Arkansas Council of Teachers of Mathematics
2019 Precalculus Regional Exam

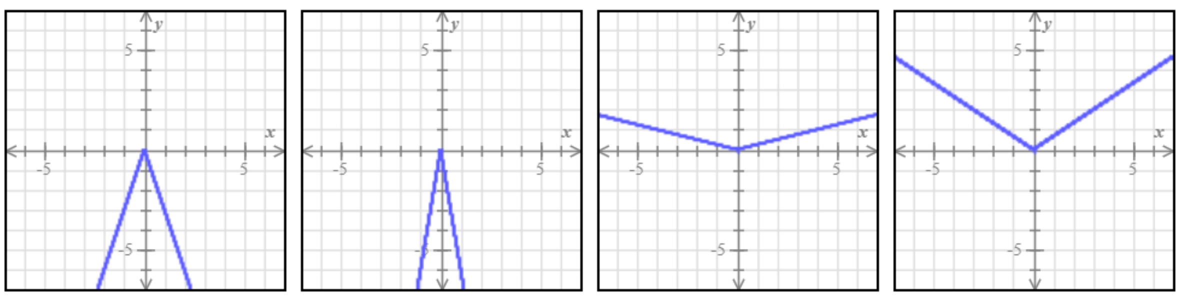
Work the multiple choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then, attempt the tiebreaker questions at the end starting with Tie Breaker #1, then #2, and finally #3. Turn in your answer sheet and the tiebreaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1. Given $f(x) = x^2 - 6x$, expand $f\left(x - \frac{1}{2}\right)$.

   a. $x^2 - 6x - \frac{1}{2}$
   b. $x^2 - 5x - \frac{11}{4}$
   c. $x^2 - 6x + \frac{11}{4}$
   d. $x^2 - 7x + \frac{13}{4}$

2. Consider the graphs and their equations below.



$y = A|x|$ $\quad$ $y = B|x|$ $\quad$ $y = C|x|$ $\quad$ $y = D|x|$

   Which coefficient (A, B, C, or D) has the least value?

   a. A
   b. B
   c. C
   d. D
   e. Unable to determine

3. Identify the vertical asymptote(s) of the function $f(x) = \frac{5x}{x^2-64}$

   a. $x = 0$
   b. $x = 8$
   c. $x = \pm 8$
   d. $x = 64$
   e. There are no vertical asymptotes.

4. Given $f(x) = \sqrt{x^2 - 4}$ and $g(x) = \frac{1}{x}$, what is the domain of the function $f\big(g(x)\big)$.

    a. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
    b. $[-2,2]$
    c. $\left[-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$
    d. $(-\infty, 0) \cup (0, \infty)$
    e. $(-\infty, -2) \cup (2, \infty)$

5. A homeowner mows the lawn every Wednesday afternoon. Let h be a function that determines the height of the grass (in inches) in terms of the amount of time that has passed since Wednesday afternoon (in days). Which of the following is the correct interpretation of the equation $h(2) = 5$?

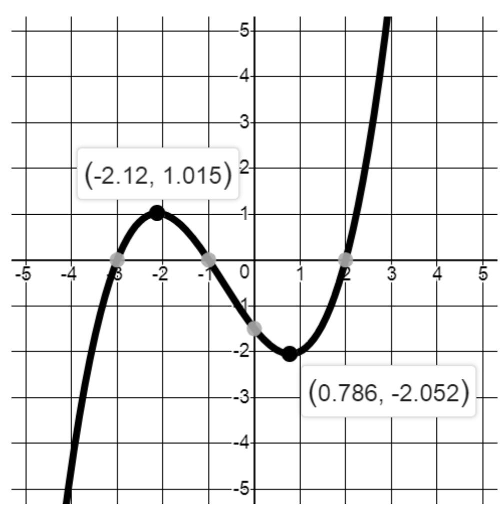
    a. On Monday afternoon, the grass is 2 inches tall.
    b. On Friday afternoon, the grass is 5 inches tall.
    c. On Wednesday afternoon, the grass has just been cut.
    d. Two times the height of the grass is 5 inches.

6. Determine the value of $\sin(\theta)$ if $\tan(\theta) = -\frac{12}{5}$ and $\cos(\theta) > 0$.

    a. $\sin(\theta) = -\frac{5}{12}$
    b. $\sin(\theta) = -\frac{13}{12}$
    c. $\sin(\theta) = -\frac{5}{13}$
    d. $\sin(\theta) = -\frac{12}{13}$
    e. Cannot be determined

7. The graph of $f(x)$ is shown to the right. State the interval(s) of x on which f is increasing.

    a. $(-2.12, 0.786)$
    b. $(-\infty, 1.015) \cup (-2.052, \infty)$
    c. $(1.015, -2.052)$
    d. $(-\infty, -2.12) \cup (0.786, \infty)$
    e. Unable to determine



8. Find all valid values for θ in the function below. Assume that k is an integer.

$$W(\theta) = \frac{\sin(\theta)}{\cos(\theta) + 2}$$

   a. $\theta \neq -2$
   b. $\theta = -2$
   c. $\theta \neq k\pi$
   d. $\theta = k\pi$
   e. None of the above.

