Arkansas Council of Teachers of Mathematics
Regional Exam – Algebra II

You have one hour to complete the exam. Students should answer the 25 multiple choice items first by marking your response on the answer sheet, then move to the tie breaker items. Tie breaker items require you to write the solution on the pages provided. Be sure that your name is at the top of each sheet. Tie breaker items will only be scored in the case of a tie in the number of correct responses on the multiple choice items.

1.)   Find the solution of the following system of equations.
$$\begin{cases} x + 2y - z = 8 \\ 2x + 4y + 3z = 11 \\ 4x - 2y + z = -3 \end{cases}$$
The sum of the x, y and z coordinates of the solution is:
a.)   2   b.)   3   c.)   4   d.)   5
e.)   Each of the other answers is incorrect.

2.) Tom was driving 20.7 meters per second in a 45 mile per hour zone. Was Tom speeding?   [1 mile = 1.6093 kilometers]
a.)   Yes, by more than five miles per hour.
b.)   Yes, but by less than five miles per hour.
c.)   No, he is less than five miles per hour below the limit.
d.)   No, he is more than five miles per hour below the limit.
e.)   Each of the other answers is incorrect.

3.)   Bob made $40,000 in 2004. For 2005, his salary went up 5%. For 2006, his salary went up 5% again. What was his salary in 2006?
a.) $50,000   b.) $44,100   c.) $48,000   d.) $44,000
e.)   Each of the other answers is incorrect.

4.)   You put $2500 in a certificate of deposit account for 3 years. It receives 4.5% interest compounded quarterly. You neither take out any money nor put in any more money. How much money will you have after 3 years?
a.) $2859.19   b.) $2852.92   c.) $2837.50
d.) $21195673.52   e.)   Each of the other answers is incorrect.

5.)   In 2004, Bobby Abreu of the Philadelphia Phillies made a salary of $10,600,000. That was an increase of 16.4835% from his 2003 salary. What was Abreu's 2003 salary (round your answer to the nearest thousand dollars? (data from http://www.businessofbaseball.com/)
a.) $8,853,000   b.) $9,100,000   c.) $12,347,000
d.) $6,430,000   e.)   Each of the other answers is incorrect.

6.) Consider the sequence where $a_1 = 1$, $a_2 = 3$, and $a_n = 2a_{n-1} + a_{n-2}$, $n > 2$. Find a_5.
   a.) 4π b.)   9   c.)   21   d.)   41
   e.)   Each of the other answers is incorrect.

7.) Bubba's All You Can Stomach Exotic Animal Eatery has a buffet price of $82.99. They give a 42% discount to college students. Once you get the discount, you have to pay 8.45% sales tax. How much will it cost for you to eat at Bubba's?
   a.) $127.80   b.) $37.80   c.) $52.20   d.) $88.81
   e.)   Each of the other answers is incorrect.

8.) Suppose the function f(x) is given by the graph below (each tick mark denotes 1 unit) and g(x) is defined by the table below.

x	g(x)
-2	2
-1	1
0	3
1	2
2	4
3	3
4	2
5	1
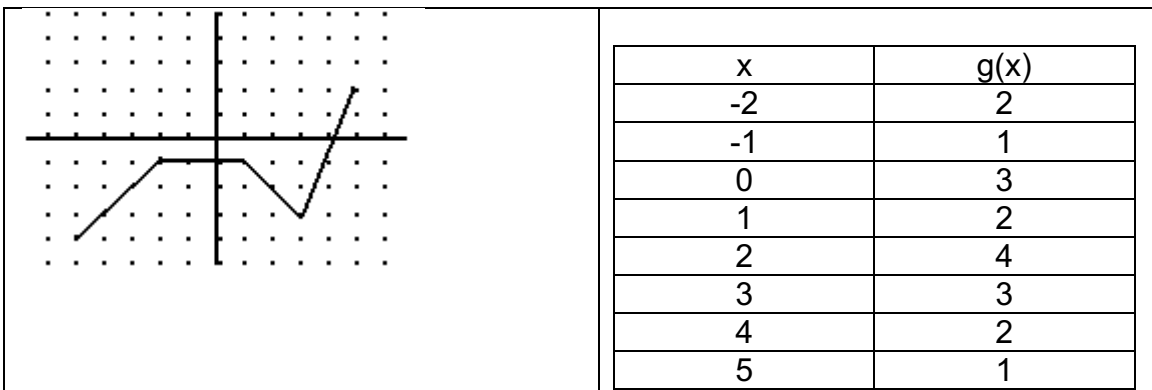
If $h(x) = x^2 + 1$, then what is $g \circ h \circ f\,(1)$.
   a.)   1   b.)   0   c.)   2   d.)   4
   e.)   Each of the other answers is incorrect.

