Arkansas Council of Teachers of Mathematics
Algebra II 2019 Regional Exam

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

1. Determine the real numbers D, E, and F so that the equation
$$D(3x-5)+E(2x-1)+Fx^2=-6+5x$$
is an identity.

   A. $D=1, E=1, F=0$
   B. $D=0, E=2, F=1$
   C. $D=-1/2\,, E=6, F=0$
   D. $D=1, E=4/5\,, F=-3$

2. What is the product of $(3-2i)$ and $(7+6i)$?

   A. $9+4i$
   B. $21+16i$
   C. $33+4i$
   D. $21-12i$

3. Find the quotient $q(x)$ and the remainder $r(x)$ if $f(x)=2x^4-x^3+7x+3$ is divided by $g(x)=x^2+2x-5$.

   A. $q(x)=x^2+x-1, r(x)=-x+3$
   B. $q(x)=x^2-7, r(x)=-10x+2$
   C. $q(x)=2x^2-5x+20, r(x)=-58x+103$
   D. $q(x)=9x^2-1, r(x)=-12x+1$

4. Rewrite the product of the following two polynomials in expanded notation:
$$2x^3-x^2 \qquad \text{and} \qquad 3x^4-x^3+x$$

   A. $6x^7-5x^6+x^5+2x^4-x^3$
   B. $6x^7+x^6-x^5+2x^4+x^3$
   C. $6x^7+x^6-x^5-2x^4-x^3$
   D. $6x^7+5x^6+x^5-2x^4-x^3$

5. Line A goes through the point $(1, 5)$ and is perpendicular to Line B, which is given by $x + 3y = 6$. What is the slope-intercept form of the equation representing Line A?

   A. $y = -\frac{1}{3}x - 4$
   B. $y = 2x + 1$
   C. $y = \frac{2}{3}x + 6$
   D. $y = 3x + 2$

6. Determine the quotient of the complex numbers $3 + 4i$ and $8 - 2i$, using the latter as the divisor.

   A. $\frac{3}{10} + \frac{11}{20}i$
   B. $-\frac{3}{7} + \frac{5}{9}i$
   C. $\frac{4}{17} + \frac{19}{34}i$
   D. $-\frac{11}{32} - \frac{23}{38}i$

7. How many real roots does the following quadratic equation contain?

   $$f(x) = -4x^2 + 12x - 9$$

   A. 0
   B. 1
   C. 2
   D. 3

8. A circle with a radius of 4, translated 2 to the left, and 5 up, can be described by which of the following equations?

   A. $(x - 2)^2 + (y + 5)^2 = 4$
   B. $(x + 2)^2 + (y - 5)^2 = 4$
   C. $(x - 2)^2 + (y + 5)^2 = 16$
   D. $(x + 2)^2 + (y - 5)^2 = 16$

9. If g varies inversely as the square root of h, and $g = 9$ when $h = 121$, find g when $h = 81$.

   A. $g = \frac{1}{11}$
   B. $g = 11$
   C. $g = 13.4$
   D. $g = 99$

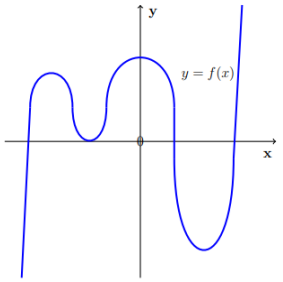
10. A butcher has some hamburger that is 80% lean and some that is 88% lean. He wishes to make 800 pounds of a burger mix that is 83% lean. How much of each type should he use?

    A. 300 pounds at 80%, 500 pounds at 88%
    B. 400 pounds at 80%, 400 pounds at 88%
    C. 500 pounds at 80%, 300 pounds at 88%
    D. 550 pounds at 80%, 250 pounds at 88%

11. Select the correct interval of x values obtained from the inequality: $x^2 - 2x - 8 \geq 0$.

    A. $(-\infty, -2] \cup [4, \infty)$
    B. $[-2, 4]$
    C. $(-\infty, -4] \cup [2, \infty)$
    D. $[-4, 2]$

12. Which of the following functions could represent the polynomial graph shown?



    A. $y = (x + 4)(x + 2)^2(x - 1)(x - 3)$
    B. $y = (x + 7)^3(x + 2)^4(x - 1)(x - 3)$
    C. $y = (x + 5)^2(x + 2)(x - 2)^2(x - 3)^2$
    D. $y = (x + 3)(x + 1)(x - 1)(x - 4)^4$

13. If y varies directly as x and inversely as the square of z and $y = 1/6$ when $x = 20$ and $z = 6$, determine y when $x = 14$ and $z = 5$.