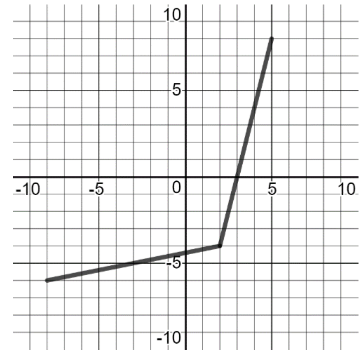
9. For the function $f(x) = 2 - \frac{2}{3}x$, find the average rate of change of $f(x)$ as x increases from -3 to 1.

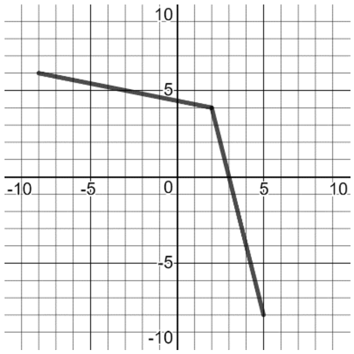
   a. 2
   b. $-\frac{2}{3}$
   c. 4
   d. $\frac{4}{3}$
   e. Unable to determine

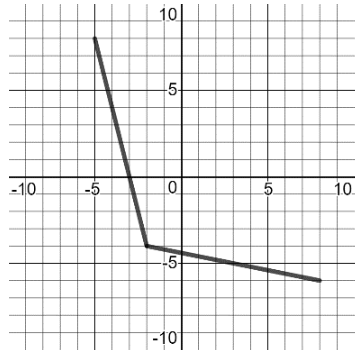
10. Mark works as a cook and spends all the money he earns on graphic tees. Let w be a function that determines the amount of money Mark has earned (m) in terms of the number of hours he has worked (h). Let g be a function that determines the number of graphic tees Mark can buy (n) in terms of the amount of money Mark has earned (m). Which of the following compositions of functions makes sense?

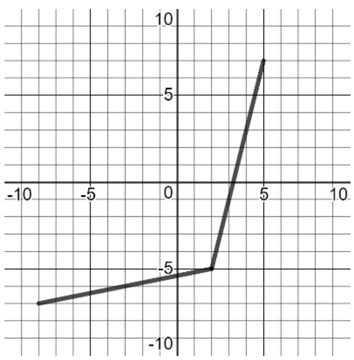
   a. $w\big(m(n)\big)$
   b. $w\big(g(m)\big)$
   c. $g\big(w(h)\big)$
   d. $h\big(m(n)\big)$
   e. Unable to determine with the information given.

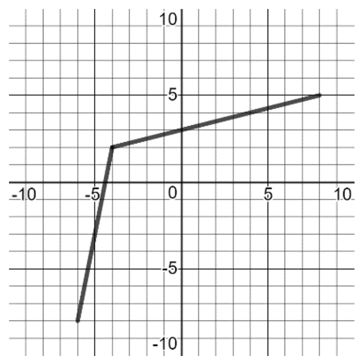
11. The graph of f is shown to the right. Choose the graph closest to f^{-1}.



a. 

b. 

c. 

d. 

e. None of the above.

12. Determine if the function $f(x) = \frac{2x+3}{5x+1}$ is one-to-one. If it is one-to-one, find $f^{-1}(x)$.

a. $f^{-1}(x) = -\frac{2x+3}{5x+1}$
b. $f^{-1}(x) = \frac{3x+2}{x+5}$
c. $f^{-1}(x) = \frac{-3x+1}{5x+2}$
d. $f^{-1}(x) = \frac{-x+3}{5x-2}$
e. This function is not one-to-one

For problems 13 and 14, consider the following rational function.

