2018 – Regional Mathematics Contest

Geometry Test

In each of the following, choose the BEST answer and record your choice on the answer sheet provided. To insure correct scoring, be sure to make all erasures completely. The tie-breaker questions at the end will only be used to resolve ties in first, second, and/or third place. They will be used in the order given. Complete the first 25 questions before attempting the tie-breaker questions. Figures are not necessarily drawn to scale.

1. In order to construct the incenter of a triangle, which of these steps is necessary.

A. Construct the perpendicular bisector of the sides of the triangle.

B. Construct the Angle Bisectors for the angles of the triangle.

C. Construct the Medians of the triangle.

D. Construct the Altitudes of the triangle.

E. None of these.

2. Given ΔABC. D is the midpoint of AB, E is the midpoint of AC, and F is the midpoint of BC. Which of the following statements cannot be proven.

A. $\Delta ABC \cong \Delta EFC$

B. *DBFE* is a parallelogram

C. $\angle DAE \cong \angle BFE$

D. $\angle ECF$ and $\angle DBF$ are supplementary angles

E. None of these.

3. Given the statement: “If a figure is a rectangle, then it is a parallelogram.” If the statement is made that “If the figure is not a parallelogram, then it is a rectangle” would be which of the following?

A. Inverse

B. Converse

C. Contrapositive

D. Syllogism

E. None of these

4. The sides of a parallelogram are 12cm and 20 cm. One diagonal makes an angle of 30 degrees with the 20cm side. Find the length of the diagonal.

A. 23.3 cm

B. 24.0 cm

C. 11.3 cm

D. 16.7 cm

E. None of these.

5. Given the Venn Diagram, which statement(s) are true?

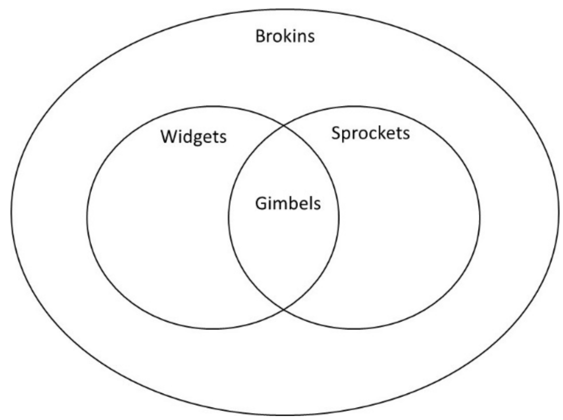
A. All Gimbels are Sprockets

B. All Widgets are Sprockets

C. All Sprockets are Gimbels

D. All Brokins are Gimbels

E. None of these



6. The area of the sector of a circle is known to be $24\pi\, in^2$. If the diameter of the circle is 16*in*, then find the measure of the central angle of the sector.

A. 135°

B. 33.75°

C. 67.5°

D. 90°

E. None of these.

7. Which angle has the largest measure in the following picture.

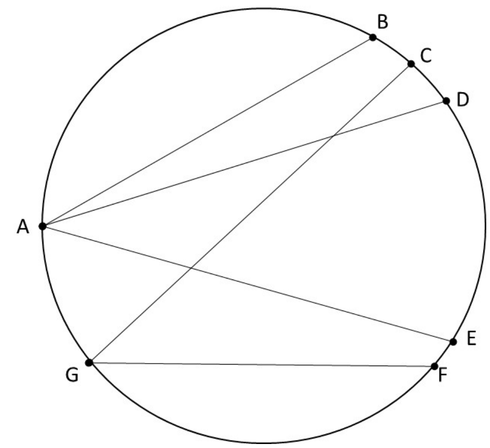
A. BAE

B. DAE

C. CGF

D. BAD

E. Not enough information.



8. Given: segment CD is a perpendicular bisector of segment AC and the $m\angle BAD = m\angle ABC$. What type of figure is ADBC?

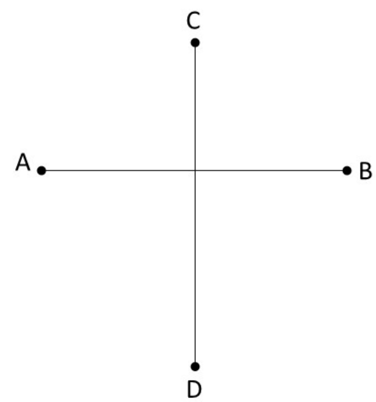
A. Chevron

B. Rectangle

C. Rhombus

D. Square

E. None of these



9. Given that the segment AB is tangent to the circle and BC is a diameter of the circle. Segment BD is 5 units long. The radius of the circle is 6.5 units. Find the length of segment AC.

