For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items. Be sure that your name is printed on each of the tiebreakers.

1. If $f(x) = \begin{cases} x \ln x, & 0 < x \le 2 \\ \ln x^2, & 2 < x \le 4 \end{cases}$, then $\lim_{x \to 2} f(x) =$

(A) ln 4
(B) ln 8
(C) ln 16
(D) 4
(E) Does Not Exist.

2. $\lim_{x \to -\infty} \dfrac{5x^2 + 7x^3 - 3}{2 + 3x - 11x^2} =$

(A) $-\infty$
(B) -5/11
(C) 0
(D) ∞
(E) Does Not Exist.

3. Find the average rate of change of $f(x) = x^2 + 5x + 14$ over $[-1, 2]$?

(A) 6
(B) 8
(C) 18
(D) 10
(E) 14

4. Find $\lim_{x \to 0} x \sin(\dfrac{4}{x}) =$

(A) 0
(B) ∞
(C) $-\infty$
(D) 4
(E) Does Not Exist.

5. $\lim_{x \to 0} \frac{tan3x}{sin5x}$

(A) $\frac{0}{0}$

(B) $\frac{3}{5}$

(C) $-\frac{3}{5}$

(D) $\frac{5}{3}$

(E) Does Not Exist.

6. The slope of an equation of the line tangent to the curve $y = x^2 - 4\sqrt{x}$ at the point where x = 4 is:

(A) 6
(B) 7
(C) 8
(D) 9
(E) 12

7. Suppose g(x) is a differentiable function with $g(2) = 5$ and $g'(2) = -3$. If $y = x^3 g(x)$ what is $y'|_{x=2}$?
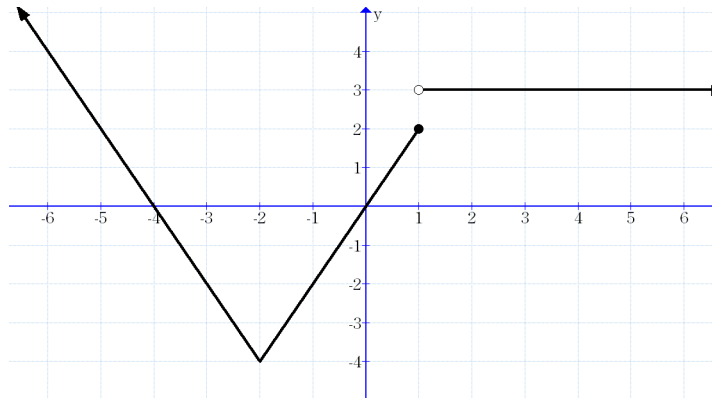
(A) −36
(B) −12
(C)　10
(D)　20
(E)　36

8. Find $\cos^{(256)}(x)$, where (256) denotes the 256th derivative.

(A) $cos\ x$
(B) $-cosx$
(C) $sinx$
(D) $-sinx$
(E) None of the Above

9. If $f(x) = \dfrac{\cos^2 x}{1 + \sin x}$, then $f'(x) =$

(A) $\cos x$

(B) $\sin x$

(C) $- \sin x$

(D) $2 \sin x$

(E) $- \cos x$

10. Consider the graph of $y = f(x)$ shown below.

At which of the following value(s) of x will the derivative of **f(x)** not exist?

    (A) $x = -4$ and $x = 0$

    (B) $x = -4$ and $x = -2$

    (C) $x = -2$ and $x = 0$

    (D) $x = -2$ and $x = 1$

    (E) $x = -4$, $x = -2$, and $x = 1$

11. The vertical and horizontal asymptotes of $f(x) = \dfrac{(x-2)(x+3)}{x^2-4}$ are:

(A) $x = -2$ and $y = 0$

(B) $x = -2$ and $y = 1$

(C) $x = -2$, $x=2$ and $y = 0$

(D) $x = -2$, $x=2$ and $y = 1$

(E) $x = -4$, $x = -2$, $y=-3$ and $y= 1$

12. Suppose $f(x) = \frac{x^2+x-20}{x-4}$ except for $x=4$. What value should be assigned to $f(4)$ to make $f(x)$ continuous at $x=4$?

