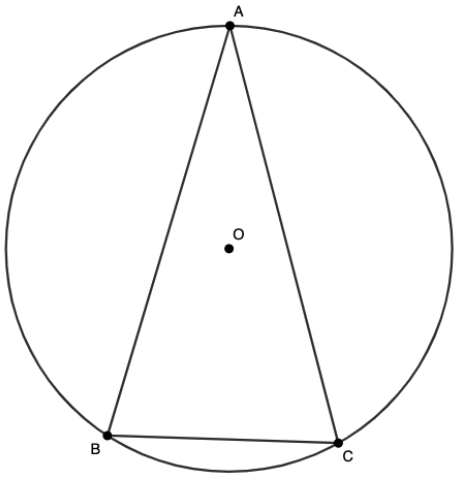
(a) 2 centimeters

(b) $2\sqrt{4}$ centimeters

(c) $2\sqrt[3]{4}$ centimeters

(d) $2\sqrt[4]{4}$ centimeters

14. Points A, B, and C are points on a circle with center O, and $\overline{AB} \cong \overline{AC}$. If $\triangle OBC$ is equilateral, what is the measure of $\angle BCA$?



(a) $60°$

(b) $75°$

(c) $80°$

(d) It cannot be determined from the information given.

15. In the xy plane, point K lies on l and has coordinate $(-3, 5)$. If l is rotated counterclockwise $90°$ about the origin, then translated 4 units right and 7 units down, what will be the coordinates of K', the image of K under this series of transformations?

(a) $(-1, -10)$

(b) $(9, -4)$

(c) $(1, -12)$

(d) $(9, -10)$

16. What is the value of $\cos(\tan^{-1}(\frac{7}{3}))$?

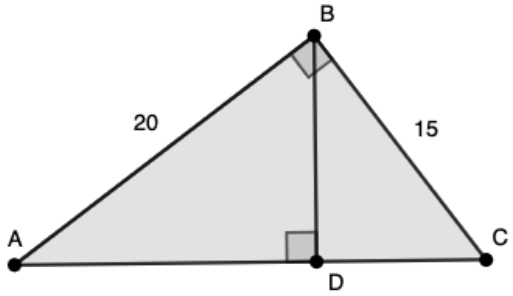
(a) $\frac{7}{3}$

(b) $\frac{3}{\sqrt{58}}$

(c) $\frac{7}{\sqrt{58}}$

(d) It cannot be determined from the information given.

17. In the triangle below, $AB = 20$, $BC = 15$, and both $\angle ADB$ and $\angle ABC$ are right angles. What is the length of $\overline{AD}$?



(a) 9

(b) 15

(c) 16

(d) 25

18. A triangle is inscribed in a circle. Which of the following *must* be true about the center of the circle?

    (a) It lies on the perpendicular bisector of at least one side of the triangle.

    (b) It is in the interior of the triangle.

    (c) It lies on the angle bisector of at least one angle of the triangle.

    (d) It lies on at least one median line of the triangle.

19. Line m is tangent to a circle with center $(2, 3)$ at the point $(3, -2)$. Which of the following is an equation for line m?

    (a) $y = -5x + 13$

    (b) $y = \frac{1}{5}x - \frac{13}{5}$

    (c) $y = \frac{-1}{5}x - \frac{7}{5}$

    (d) $y = 5x - 17$

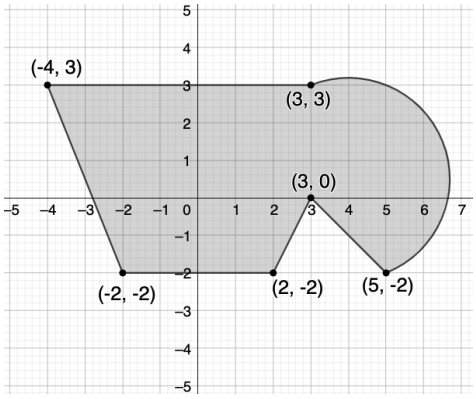
20. What is the area of the shaded region shown below? You may assume that the line segment with endpoints $(3, 3)$ and $(5, -2)$ is a diameter of the semicircular region.

    (a) $7\sqrt{29} - 3 + \frac{29\pi}{8}$

    (b) $32 + \frac{29\pi}{4}$

    (c) $32 + \frac{29\pi}{8}$

    (d) None of the above



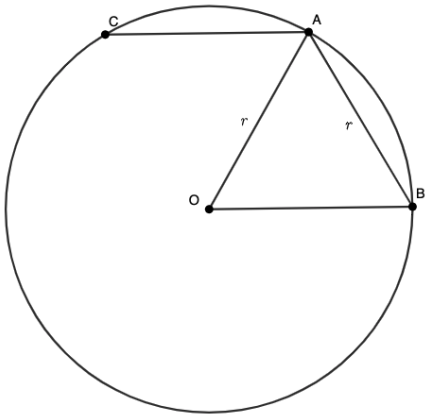
21. In the diagram below, $\angle AOB$ subtends a chord with length equal to the radius r of the circle. Which of the following *must* be true?