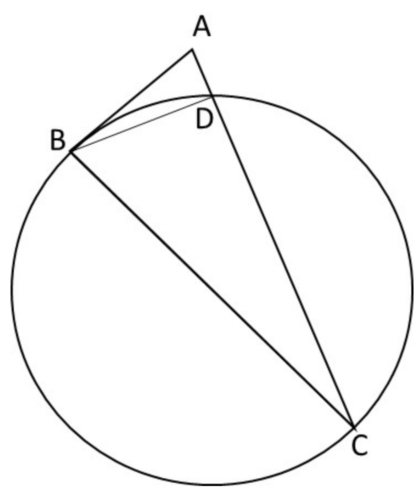
A. 14.1 units

B. 12 units

C. 17 units

D. 33.8 units

E. Not enough information.



10. The vertices of a parallelogram lie on the coordinate plane at (0, 4), (1, 1), (6, 5) and (7, 2). Find the perimeter of the parallelogram.

A. $2\left(\sqrt{53}+\sqrt{41}\right)$

B. $\left(\sqrt{73}+\sqrt{26}\right)$

C. 9.24

D. $2\left(\sqrt{10}+\sqrt{37}\right)$

E. None of these.

11. The points (1, 1), (1, 3) and (4, 1) enclose a triangular region. Which of the following sets of points does not enclose a congruent triangular region if the original triangular region undergoes a single rigid transformation or a series of rigid transformations?

A. (– 2, – 5), (– 5, – 3), and (– 5, – 5)

B. (– 4, – 2), (– 7, – 2), and (– 7, – 3)

C. (3.5, – 1.2), (6.5, – 1.2), and (6.5, – 3.2)

D. (– 2, 3), (– 5, 3), and (– 5, 1)

E. None of these.

12. You are given a circle with radius of 5m. Find the length of side b in the following figure.

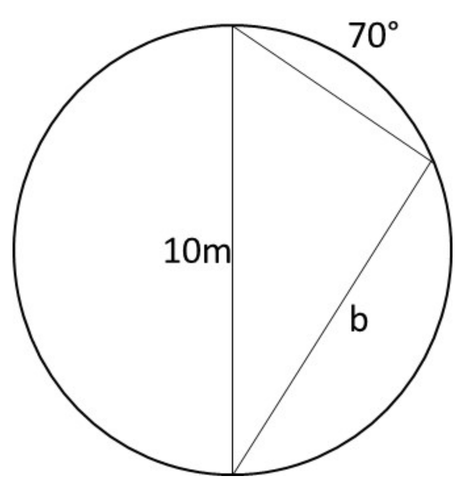
A. 8.7m

B. 5.7m

C. 3.4 m

D. 8.2m

E. None of these.



13. Given that 1 side of a rectangle lies on the line $y = \frac{2}{3}x + 2$. Which line cannot contain a side of the rectangle?

A. $y = \frac{2}{3}x - 4$

B. $y = -\frac{3}{2}x + 6$

C. $y = -\frac{6}{4}x - 5$

D. $y = \frac{2}{3}x + 8$

E. None of these.

14. Given a circle with center at (3, 2) and radius of 3 units. Which of the following points does not lie on the circle?

A. $(3,5)$

B. $(1, 2+\sqrt{5})$

C. $(5, 2+\sqrt{3})$

D. $(4, 2-2\sqrt{2})$

E. None of these.

15. Given the inscribed quadrilateral ABCD. Segment BD bisects angle ABC. If the $m\angle BAD = 80°$ and the $m\angle BDC = 47°$, then find the $m\angle ADB$.

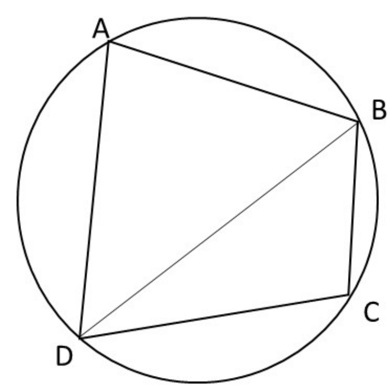
A. 33°

B. 47°

C. 67°

D. 53°

E. None of these.



16. Given that the central angle in a circle has a measure of 2.5 radians. Find the approximate length of the arc intercepted by that angle when the radius of the circle is 8 cm.

A. 20 cm

B. 0.35 cm

C. 6.4 cm

D. 50.2 cm

E. Not enough information.

17. Given ΔABC with $\overline{ED} \| \overline{CB}$. The length of AB is 9 inches, the length of AD is 3 inches, the length of DE is 5.2 inches, and the length of BC is 15.6 inches. Find the length of side AC.