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

13. If $f(x) = 6x^3 - 4x$, then $\frac{d}{dx}\left(f(lnx)\right) =$

(A) $\frac{2}{x}[9(lnx)^2 - 2]$

(B) $\frac{2}{x}[9(lnx)^2 - 4x]$

(C) $18(lnx)^2 - \frac{4}{x}$

(D) $18x^2 - 4$

(E) $18(\ln x)^2 - 4$

14. Which of the following functions is continuous at $x = 0$ but not differentiable at $x = 0$?

(A) $f(x)=x^{-4/3}$

(B) $f(x)=x^{-3/4}$

(C) $f(x)=x^{3/4}$

(D) $f(x)=x^{4/3}$

(E) $f(x)=x^4$

15. Let f and g be differentiable functions. If g is the inverse function of f and if $g(-2) = 5$, $f'(5) = -\frac{1}{2}$, $f'(-2) = 5$, find $g'(-2)$.

(A) 2

(B) 0.5

(C) 0.2

(D) -0.2

(E) -2

16. Evaluate $\int x^2 \cos(x^3)dx$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

17. Evaluate $\lim_{x\to\infty} \frac{x^3+2x}{-1+4x-3x^3}$

(A) 1/6
(B) 1/3
(C) -1/3
(D) 3
(E) Does Not Exist

18. Find the derivative dy/dx for the curve $x^2+xy+1=y$ at $x=-1$.

(A) -2
(B) -1/2
(C) 1/2
(D) -1
(E) Does Not Exist

19. Evaluate $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

(A) 0
(B) 1/4
(C) 1/2
(D) 0/0
(E) Does Not Exist

20. Suppose that $f(x) = \frac{x}{x^2+1}$. Find $\int_0^2 f'(x)dx$.

(A) 2/5
(B) -28/25
(C) 28/25
(D) 0
(E) None of the above

21. Which of the following gives the area between the curves $y = x^2$ and $y = 2x$ over the interval $[-2, 2]$?

(A) $\int_{-2}^{2}(x^2 - 2x)dx$

(B) $\int_{-2}^{2}(2x - x^2)dx$

(C) $\int_{-2}^{0}(x^2 - 2x)dx + \int_{0}^{2}(2x - x^2)dx$

(D) $|\int_{-2}^{2}(x^2 - 2x)dx|$

(E) None of the above

22. Let $y = 4e^{tanx}$. Evaluate y'.

(A) $4e^{tanx} \cdot sec^2 x$

(B) $4e^{tanx}$

(C) $4e^{tanx} \cdot cotx$

(D) $4sec^2 x$

(E) $4e^{tanx} \cdot \frac{1}{1+x^2}$

23. Let $f(x) = x^3 - 3x$. Which of the following statements are true?

I. f(x) has local maxima at both x = −1 and x = 1.

II. f(x) has a local minimum at x = 1 and an inflection point at x = 0.

III. f(x) has both a local minimum and an inflection point at x = 0.

(A) only I is true

(B) only II is true

(C) only III is true

(D) only I and III are true

(E) None of the statement is true

24. Suppose that $f''(x) = cos\ x$, $f'(\pi) = 2$ and $f(0) = 4$. What is $f(\pi)$?

(A) 2

(B) 2π

(C) $\pi + 2$

(D) $6 + 2\pi$

(E) None of the above

25. Evaluate $\lim_{x \to 1^-} \frac{x^3 + x}{-1 + x}$

(A) 1
(B) 4
(C) ∞
(D) $-\infty$
(E) Does Not Exist

Name:

Tiebreaker 1

Evaluate the following limit. If the limit does not exist, write "$+\infty$," "$-\infty$," or "DNE" (Does Not Exist) whichever is most appropriate.

$$\lim_{x \to 0} \frac{3}{e^{x^2} - 1} - \frac{3}{x^2}$$

You must show your work. An answer without explanation is not sufficient.

Name:

Tiebreaker 2

A boat is pulled by a rope, attached to the bow of the boat, and passing through a pulley on a dock that is 3 meter higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/sec, how fast is the boat approaching the dock when it is 4 meters from the dock?

Your answer should include appropriate units.

Name:

Tiebreaker 3

A closed rectangular container with a square base is to have a volume of 2000 in^3. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$1 per in^2. Find the dimensions of the container of least cost?

   Your answer should include appropriate units.

ACTM REGIONAL 2016 FOR CALCULUS (KEY)

1. A
2. D
3. A
4. A
5. B
6. B
7. E
8. A
9. E
10. D
11. B
12. D
13. A
14. C
15. E
16. B
17. C
18. B
19. B
20. A
21. C
22. A
23. B
24. D
25. D

Tiebreaker 1: $-\frac{3}{2}$ (no credit for just an answer, must show work)
Tiebreaker 2: 1.25 m/sec
Tiebreaker 3: $10in \times 10in \times 20in$