9.) Suppose $\log_a b = r$ and $\log_a c = s$, then what is $\log_a(a^2 b/c)$?
   a.) $2 + r - s$ b.) $2r/s$ c.) $2 + r/s$ d.) $2r - s$
   e.)   Each of the other answers is incorrect.

10.) Find an equation for the line on the point (5, -2) and perpendicular to the line $3x - 6y = 1$.
   a.) $y - 2x = -12$ b.) $2x + y = 8$ c.) $x - 2y = -9$ d.) $2x + y = -3$
   e.)   Each of the other answers is incorrect.

11.) If $i = \sqrt{-1}$, then $\dfrac{-1}{i^{321,654,978}} =$ _____ .
   a.)   i   b.)   -i   c.)   1   d.)   -1
   e.)   Each of the other answers is incorrect.

12.) Find the domain of the real valued function $f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$.
   a.)   [-1, 3]   b.)   [-1, 2]   c.)   [2, 3]   d.)   (-∞, 3]
   e.)   Each of the other answers is incorrect.

13.) Simplify the following expression.
$$\sqrt{x^4 - 4x^2 + 4}$$
  a.) $x^2 - 2x + 2$ b.) $|x^2 - 2|$ c.) $x^2 - 2$
  d.) $x^2 + 2$ e.) Each of the other answers is incorrect.

14.) A box contains four copperheads, three scorpions and five cats. You pull two creatures out of the box without replacement. What is the probability they are both cats?
  a.) 5/33   b.) 5/12   c.) 25/144   d.) 1/9
  e.) Each of the other answers is incorrect.

15.) Everything is back in the box from #14. You pull out two creatures without replacement. What is the probability that they are different kinds of creatures?
  a.) 47/66   b.) 47/72   c.) 4/9   d.) 1/2
  e.) Each of the other answers is incorrect.

16.) Everything is back in the box from #14. You pull out two creatures with replacement. What is the probability that one is a cat and one is a scorpion?
  a.) 5/48   b.) 5/24   c.) 5/44   d.) 5/22
  e.) Each of the other answers is incorrect.

17.) If angle A is not in quadrant 2 and cos A = -3/5, what is sin A?
  a.) -4/5   b.) 4/5   c.) 3/5   d.) -3/5
  e.) Each of the other answers is incorrect.

18.) Evaluate the sum $\sum_{n=1}^{7} 4\left(\frac{-2}{3}\right)^n$. Round your answer to the nearest thousandth.
  a.) -1.694   b.) -1.600   c.) -8.468   d.) 7.532
  e.) Each of the other answers is incorrect.

19.) A company finds it can produce 25 heaters for $5950, while producing 35 heaters costs $8250. Express the cost, y, as a linear function of the number of heaters, x.
  a.) y = 23x + 5375   b.) y = 23x - 6525   c.) y = 230x + 200
  d.) y = 230x - 11700   e.) Each of the other answers is incorrect.

20.) If $i = \sqrt{-1}$, then $\frac{5-2i}{4+3i} = ?$
  a.) $\frac{26}{25} - \frac{23}{25}i$ b.) $\frac{26}{25} + \frac{7}{25}i$ c.) $\frac{14}{25} - \frac{23}{25}i$ d.) $\frac{5}{4} - \frac{2}{3}i$
  e.) Each of the other answers is incorrect.

21.) Find equations for any vertical asymptotes for $f(x) = \frac{3x^3 - 2x^2 - 5x + 1}{x^2 + 5x - 6}$.
  a.) x = 2, x = 3   b.) x = -6, x = 1   c.) x = 6, x = -1   d.) x = -2, x = -3
  e.) Each of the other answers is incorrect.

