## Arkansas Council of Teachers of Mathematics Algebra I 2018 Regional Exam

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker \#1, then \#2, and then \#3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions. Figures aren't necessarily drawn to scale.

1. Given the numbers $m=\sqrt{2}$ and $n=2$, which of the following is true?
a. The sum of $m$ and $n$ is irrational and their product is irrational
b. The sum of $m$ and $n$ is irrational and their product is rational
c. The sum of $m$ and $n$ is rational and their product is irrational
d. The sum of $m$ and $n$ is rational and their product is rational
2. A pipe is leaking at the rate of 8 fluid ounces per minute. How many gallons is the pipe leaking per hour?
a. $0.02 \mathrm{gal} / \mathrm{h}$
b. $3.75 \mathrm{gal} / \mathrm{h}$
c. $\quad 17.07 \mathrm{gal} / \mathrm{h}$
d. $3,840 \mathrm{gal} / \mathrm{h}$
3. A proposed procedure to solve to the equation $3(x+5)=2 x+35$ is given below.

Step 1: $\quad 3 x+15=2 x+35$
Step 2: $\quad 5 x+15=35$
Step 3: $\quad 5 x=20$
Step 4: $\quad x=4$
Which is the first incorrect step in the solution shown?
a. Step 1
b. Step 2
c. Step 3
d. Step 4
4. The data in the table show the cost of renting a bicycle by the hour, including a deposit.

Renting a Bicycle

| Hours $(h)$ | Cost in Dollars $(c)$ |
| :---: | :---: |
| 2 | 15 |
| 5 | 30 |
| 8 | 45 |

If hours, $h$, were graphed on the horizontal axis and cost, $c$, were graphed on the vertical axis, what would be the equation of a line that fits the data?
a. $c=5 h$
b. $c=\frac{1}{5} h+5$
c. $c=5 h+5$
d. $c=5 h-5$

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5. Is the equation $3(2 x-4)=-18$ equivalent to $6 x-12=-18$ ? If yes, explain your answer.
a. Yes, the equations are equivalent by the Associative Property of Multiplication.
b. Yes, the equations are equivalent by the Commutative Property of Multiplication.
c. Yes, the equations are equivalent by the Distributive Property of Multiplication over Addition.
d. No, the equations are not equivalent.
6. Which mathematical sentence represents the solution for $d$ in the equation $6 e=e f+3 d$ ?
a. $d=\frac{6 e-e f}{3}$
b. $d=\frac{6 e+e f}{3}$
c. $d=\frac{2 e-e f}{3}$
d. $d=\frac{2 e+e f}{3}$
7. The scatterplot below shows data for groups $R$ and $S$. Which of the following statements is true about the correlation between the $x$ and $y$ values of group $R$ and the correlation between the $x$ and $y$ values of group S?

a. The $x$ and $y$ values appear to be negatively correlated in both $R$ and $S$.
b. The $x$ and $y$ values appear to be positively correlated in both $R$ and $S$.
c. The $x$ and $y$ values appear to be negatively correlated in $R$, but positively correlated in $S$.
d. The $x$ and $y$ values appear to be positively correlated in $R$, but negatively correlated in $S$.
8. Which is one of the solutions to the equation $2 x^{2}-x-4=0$ ?
a. $\frac{1}{4}-\sqrt{33}$
b. $-\frac{1}{4}+\sqrt{33}$
c. $\frac{1+\sqrt{33}}{4}$
d. $\frac{-1-\sqrt{33}}{4}$

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9. A random sample of graduates from a particular college program reported their ages and incomes in response to a survey. Each point on the scatterplot below represents the age and income of a different graduate.


Of the following equations, which best fits the data above?
a. $y=1,000 x$
b. $y=1,000 x+15,000$
c. $y=10,000 x$
d. $y=10,000 x+15,000$

10 . The point $(4, k)$ is a solution to the equation $3 x+2 y=12$. What is the value of $k$ ?
a. 0
b. 2
c. 4
d. 12
11. The circle graph shown below represents the distribution of the grades of 40 students in a certain geometry class. How many students received As or Bs?
a. 6
b. 10
c. 15
d. 25


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12. Which statement is true about the value of $(\sqrt{27}-3) \cdot 9$ ?
a. It is rational, because the product of two rational numbers is rational.
b. It is rational, because the product of a rational number and an irrational number is rational.
c. It is irrational, because the product of two irrational numbers is irrational.
d. It is irrational, because the product of an irrational number and a rational number is irrational.
13. Tonya's class planted sunflowers and the students are tracking the growth of their individual plants. The table shows the height of Tonya's plant $t$ days after she planted her sunflower seed.

| Time (days) | Height (inches) |
| :---: | :---: |
| 10 | 4 |
| 20 | 8 |
| 30 | 12 |
| 40 | 16 |