    (a) Point A lies on the perpendicular bisector of $\overline{OB}$

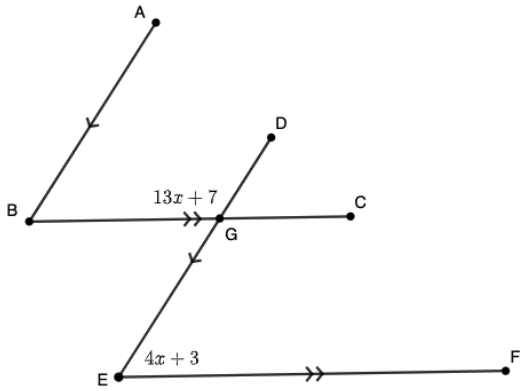
    (b) $m\angle OBA = 60°$

    (c) If $\overline{CA} \parallel \overline{OB}$, then $CA = r$.

    (d) All of the above must be true.



22. In the figure below, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$, $m\angle BGD = 13x + 7$, and $m\angle GEF = 4x + 3$. What is the measure of $\angle ABC$?



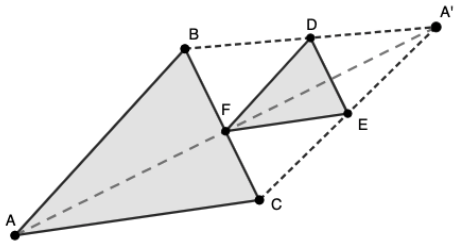
(a) 10°

(b) 43°

(c) 45°

(d) 137°

23. In the diagram below, point A' is the image of point A reflected over line $\overleftrightarrow{BC}$, and points D and E are the midpoints of BA' and CA', respectively. Point F is on the line segment $\overline{AA'}$. Which of the following *must* be true?



(a) $\triangle ABC$ is the image of $\triangle FDE$ under a dilation of magnitude 2 centered at A'

(b) The area of $\triangle FDE$ is half the area of $\triangle ABC$.

(c) F is the midpoint of $\overline{BC}$

(d) All of the above must be true.

24. Which of the following transformations will carry a regular octagon onto itself? Consider the *center* to be the midpoint of any diagonal connecting opposite vertices.

(a) A rotation of 60° about the center.

(b) A translation by the length of one side.

(c) A reflection over a diagonal connecting two opposite vertices.

(d) All of the above

25. A sector of a circle has an area of $\frac{17\pi}{2}$ square centimeters and a central angle with measure 85°. What is the radius of the circle, in centimeters?

