ACTM Regional Pre-Calculus/Trig Exam 2018

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and then #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1. Simplify the expression $6i^3 - 4i^9$.
   a. $-10i$
   b. $10i$
   c. $-2i$
   d. $2i$

2. Find the rectangular coordinates of the point with polar coordinates $\left(-1, -\frac{\pi}{3}\right)$.
   a. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
   b. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
   c. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
   d. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

3. Convert 29.411° to $D^{\circ}M'S''$ (degrees, minutes, seconds). Round to the nearest second.
   a. $29^{\circ}23'53''$
   b. $29^{\circ}24'53''$
   c. $29^{\circ}24'40''$
   d. $29^{\circ}23'40''$

4. Suppose that the x-intercepts of the graph of $y = f(x)$ are $(-9, 0)$ and $(2, 0)$. What are the x-intercepts of the graph of $y = 5f(x + 4)$?
   a. $(-13, 0)$ and $(-2, 0)$
   b. $(-5, 0)$ and $(6, 0)$
   c. $(-9, 0)$ and $(2, 0)$
   d. $(-45, 0)$ and $(10, 0)$

5. What are the foci of this hyperbola?
$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$
   a. $(0, \pm 2)$
   b. $(\pm 2, 0)$
   c. $(0, \pm 3)$
   d. $\left(0, \pm\sqrt{5}\right)$

6. Let $f(x) = \ln x$ and $g(x) = \frac{x}{x-1}$. Find the domain of $(f \circ g)(x)$.
   a. $(-\infty, 1) \cup (1, \infty)$
   b. $(0, \infty)$
   c. $(-\infty, \infty)$
   d. $(-\infty, 0) \cup (1, \infty)$

7. Suppose f is an odd function. What is an equivalent expression for $f(-x)$?
   a. $f(x)$
   b. $-f(x)$
   c. $-f(-x)$
   d. None of these

8. A farmer is fencing a triangular field. The sides of the field are 45 yards, 53 yards, and 72 yards respectively. In order to make the fence perfect, the farmer needs to know the angle opposite the shortest side. Find this angle to the nearest degree.
   a. $36°$
   b. $47°$
   c. $42°$
   d. $39°$

9. Convert $-\frac{11\pi}{9}$ to degrees and write it as the least possible positive coterminal angle.
   a. $-220°$
   b. $120°$
   c. $140°$
   d. $130°$

10. Find the following value.

$$\sum_{k=1}^{26}(3k-7)$$

   a. 871
   b. 867
   c. 791
   d. 855

11. Consider the following diagram:

If $\overline{AB} = 300$ miles, $\angle BAC = 10°$, and $\overline{AC} = 50$ miles, find the length of $\overline{BC}$ to the nearest mile.
   a. 343
   b. 279
   c. 251
   d. 250

12. Find the exact value of $\log_2 3 \times \log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8$.
   a. $\log_2 3$
   b. 3
   c. $\log 8$
   d. $\log 3$

13. If $\sin A = -3/5$ and $\pi < A < 3\pi/2$, find $\tan A$.
    a. $4/3$
    b. $-4/5$
    c. $3/4$
    d. $-3/4$

14. If $f(x) = a^x$ then $f(\alpha x) =$
    a. $\alpha f(x)$
    b. $(f(x))^\alpha$
    c. $f(x)f(\alpha)$
    d. None of these

15. Determine the solution set of the equation $\log(x-2) = 1 - \log(x+1)$.
    a. $\{4\}$
    b. $\{-3, 4\}$
    c. $\{-3\}$
    d. none of these

16. Find the oblique (slant) asymptote of the following function.
$$f(x) = \frac{3x^4 + x^2 - 7}{2x^3 - 7}$$
    a. $y = \frac{3}{2}x$
    b. $y = \frac{3}{2}x - 7$
    c. $y = 3x - 2$
    d. $y = 2x - 3$

17. Find the inverse function of $f(x) = x^{-1} + 1$.
    a. $f^{-1}(x) = x^{-1} - 1$
    b. $f^{-1}(x) = (x-1)^{-1}$
    c. $f^{-1}(x) = x - 1$
    d. $f^{-1}(x) = x + 1$

18. Find the range of the function $f(x) = \cos^{-1}(x^2 - 4)$.
    a. $[\sqrt{3}, \sqrt{5}]$
    b. $[-\sqrt{5}, -\sqrt{3}]$
    c. $[-1, 1]$
    d. $[0, \pi]$

19. If $\theta = \sin^{-1}\left(\frac{1}{2}\right)$, find $\sin(2\theta)$.
    a. $\frac{\sqrt{2}}{2}$
    b. $\frac{\sqrt{3}}{2}$
    c. $\frac{1}{2}$
    d. 1

20. Given that $\alpha = \cos^{-1}(\cos\theta)$ and that θ is in quadrant IV, what quadrant is α in?
    a. Quadrant I
    b. Quadrant II
    c. Quadrant III
    d. Quadrant IV