Based on the data in the table, which function is an appropriate model for the height, $h(t)$, in inches?
a. $h(t)=4 t$
b. $h(t)=\frac{1}{4} t$
c. $h(t)=\frac{5}{2} t$
d. $h(t)=\frac{2}{5} t$
14. The sum of the first $n$ consecutive even numbers can be found using $S=n^{2}+n$, where $n \geq 2$. What is the value of $n$ when the sum is 156 ?
a. 6
b. 12
c. 26
d. 39
15. How does the graph of $f(x)=2 x+10$ compare to the graph of $f(x)=-2 x+10$ ?
a. The graph does not change at all.
b. The $y$-intercept would be different, but the slope would remain the same.
c. The slope would be different, but the y-intercept would remain the same.
d. Both the $y$-intercept and the slope of the graph would change.
16. Let $h(x)=2(3)^{x}$ and $k(x)=3(2)^{x}$. Which statement about the graphs of these functions is true?
a. $h(x)$ and $k(x)$ rise at the same rate.
b. $k(x)$ rises at a faster rate than $h(x)$.
c. $h(x)$ rises at a faster rate than $k(x)$.
d. The $y$-intercept of $h(x)$ is above the $y$-intercept of $k(x)$.

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17. Vinny collects population data, $P(h)$, about a specific strain of bacteria over time in hour, $h$, as shown in the graph below.


Which equation represents the graph of $P(h)$ ?
a. $P(h)=4(2)^{h}$
b. $\quad P(h)=\frac{46}{5} h+\frac{6}{5}$
c. $P(h)=3 h^{2}+.2 h+4.2$
d. $P(h)=\frac{2}{3} h^{3}-h^{2}+3 h+4$
18. The sum of the perimeters of two different squares is 32 centimeters, and the difference between their perimeters is 8 centimeters. If $x$ represents the side length of the larger square and $y$ represents the side length of the smaller square, which of the following systems of equations could be used to find the dimensions of the squares?
a. $x+y=32$
$x-y=8$
b. $4 x+4 y=32$
$4 x-4 y=8$
c. $2 x+2 y=32$
$2 x-2 y=8$
d. $4 x+2 y=32$
$4 x-2 y=8$
19. At the school bookstore, a pencil costs $25 ¢$, a notebook costs $\$ 1.75$, and a piece of graph paper costs $5 \not \subset$. Which formula below could be used to determine the total cost $c$, in cents, of purchasing $p$ pencils, $n$ notebooks, and $g$ pieces of graph paper?
a. $c=25 p+1.75 n+5 g$
b. $c=0.25 p+1.75 n+0.05 g$
c. $c=25 p+175 n+5 g$
d. $c=0.25 p+1.75 n+0.5 g$

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20. Baseball fans can buy tickets for seats in the lower deck or upper deck of the stadium. Tickets for the lower deck cost $\$ 42$ each. Ticket prices for the upper deck are $75 \%$ of the cost of tickets for the lower deck. Which inequality represents all possible combinations of $x$, the number of tickets for the lower deck, and $y$, the number of tickets for the upper deck, that someone can buy for no more than $\$ 800$ ?
a. $42 x+56 y \leq 800$
b. $42 x+31.5 y \leq 800$
c. $42 x+56 y>800$
d. $42 x+31.5 y>800$
21. Simplify the following expression completely:

$$
\frac{a^{2}-b^{2}}{a-b} \cdot \frac{a-b}{a+b}
$$

a. $\quad a-b$
b. $\frac{a^{3}-a^{2} b-a b^{2}+b^{3}}{a^{2}-b^{2}}$
c. $\frac{a^{2}-b^{2}}{a+b}$
d. $\frac{a^{3}-2 a^{2} b^{2}+b^{3}}{a^{2}-b^{2}}$
22. For which values of $a$ and $b$ will the following inequality always be true?

$$
-1<a b<1
$$

a. $a>0 ; b>0$
b. $\frac{a}{b}<0 ; b \neq 0$
c. $|a|<\left|\frac{1}{b}\right| ; b \neq 0$
d. $-\frac{1}{a}<\frac{a}{b}<\frac{1}{b} ; a, b \neq 0$
23. The formula for blood flow rate is given by $F=\frac{p_{1}-p_{2}}{r}$, where $F$ is the flow rate, $p_{1}$ the initial pressure, $p_{2}$ the final pressure, and $r$ the resistance created by blood vessel size. Which formula can not be derived from the given formula?
a. $\quad p_{1}=F r+p_{2}$
b. $p_{2}=p_{1}-F r$
c. $\quad r=F\left(p_{2}-p_{1}\right)$
d. $r=\frac{p_{1}-p_{2}}{F}$

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24. Last month Bonnie recorded the weights of 11 dogs. Statistics for these weights are given below.