    A. $^{3}/_{10}$
    B. $^{14}/_{25}$
    C. $^{21}/_{25}$
    D. $^{21}/_{125}$

14. The following conic section equation is written in standard form. What conic section does this formula create?

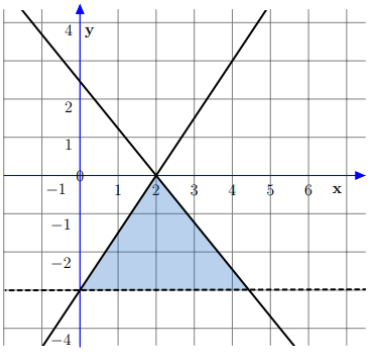
$$x^2 + x \cdot y + y^2 - 6x - 4y - 3 = 0$$

A. Circle
B. Parabola
C. Ellipse
D. Hyperbola

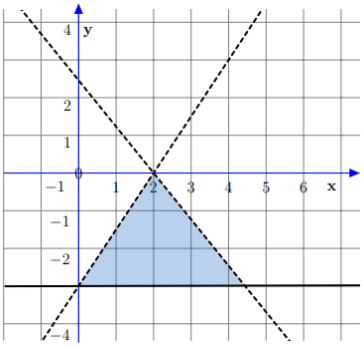
15. Which of the graphs depict the solution to the following systems of inequalities?

$$3x - 2y \geq 6$$
$$5x + 4y \leq 10$$
$$y > -3$$

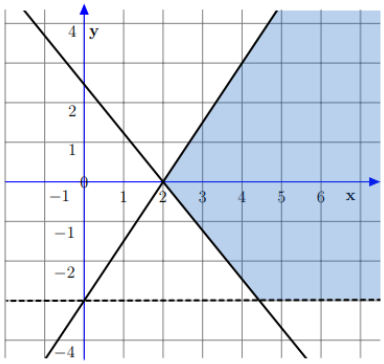
A)



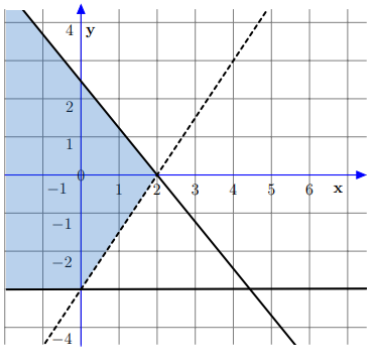
C)



B)



D)



16. Assuming that if $f(x) = 3x - 1$ and $g(x) = x^2 - 2$, find the composition of functions given as $g(f(x))$.

A. $g(f(x)) = 6x^2 - 6x + 1$
B. $g(f(x)) = 9x^2 - 6x + 3$
C. $g(f(x)) = 9x^2 - 3x - 1$
D. $g(f(x)) = 9x^2 - 6x - 1$

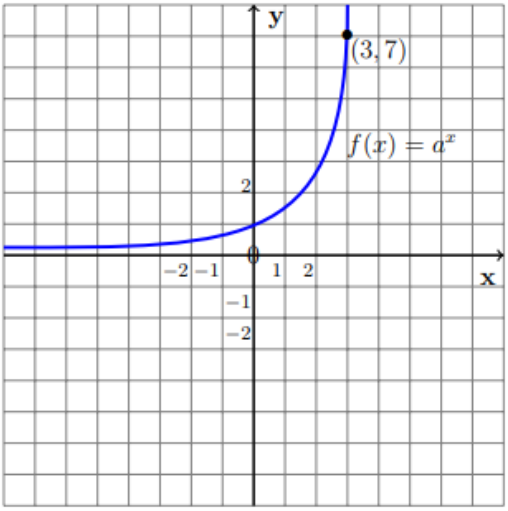
17. Suppose you wanted to solve the following equation by completing the square.

$$x^2 - 6x = 3$$

The next step would be to add a value p to both sides of the equal sign. What is this value?