(a) $\sqrt{\frac{17}{2}}$

(b) 3

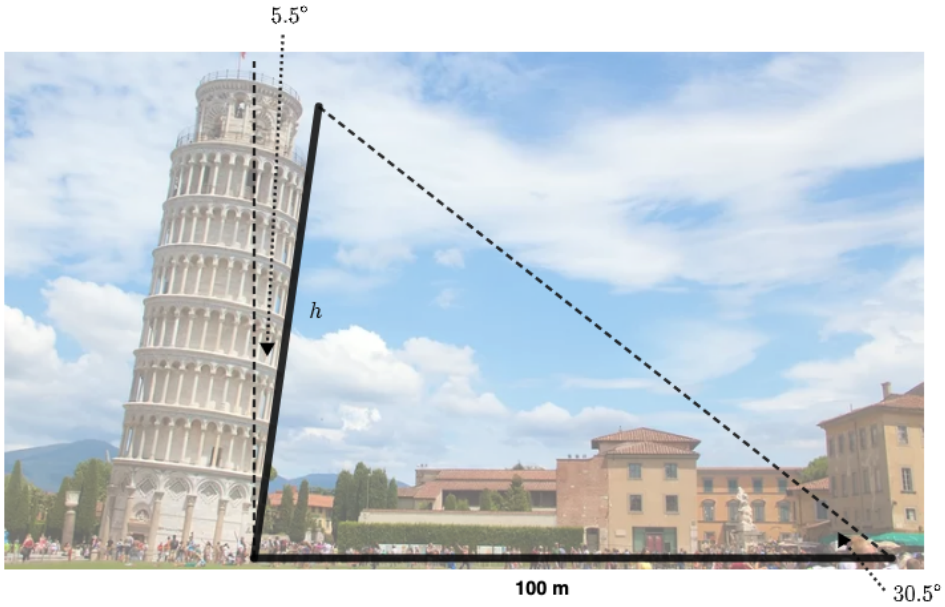
(c) 6

(d) $\frac{17\sqrt{10}}{20}$

Tiebreaker Question #1

NAME_____ SCHOOL_____

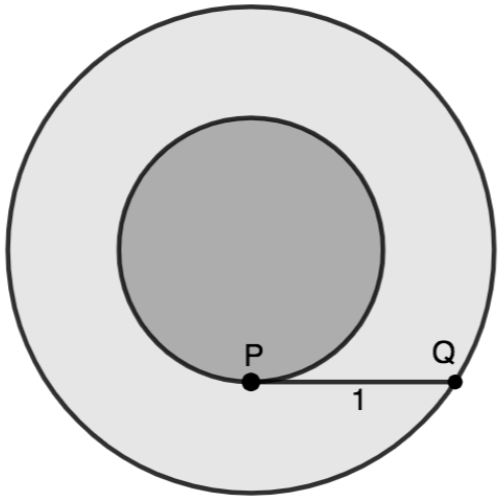
The Leaning Tower of Pisa leans 5.5° from the vertical. At a distance of 100 meters from the base of the tower, the angle of elevation to the top is 30.5°. What is the height of the tower, h, as measured along the side, as shown in the image below? Round your answer to the nearest hundredth of a meter.



Tiebreaker Question #2

NAME_____ SCHOOL_____

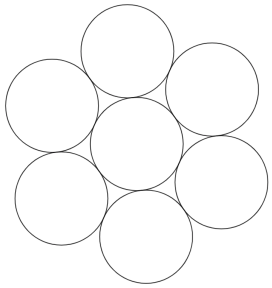
In the figure below, segment $\overline{PQ}$ is 1 unit long. $\overline{PQ}$ is tangent to the inner circle at point P and intersects the outer circle at point Q. The two circles are concentric. What is the area of the annular region (the light gray region) between the circles?



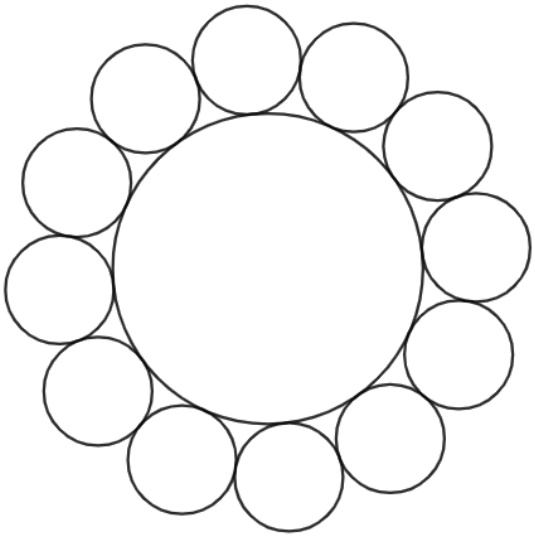
Tiebreaker Question #3

NAME_____ SCHOOL_____

Seven circles of the same radius can be arranged so that a central circle is tangent to the other six and each outer circles is tangent to the central circle and its two neighbors, as shown below:



Twelve circles are placed around a central circle, as pictured below. If the radius of the central circle is 1, what is the radius of each of the outer circles?



ACTM 2019 State Geometry Exam

ANSWER KEY

MULTIPLE CHOICE

1. c	6. a	11. a	16. b	21. d
2. d	7. b	12. c	17. c	22. b
3. c	8. d	13. c	18. a	23. a
4. a	9. b	14. b	19. b	24. c
5. b	10. d	15. a	20. c	25. c

TIEBREAKER 1

By subtraction, the angle at the base of the tower is $90° - 5.5° = 84.5°$. These leaves the third angle of the triangle to be $180° - (84.5° + 30.5°) = 65°$.

Utilizing the Law of Sines, we have $\frac{h}{\sin 30.5°} = \frac{100}{\sin 65°}$. Thus, $h = \frac{100 \sin 30.5°}{\sin 65°} \approx 56$ m.

TIEBREAKER 2

Let r be the radius of the inner circle, and the s be the difference in the two radii. That is, the radius of the outer circle is $r + s$. The area of the annular region is thus

$$\pi(r+s)^2 - \pi r^2 = \pi(r^2 + 2rs + s^2) - \pi r^2 = \pi(2rs + s^2)$$

Note that $\overline{PQ}$ is tangent to the inner circle, so it forms a right angle with a segment from P to the center. The center, P, and Q thus form a right triangle, so

$$r^2 + 1^2 = (r+s)^2 \iff 1 = 2rs + s^2$$

Substituting into the area found above, we can see that the area of the annular region must be π.

TIEBREAKER 3

Consider a triangle formed by the center of the central circle (let's call it O) and the centers of two adjacent circles around the outside. The angle at O would have measure $\frac{360°}{12} = 30°$. Since the two outside circles are tangent, a segment from O to the midpoint of the opposite side is tangent to both circles and thus perpendicular to the other side of the triangle. This yields a right triangle with angle $15°$. If the radius of one of the outside circles is r, the right triangle has hypotenuse of length $1 + r$ and opposite side of length r. Thus, we have

$$\sin 15° = \frac{r}{1+r}$$

.

Multiply both sides by $1 + r$ and solving for r yields $r = \frac{\sin 15°}{1 - \sin 15°} \approx 0.35$.