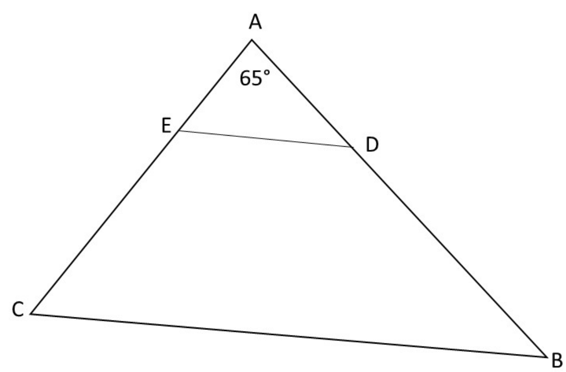
A. 5.6 inches

B. 8 inches

C. 8.4 inches

D. 12.7 inches

E. None of these.



18. Find the center and radius of the circle whose equation is given by: $x^2 + y^2 = 2(3x - 2y + 6)$

A. Center = (3, – 1), radius = 4 units

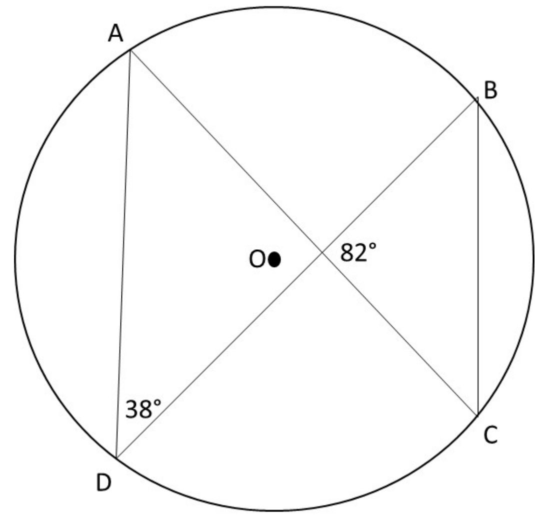
B. Center = (3, – 2), radius = 5 units

C. Center = (3, – 2), radius = $2\sqrt{3}$

D. Center = (3, 2), radius = 5

E. None of these.

19. Given Circle O with angular measures as shown. Find the $m\angle AOD$.



A. 82°

B. 90°

C. 164°

D. 76°

E. Not enough information

20. A woman stands on top of a building directly over the base of the building closest to a tree. She notices that the angle of depression to the base of that tree is 38°. She also notices that the angle of elevation to the top of that same tree is 15°. If the base of the tree is 15 m from the base of the building, then how tall is the tree?

A. 15.7 m

B. 82.4 m

C. 26.3 m

D. 34.6 m

E. None of these.

21. The distance (apothem) from the center of a regular hexagon to the midpoint of a side of that hexagon has a length of 4 units. Find the circumference of the circle that circumscribes the hexagon.

A. 8π

B. 16π

C. $16\sqrt{3}\pi$

D. $\frac{16\sqrt{3}}{3}\pi$

E. None of these.

22.The volume of a sphere is $\frac{256}{3}\pi\ in^3$. Find the area of a circle that has the same radius as the sphere.

A. $8\pi\ in^2$

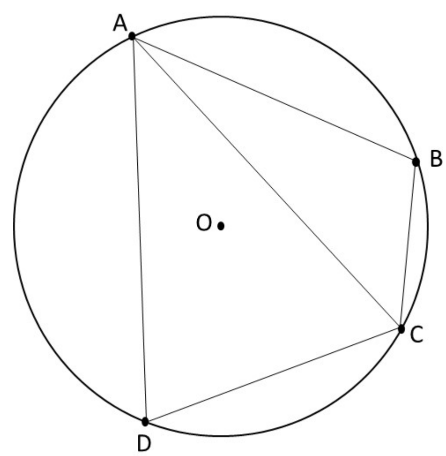
B. $16\pi\ in^2$

C. $\frac{16\sqrt[3]{3}}{3}\pi\ in^2$

D. $64\pi\ in^2$

E. None of these.

23. Given that ΔABC and ΔADC are both inscribed in circle O. Which of the following pieces of information is sufficient to prove that $\Delta ABC \cong \Delta ADC$ without any additional information?