22.) Suppose you're looking at the graph of f(x). Suppose g(x) = 3f(x - 2) + 4. How does the graph of g(x) compare to the graph of f(x)?
    a.)   stretched vertically, moved left and up
    b.)   stretched vertically, moved right and down
    c.)   stretched vertically, moved right and up
    d.)   stretched vertically, moved left and down
    e.)   Each of the other answers is incorrect.

23.) The Empire State Building is 1454 feet tall (to the top of the radio tower). If a ball is thrown from the top of the tower with an downward velocity of 30 feet per second, its position at time, t, is given by $s(t) = -16t^2 - 30t + 1454$. Find the time it takes for the ball to get halfway down to the ground and the time it takes to reach the ground and add the numbers together. Round all answers to the nearest thousandth.
    a.)   17.604   b.)   12.962   c.)   13.858   d.)   14.509
    e.)   Each of the other answers is incorrect.

24.) Consider the parabola given by $y = x^2 + x + 1$ and the line given by y = 5x - 3. Find any points of intersection.
    a.)   The only point of intersection is (2, 7).
    b.)   The two curves do not intersect.
    c.)   There are exactly two points of intersection, one of which is (2, 7).
    d.)   There are at least three points of intersection, one of which is (2, 7).
    e.)   Each of the other answers is incorrect.

25.) What is the coefficient of the x^4 term in the expansion of $(2x^2 - 3)^5$.
    a.)   5   b.)   -1080   c.)   -240   d.)   80
    e.)   Each of the other answers is incorrect.

Algebra II Tiebreakers

Student Name: _____

Tiebreaker #1 - Without a calculator and showing all necessary work, find the exact

value of $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}}$

Tiebreaker #2 - The population of Bubbaville increases at fixed percentage each year. The population on January 1, 2008 was 2000. If the population on January 1, 2010 was 2250, during what month/year will the population reach 4000?

Student Name: _____

Tiebreaker #3 - Tom can paint a room in seven hours. Dave can paint the same room in nine hours. Mary can paint the same room in ten hours. Assuming they work with the same efficiency as when they work alone, how long will it take for the three working together to finish painting the room?

Answers
1.)   b
2.)   b
3.)   b
4.)   a
5.)   b
6.)   d
7.)   c
8.)   d
9.)   a
10.) b
11.) c
12.) b
13.) b
14.) a
15.) a
16.) b
17.) a
18.) a
19.) c
20.) c
21.) b
22.) c
23.) d
24.) a
25.) b

Tiebreaker #1

Let $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}}$

$x^2 = 2 + x$

$x^2 - x - 2 = (x - 2)(x + 1) = 0$ so $x = 2$ or $x = -1$. But since $x > 0$, $x = 2$ is the only solution.

Tiebreaker #2 - The population of Bubbaville increases at fixed percentage each year. The population on January 1, 2008 was 2000. If the population on January 1, 2010 was 2250, during what month/year will the population reach 4000?

$P = P_0 (1 + r)^t$

$2000 = P_0 (1 + r)^0$

$2250 = 2000 (1 + r)^2$

$9/8 = (1 + r)^2$

$1.6066 = 1 + r$

$R = .06066$

P = 2000 (1.06066)^t
4000 = 2000 (1.06066)^t
2 = (1.06066)^t
ln 2 = ln(1.06066)^t
ln 2 = t ln(1.06066)
t = ln 2/ ln 1.06066
t = 11.7699

month	days	cumulative fraction of year
January	31	0.084932
February	59	0.161644
March	90	0.246575
April	120	0.328767
May	151	0.413699
June	181	0.49589
July	212	0.580822
August	243	0.665753
September	273	0.747945
October	304	0.832877
November	334	0.915068
December	365	1

Population will reach 4000 in October 2019.

Tiebreaker #3 - Tom can paint a room in six hours. Dave can paint the same room in eight hours. Mary can paint the same room in ten hours. Assuming they work with the same efficiency as when they work alone, how long will it take for the three working together to finish painting the room?

$$\frac{x}{6} + \frac{x}{8} + \frac{x}{10} = 1$$

$$\frac{40x}{240} + \frac{30x}{240} + \frac{24x}{240} = 1$$

$$\frac{94x}{240} = 1$$

x = 240/94 = 120/47 hours