21. Find the largest angle possible in a triangle with sides $a = 1$ and $b = 2$ and $\angle A = 10^{\circ}$, where side a is opposite of $\angle A$. Round to the nearest degree.
    a. 20°
    b. 120°
    c. 150°
    d. 160°

22. Given that $\sin\alpha = 3/5$ and that $\sin\beta = -4/5$ find a possible value of $\sin(\alpha + \beta)$.
    a. $-1/5$
    b. $-7/25$
    c. $-12/25$
    d. $1/5$

23. Solve $5 \cdot 2^{x+1} = 3 \cdot 5^{3-x}$ for x. Which interval contains the solution?
    a.) $[-3, -1.5]$
    b.) $[-1.5, 0.5]$
    c.) $[0.5, 1.5]$
    d.) $[1.5, 3]$

24. For all θ for which the expression is defined, is equivalent to which of the following?
$$\frac{\cot(\theta + \pi)\cos\theta}{\csc(\theta + 2\pi)}$$
    a. $\cos^2\theta$
    b. $\sec(\theta + \pi)$
    c. $\csc^2\theta$
    d. none of these

25. Which interval is the sum of the following infinite series in?
$$\sum_{n=1}^{\infty}[\tan^{-1}(\sin\theta)]^n$$
    a. $[-5, -0.5]$
    b. $[-5, 0.5]$
    c. $[-0.5, 5]$
    d. $[0.5, 5]$

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TIEBREAKER #1

Name______________________________

Show all your work.

An airplane departs from city A. It heads north at a speed of 100 miles per hour and travels for 30 minutes. It then turns 15° to the east (clockwise) and continues at the same speed for 90 more minutes to arrive at city B. If the plane traveled the direct straight line path from city A to city B, how much time would have been saved?

TIEBREAKER #2

Name______________________________

Show all your work.

A camera with a viewing angle of $80°$ is mounted 3 feet above the ground. It is focused on a 10-foot tall vertical monument. How far away from the monument does the camera need to be in order to capture it completely? Round your answer to the nearest inch.

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TIEBREAKER #3

Name______________________________

Show all your work.

An object whose temperature is different than that of the ambient temperature around it will either warm up or cool down to reach that ambient temperature. The difference between the object's temperature and the ambient temperature decays exponentially. A loaf of bread is removed from an oven whose temperature is fixed at 475°F into a room that is a constant 80°F. After 5 minutes, the loaf's temperature 350°F. Determine how long it will take for the loaf to reach 100°F.

ANSWERS

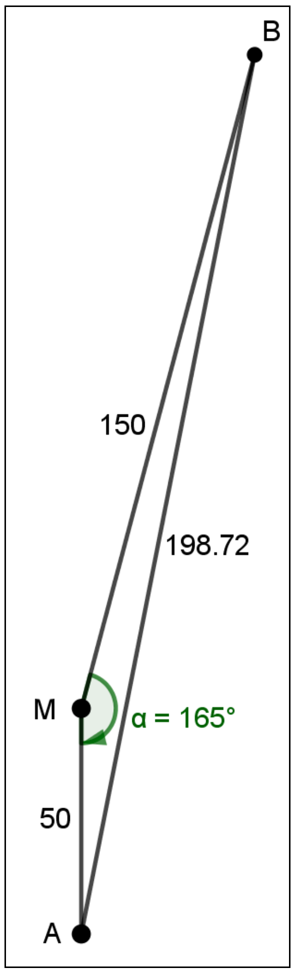
1) A
2) B
3) C
4) A
5) C
6) D
7) B
8) D
9) C
10) A
11) C
12) B
13) C
14) B
15) A
16) A
17) B
18) D
19) B
20) A
21) D
22) B
23) D
24) A
25) C

Tiebreaker 1:

Sketch 50 miles north, then turn 15º clockwise, then fly another 150 miles. Total travel time is 2 hours.