$$R(x) = \frac{3x^2}{x + x^2} + 50$$

13. Finish the statement: As x increases without bound, $R(x)$ approaches…

    a. $-1, 0$
    b. 3
    c. 50
    d. 53
    e. $+\infty$

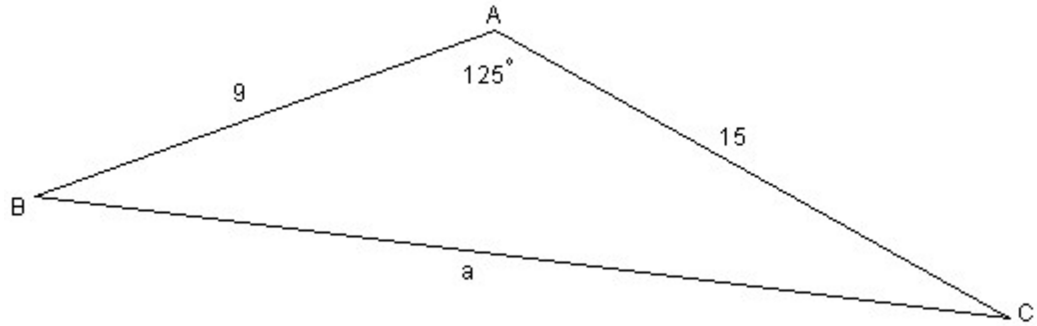
14. Finish the statement: As x approaches -1 from the left, $R(x)$ approaches…

    a. $+\infty$
    b. $-\infty$
    c. 50
    d. 0
    e. Unable to determine.

15. Mary bought a used car for \$5000 and the sales person told her that the value of the car is expected to depreciate at a rate of 8% per year. Which of the following models the value of Mary's car with respect to the number of years that have passed since she bought it?

    a. $M(n) = 5000 - 0.08n$
    b. $M(n) = 5000(0.92)^n$
    c. $M(n) = 5000(0.08)^n$
    d. $M(n) = 5000 - 0.92n$
    e. Unable to determine

Use the following triangle figure in problems 16 and 17.



16. The law of cosines and the law of sines can be used to find the length of the sides of an arbitrary (non-right) triangle. Apply one of these laws to find the length of side a in the above figure.

    a. 17.5 $units$
    b. 21.5 $units$
    c. 306.0 $units$
    d. 460.9 $units$
    e. Unable to determine

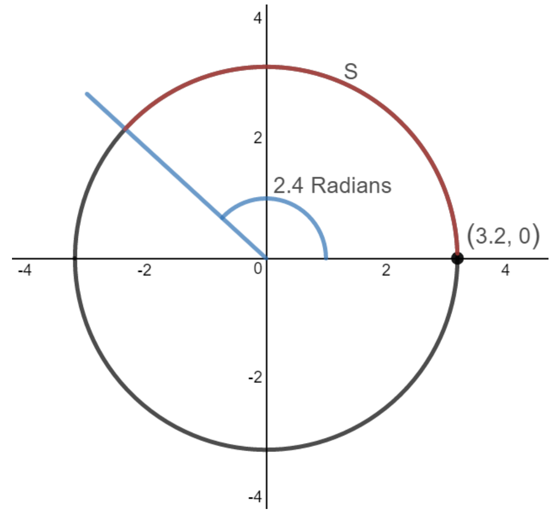
17. Determine the area of the above triangle.

    a. 55.3 $units^2$
    b. 67.5 $units^2$
    c. 96.6 $units^2$
    d. 161.0 $units^2$
    e. The area cannot be found.

18. Define $\log_b(A) = 10$ and $\log_b(B) = 2$. Use these to evaluate $\log_b\left(\frac{A^2}{B}\right)$.

    a. $x = 5$
    b. $x = 8$
    c. $x = 18$
    d. $x = 50$
    e. $x = 98$

19. An angle of 2.4 radians intercepts a circle of radius 3.2, as shown in the figure to the right. Find the length of the arc S.



a. 1.33
b. 2.00
c. 7.49
d. 7.68

20. Which of the following is a point on the unit circle and the terminal side of the angle $-\frac{4\pi}{3}$?

a. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
b. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
c. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
d. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
e. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

21. Use the properties of logarithms to combine the following expression into a single logarithm.

$$2\log z - 3\log(x+1)$$

a. $\log\left(z^2(x+1)^3\right)$
b. $\log\left(\frac{z^2}{(x+1)^3}\right)$
c. $\log\left(\frac{2z}{3x+3}\right)$
d. $\log\left(2z - 3x - 3\right)$
e. None of the above.

22. Solve the following equation. Give all solutions in the interval $0 \leq \theta < 2\pi$.

$$\tan\left(\theta + \frac{\pi}{3}\right) = 1$$

a. $\theta = \frac{11\pi}{12}, \frac{23\pi}{12}$
b. $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$
c. $\theta = \frac{\pi}{4}$
d. $\theta = -\frac{\pi}{12}$
e. None of the above.

23. A fireman has a 20-foot ladder that she can use to rescue someone from a window. The safety officer determines that this ladder can be leaned against a vertical wall with a minimum angle of elevation of 50° and a maximum angle of elevation of 75°. Given these two constraints, what is the difference in minimum and maximum heights that the ladder can safely reach?

a. 4.0 feet
b. 8.5 feet
c. 15.3 feet
d. 19.3 feet
e. Unable to determine.