A. $\overset{\frown}{BC} \cong \overset{\frown}{DC}$

B. $\overset{\frown}{AD} \cong \overset{\frown}{AB}$

C. $\overline{AC}$ passes through point O

D. $\overline{AD} \cong \overline{AB}$

E. None of these.

24. You have a cylinder with a surface area of $48\pi cm^2$ and a volume of $40\pi cm^3$. If you double the height and the diameter of the base of the cylinder, then what is the new volume?

A. $80\pi cm^3$

B. $160\pi cm^3$

C. $320\pi cm^3$

D. $384\pi cm^3$

E. Not enough information.

25. Two airplanes leave the airport at the same time. One plane flies due east at 420 mph. The other plane flies 20°West of North at 450 mph. In 2 hours, how far apart are the planes?

A. 1426 miles

B. 1231 miles

C. 999 miles

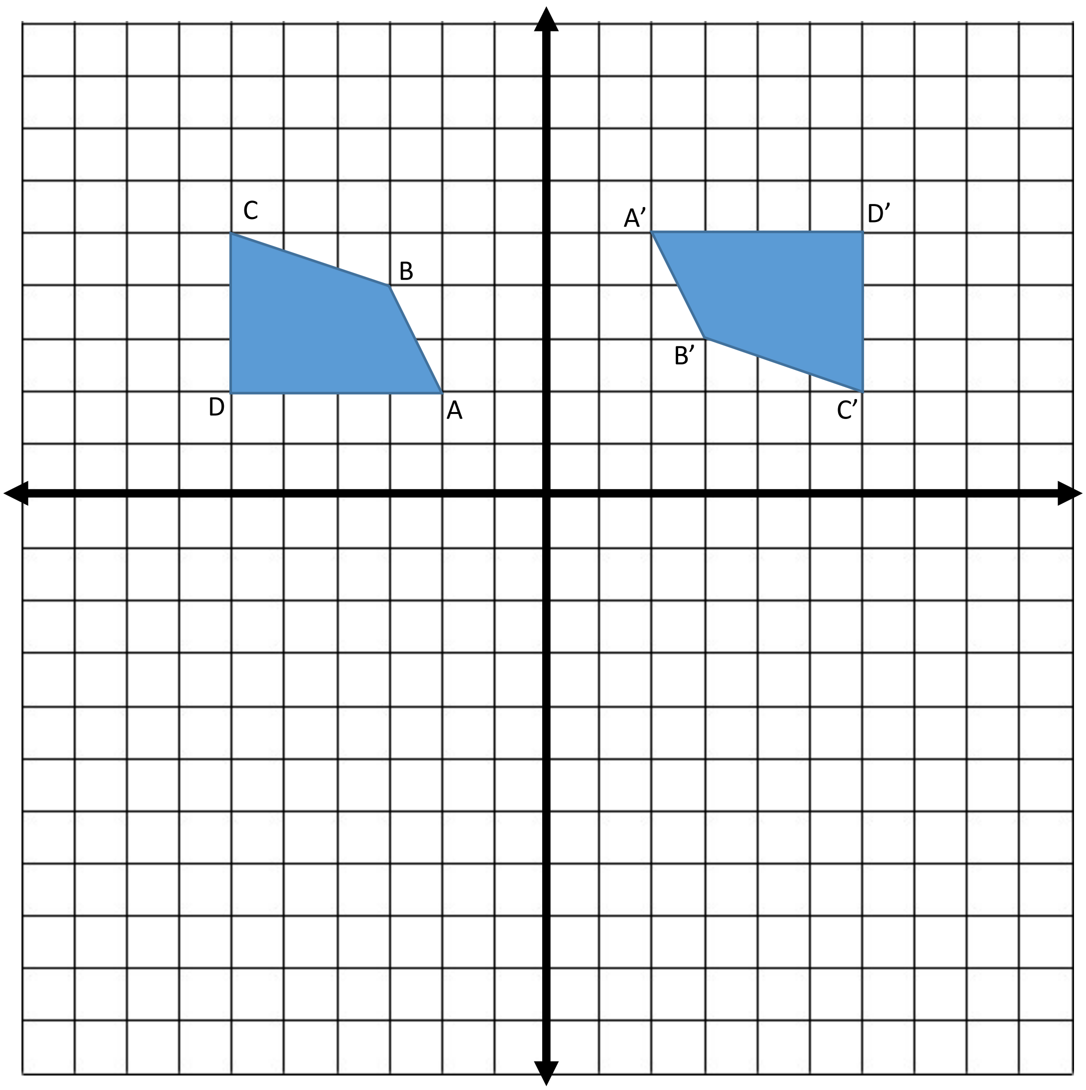
D. 1331 miles

E. None of these

Tie Breaker #1

Name:_________________________________

Identify a sequence of transformations that are needed to carry ABCD onto A’B’C’D’.