A. $p = 3$
B. $p = 9$
C. $p = -3$
D. $p = -9$

18. Which equation matches the exponential graph shown below?



A. $a = \sqrt[7]{3}$
B. $a = \sqrt[3]{7}$
C. $a = 3^7$
D. $a = 7^3$

19. Find a polynomial of degree 8 such that -1 is a zero of multiplicity three and 0 is a zero of multiplicity five.

A. $f(x) = x^8 + 3x^7 + 3x^6 + x^5$
B. $f(x) = 2x^8 + 5x^7 + 5x^6 - x^5$
C. $f(x) = x^8 + 3x^7 + 3x^6 + x^5 + x^4$
D. $f(x) = x^8 - 3x^7 - 3x^6 + x^5$

20. Name any vertical asymptotes and/or holes found in the following rational expression:

$$f(x) = \frac{2x - 8}{x^2 - 16}$$

A. Asymptote at $x = 4$; there are no holes
B. There are no asymptotes; holes at $x = 4$ and $x = -4$
C. Asymptote at $x = -4$; hole at $x = 4$
D. Asymptotes at $x = 4$ and $x = -4$; there are no holes

21. Which of the following tables shows a relationship that is directly proportional?

A.

x	1	2	3	4
y	2	3	4	5

B.

x	1	2	3	4
y	5	4	3	2

C.

x	1	2	3	4
y	1	3	5	7

D.

x	1	2	3	4
y	2	4	6	8

22. Determine a, b, and c such that the graph of the equation $y = ax^2 + bx + c$ passes through the points (0,11), (1,5), and (2,3).

A. $a = 2, b = -8, c = 11$
B. $a = -2, b = 4, c = 11$
C. $a = -4, b = 8, c = 0$
D. $a = 1, b = 4, c = 11$

23. Find the determinant of the following matrix:

$$\begin{bmatrix} 10 & -9 \\ -7 & 3 \end{bmatrix}$$

A. 33
B. -33
C. 99
D. -99

24. Solve the following system of equations for x, y, and z.

$$\begin{aligned} -4x - 5y - z &= 18 \\ -2x - 5y - 2z &= 12 \\ -2x + 5y + 2z &= 4 \end{aligned}$$

A. $(-4, 1, 4)$
B. $(-4, 0, 2)$
C. $(-4, 0, -2)$
D. $(12, -1, -8)$

25. Find a polynomial of degree 2 with real coefficients that has the complex number $3 - 2i$ as a zero.

A. $5x^2 - 3x + 7$
B. $x^2 + x - 11$
C. $2x^2 + 1$
D. $x^2 - 6x + 13$

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Tie Breaker #1

Name: _____

School: _____

Find all roots for the equation $2x^3 - 3x^2 - 17x + 30 = 0$.

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Tie Breaker #2

Name: _____
School: _____

Using Matrix A and Matrix B, show that matrix multiplication is not commutative.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

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Tie Breaker #3

Name: _____

School: _____

If you invest $20,000 at an annual interest rate of 1% compounded continuously, calculate the final amount (to nearest cent) you will have in the account after 20 years.

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ACTM Contest Regional Algebra II Exam Answer Key

1. A 2. C 3. C 4. A 5. D 6. C 7. B 8. D 9. B 10. C 11. A 12. A 13. D 14. C 15. A 16. D 17. B 18. B 19. A 20. C 21. D 22. A 23. B 24. C 25. D	Tie-Breaker #1: $x = 2, -3, \frac{5}{2}$, using the rational zeroes (roots) theorem or factoring. Tie-Breaker #2: $A \cdot B = \begin{bmatrix} 2 & 3 \\ -2 & 2 \end{bmatrix}$ and $B \cdot A = \begin{bmatrix} 1 & 7 \\ -1 & 3 \end{bmatrix}$, which means $A \cdot B \neq B \cdot A$. Tie-Breaker #3: $\sim\$24428.06 = 20{,}000e^{0.01 \cdot 20}$