Weights of Dogs Last Month

| Minimum | 31 pounds |
| :---: | :---: |
| Maximum | 93 pounds |
| Mean | 81 pounds |
| Mode | 88 pounds |
| First Quartile | 78 pounds |
| Median | 88 pounds |
| Third Quartile | 90 pounds |

This month, Bonnie weighed the 11 dogs again. The weight of one dog changed from 31 pounds to 27 pounds. The weights of all the other dogs stayed the same. Which of the following statistics changed from last month to this month?
a. Maximum
b. Mean
c. First Quartile
d. Median
25. Solve the following inequality for $x$ :

$$
12>3\left(\frac{1}{2} x+6\right) \geq 21
$$

a. $-2 \geq x>4$
b. $-2 \leq x<4$
c. $-1 \geq x>\frac{1}{2}$
d. No Solution

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Tie Breaker \#1
Name: $\qquad$
School: $\qquad$
Robert decided to have sod laid in his new yard. After the sod is laid, Robert is planning on fencing in his yard to prevent damage to the sod. In the diagram below, the shaded area represents the yard and the unshaded region represents the house. Use the diagram to find the following information.

Expand all answers to polynomial form.

a. Find the area of Robert's house.
b. Find the area of the yard.
c. Find the perimeter of the yard.

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Tie Breaker \#2
Name: $\qquad$
School: $\qquad$
A single piece of string can be bent back and forth. The string, shown below, has been bent back and forth so that it has three "layers" that it can be cut through, shown below.


Imagine that the string has been cut along the dotted line above through the three layers so that there are four separate pieces of string.
a. Suppose instead that the string was cut along three vertical lines instead of one. How many pieces of string are there after the cuts?
b. For any string bent into three "layers", determine a function that can be used to predict the number of pieces of string, $P$, with $n$ representing the number of cuts.

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## Tie Breaker \#3

Name: $\qquad$
School: $\qquad$
You have decided to go on a road trip over the summer with some friends. You have three separate trips you want to take during the 2 months of summer break.

- One is to a town a distance of 90 miles to see an old friend,
- Another trip is to a shopping mall 120 miles away, and
- The third is to see a concert in a city 250 miles away.

Finances are tight and you need to hire a car for the tip. There are three car hire companies for you to choose from.

- Bill's Car Hire charges a base fee of $\$ 50$ and then $\$ 0.20$ per mile.
- Crazy John's Car Hire charges a base fee of $\$ 20$ and then $\$ 0.30$ per mile
- Cheap Drive Car Hire charges $\$ 0.40$ per mile with no base fee.

Your tasks:
a. Construct three tables showing the cost for hiring a car from each company for distances of 100 , $200,300,400$, and 500 miles.
b. Create three equations to show the cost of hiring a car from each company for $x$ miles.
c. At what trip distance would Bill's Car Hire and Crazy John's cost the same?

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Multiple Choice
Answers

| 1. | A |
| :--- | :--- |
| 2. | B |
| 3. | B |
| 4. | C |
| 5. | C |
| 6. | A |
| 7. | C |
| 8. | C |
| 9. | B |
| 10. | A |
| 11. | B |
| 12. | D |
| 13. | D |
| 14. | B |
| 15. | C |
| 16. | C |
| 17. | A |
| 18. | B |
| 19. | C |
| 20. | B |
| 21. | A |
| 22. | C |
| 23. | C |
| 24. | B |
| 25. | D |

## Tie Breaker \#1

a. House: $(3 x+2)(x-8)=3 x^{2}-22 x-16$
b. Yard: $(4 x-5)(2 x+3)-(3 x+2)(x-8)=5 x^{2}+24 x+1$
c. Yard Perimeter: $2(4 x-5)+2(2 x+3)=12 x-4$

Tie Breaker \#2
a. 10 pieces of string.
b. The formula is $P(n)=3 n+1$, where $n$ is the number of cuts and $P(n)$ is the number of pieces of string.

## Tie Breaker \# 3

a. Table

|  |  | Car Company |  |  |
| :---: | ---: | ---: | ---: | ---: |
|  |  | Bill's | John's | Cheap |
| $\approx$ | 100 | $\$ 70$ | $\$ 50$ | $\$ 40$ |
|  | 200 | $\$ 90$ | $\$ 80$ | $\$ 80$ |
|  | 300 | $\$ 110$ | $\$ 110$ | $\$ 120$ |
|  | 400 | $\$ 130$ | $\$ 140$ | $\$ 160$ |
|  | 500 | $\$ 150$ | $\$ 170$ | $\$ 200$ |

b. Formulas for each company.

1. Bill's: $C(x)=0.20 x+50$
2. John's: $C(x)=0.30 x+20$
3. Cheap: $C(x)=0.40 x$
c. The cost is the same when $0.2 x+50=.3 x+20$. The solution is $x=300$ miles. When driving 300 miles, they both charge a cost of $\$ 110$.