Tie Breaker # 2

Name:_________________________________

List the steps needed to construct a square using only a straight edge and a compass.

Tie Breaker # 3

Name:_________________________________

A regular hexagon is inscribed in a circle. If the area of the circle is 32π, then find the area of the hexagon.

Solutions:

1. B
2. D
3. E
4. B
5. A
6. A
7. E
8. C
9. A
10. D
11. B
12. D
13. E
14. C
15. C
16. A
17. E
18. B
19. E
20. A
21. D
22. B
23. E
24. C
25. A

Tie Breaker 1

The solution can consist of a rotation followed by a translation.

The solution can consist of vertical reflection and a horizontal reflection then a translation.

There are other combinations that could also create this transformation.

Tie Breaker 2

First you must draw a line segment of any length.

Mark a point on the line segment.

Construct a line perpendicular to the segment through that point. This is done by using the compass to mark off the line segment on each side of the chosen point a fixed distance from the point. Once the segment is marked in both of these locations, then you need to extend the compass beyond that opening. Then using the newly created marks as the fixed location for the compass, place an arc above the chosen point on the segment with arcs coming from the marks on both sides of the chosen point. Now using the straight edge, construct a line between the chosen point on the line and the intersection of the two arcs.

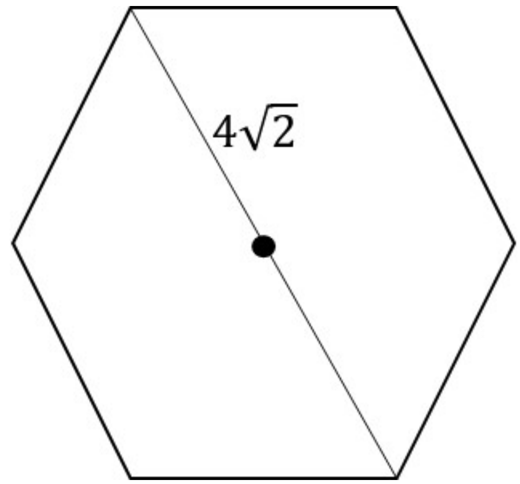
Using the intersection of the two perpendicular lines, use the compass to mark off equal segments on both lines.

Then using the compass open to the side length of the square, make an arc with a center at the endpoint of each of the perpendicular sides that are not the point of intersection. That intersection point will be the fourth vertex of the square.

Use a straight edge to connect the endpoints of the first two sides of the square to this fourth vertex.

Note: There are many ways that this can be done, so the person scoring this section must have some knowledge of constructions.

Tie Breaker 3



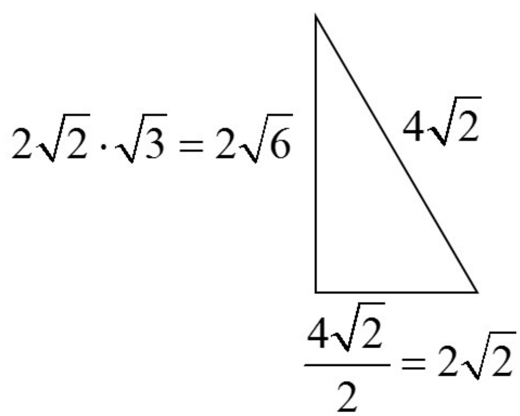
The area of the circle is 32π. So,

$\pi r^2 = 32\pi$ solving for r gives

$r = 4\sqrt{2}$

In the hexagon, the radius of the circle is shown.

The students could show that there are 6 equilateral triangles with side length of $4\sqrt{2}$ or they could use the perimeter and the apothem. In either case they must find the distance from the center of the hexagon to the midpoint of a side of the hexagon (the apothem). This can be found by using a 30-60-90 triangle (half of one of the equilateral triangles.) This would give the lengths of the 30-60-90 triangle to be:



The area of each of the 6 equilateral triangles can be found as $\frac{1}{2} 4\sqrt{2} \cdot 2\sqrt{6} = 4\sqrt{12} = 8\sqrt{3}$.

Multiplying by 6, we get the total area of the hexagon to be $48\sqrt{3}$.

If you choose to use the perimeter and the apothem, then you would get a perimeter of $6 \cdot 4\sqrt{2} = 24\sqrt{2}$ and the apothem is $2\sqrt{6}$. That gives an area of $\frac{1}{2} \cdot 24\sqrt{2} \cdot 2\sqrt{6} = 24\sqrt{12} = 48\sqrt{3}$