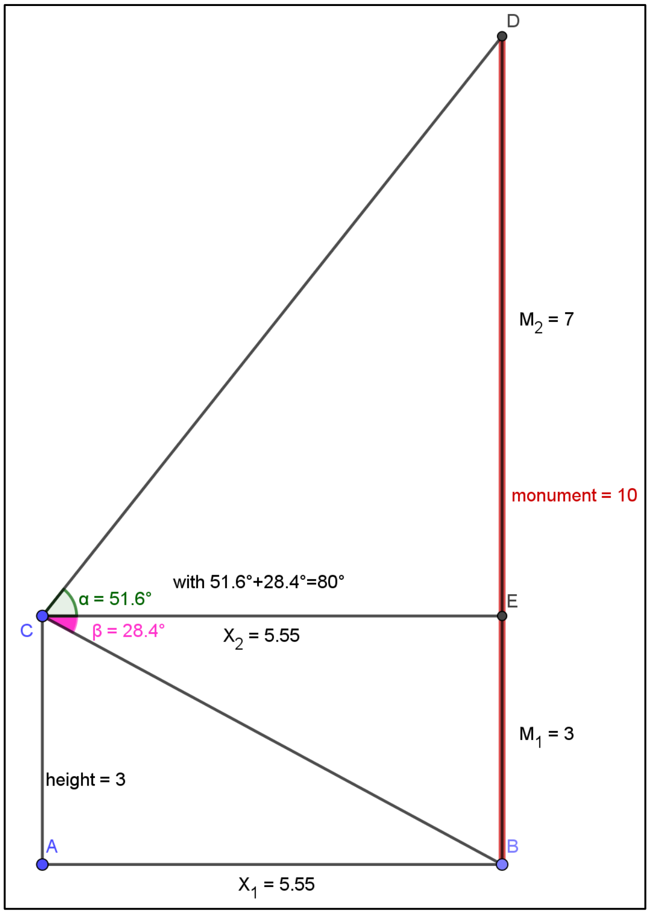
Use the Law of Cosines to calculate the other side of the triangle:
$c^2 = 50^2 + 150^2 - 2 \cdot 50 \cdot 150 \cdot \cos 165^\circ$. So $c \approx 198.7$ miles.

Using $d = rt$, with $d = 198.7$ and $r = 100$, the travel time for the direct route would be $t = 1.99$ hours. This yields a difference of ≈ 0.01 hours. This converts to ≈ 0.77 minutes or ≈ 46 seconds.



Tiebreaker 2:

Draw a sketch to set up the system of equations. We will solve by substitution.



$$\alpha + \beta = 80^\circ \implies \beta = 80^\circ - \alpha$$

$$\tan(\alpha) = \frac{7}{x} \implies x = \frac{7}{\tan(\alpha)}$$

$$\tan(\beta) = \frac{3}{x} \implies x = \frac{3}{\tan(\beta)} \implies x = \frac{3}{\tan(80^\circ - \alpha)}$$

Now since the two are equal to x, set them equal to each other.

$$\frac{7}{\tan(\alpha)} = \frac{3}{\tan(80^\circ - \alpha)}$$

$$\tan(80^\circ - \alpha) = \frac{3\tan(\alpha)}{7}$$

$$\frac{\tan(80^\circ) - \tan(\alpha)}{1 + \tan(80^\circ)\tan(\alpha)} = \frac{3\tan(\alpha)}{7}$$

$$7(\tan(80^\circ) - \tan(\alpha)) = 3\tan(\alpha) \cdot (1 + \tan(80^\circ)\tan(\alpha))$$

$$7\tan(80^\circ) - 7\tan(\alpha) = 3\tan(\alpha) + 3\tan(80^\circ)\tan^2(\alpha)$$

$$3\tan(80^\circ)\tan^2(\alpha) + 10\tan(\alpha) - 7\tan(80^\circ) = 0$$

This is a quadratic form. Make a substitution of $y = \tan(\alpha)$.

$$3\tan(80^\circ)\,y^2 + 10y - 7\tan(80^\circ) = 0$$

Now apply the quadratic formula, with $A = 3\tan(80^\circ)$, $B = 10$, and $C = -7\tan(80^\circ)$.

$$y = \frac{-(10) \pm \sqrt{10^2 - 4(3\tan(80^\circ))(-7\tan(80^\circ))}}{2 \cdot 3\tan(80^\circ)}$$

$$y_1 \approx 1.2617 \quad and \quad y_2 \approx -1.8494$$

Since we defined $y = \tan(\alpha)$, apply its inverse, $\alpha = \tan^{-1}(y)$, to both y-values to find α.

$$\alpha_1 = \tan^{-1}(1.2617) \approx 51.599^\circ$$

$$\alpha_2 = \tan^{-1}(-1.8494) \approx -61.599^\circ$$

We discard α_2 since it's a negative angle. Lastly, we need to solve for x. Apply α_1 to one of the earlier formulas, such as $x = \frac{7}{\tan(\alpha)}$.

$$x = \frac{7}{\tan(51.599)} \approx 5.55$$

Thus the camera should be 5.55 feet away from the monument. This distance is equal to $5'6.6'' \approx 5'7''$.

Tiebreaker 3:

The initial difference in temperature is $475 - 80 = 395$, after 5 min the difference in temp is $350 - 80 = 270$. $270/395 = 0.683544$ so approx. 68.35% cooling to ambient temp is completed. $100 - 80 = 20$, so we want to reach $20/395 = 0.050632911$ so about 5.06% cooling.

We solve $0.050632911 = 0.683544^x$ to get

$x = \frac{ln(0.0506)}{ln(0.6835)} \approx$ 7.84. This is how many 5 minute intervals have elapsed in order to cool to 100 degrees. 7.84 times 5 is about <u>39.2 minutes</u>.

Of course this can also be solved using Newton's Law of Cooling: $u(t) = T + (u_0 - T)e^{-kt}$.