24. Let θ be an angle in quadrant IV such that $\cos\theta = \frac{8}{9}$. Find the exact value of $\csc\theta$.

a. $\csc\theta = -\frac{\sqrt{17}}{9}$
b. $\csc\theta = \frac{9}{\sqrt{17}}$
c. $\csc\theta = \frac{\sqrt{17}}{9}$
d. $\csc\theta = -\frac{9}{\sqrt{17}}$

25. Use trigonometric identities to simplify the following expression.

$$\frac{\sin x \sec x}{\tan^2 x}$$

a. $\cot x$
b. 1
c. $\cos x$
d. $\tan^2 x$
e. None of the above.

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Tie Breaker #1

Name: ___________________

School: __________________

Given $f(x) = 3x^2 - 5x + 1$, fully simplify the difference quotient: $\frac{f(a+h)-f(a)}{h}$

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Tie Breaker #2

Name: ____________________

School: ___________________

Your grandmother has decided to push life to the edge and try bungee jumping! She is standing in the center of a bridge and jumps with no inhibitions. In the fun of it all, you notice that the relationship of her height with respect to the amount of time that has passed since she jumped forms a perfect parabolic shape.

The bridge is 100 feet in the air. Three seconds after jumping Grandma's bungee cord is completely taught and she is at her minimum height of 40 feet from the ground. Write a quadratic function that would relate Grandma's height above ground (in feet) with respect to the number of seconds that have passed since she jumped.

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Tie Breaker #3

Name: ___________________

School: __________________

Verify the following trigonometric identity:

$$\frac{1}{\sin x} - \frac{\sin x}{1 + \cos x} = \cot x$$

ACTM Precalculus Key 2019

1.	D
2.	B
3.	C
4.	C
5.	B
6.	D
7.	D
8.	E
9.	B
10.	C
11.	D
12.	D
13.	D
14.	A
15.	B
16.	B
17.	A
18.	C
19.	D
20.	B
21.	B
22.	A
23.	A
24.	D
25.	A

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Tie Breaker Number 1 Solution

Given $f(x) = 3x^2 - 5x + 1$, simplify the difference quotient: $\frac{f(a+h)-f(a)}{h}$

$$f(a+h) = 3(a+h)^2 - 5(a+h) + 1$$
$$= 3(a^2 + 2ah + h^2) - 5a - 5h + 1$$
$$= 3a^2 + 6ah + 3h^2 - 5a - 5h + 1$$

$$f(a) = 3a^2 - 5a + 1$$

$$\frac{f(a+h) - f(a)}{h} = \frac{6ah + 3h^2 - 5h}{h} = 6a + 3h - 5$$

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Tie Breaker Number 2 Solution

Your grandmother has decided to push life to the edge and try bungee jumping! She is standing in the center of a bridge and jumps with no inhibitions. In the fun of it all, you notice that the relationship of her height with respect to the amount of time that has passed since she jumped forms a perfect parabolic shape.

The bridge is 100 feet in the air. Three seconds after jumping Grandma's bungee cord is completely taught and she is at her minimum height of 40 feet from the ground. Write a quadratic function that would relate Grandma's height above ground (in feet) with respect to the number of seconds that have passed since she jumped.

$$f(t) = a(t-3)^2 + 40$$

$$100 = a(0-3)^2 + 40$$

$$100 = 9a + 40$$

$$60 = 9a$$

$$\frac{20}{3} = a$$

$$\boxed{f(t) = \frac{20}{3}(t-3)^2 + 40}$$

OR

$$f(t) = \frac{20}{3}(t-3)(t-3) + 40$$

$$= \frac{20}{3}(t^2 - 6t + 9) + 40$$

$$= \frac{20}{3}t^2 - 40t + 60 + 40$$

$$\boxed{f(t) = \frac{20}{3}t^2 - 40t + 100}$$

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Tie Breaker Number 3 Solution

Verify the following trigonometric identity:

$$\frac{1}{\sin x} - \frac{\sin x}{1 + \cos x} = \cot x$$

$$\left(\frac{1}{\sin x}\right)\left(\frac{1+\cos x}{1+\cos x}\right) - \left(\frac{\sin x}{1+\cos x}\right)\left(\frac{\sin x}{\sin x}\right)$$

$$\frac{1 + \cos x - \sin^2 x}{\sin x\,(1+\cos x)}$$

$$\frac{\cos^2 x + \cos x}{\sin x\,(1+\cos x)}$$

$$\frac{\cos x\,(1+\cos x)}{\sin x\,(1+\cos x)}$$

$$\frac{\cos x}{\sin x}$$

$$\cot x$$