Arkansas Council of Teachers of Mathematics
Algebra II 2018 Regional Exam

Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Then attempt the tie-breaker questions at the end starting with tie breaker #1, then #2, and finally #3. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

Figures aren't necessarily drawn to scale. Angles are given in radians unless otherwise stated.

1. Let $f(x) = mx + b$. If $f(x+2) = f(x) + 2$, a formula for $f(x)$ is:
   a. $f(x) = 2x + b$
   b. $f(x) = 2 + b$
   c. $f(x) = x + b$
   d. $f(x) = 2x + 1$
   e. $f(x) = 2x + 2b$

2. Find an equation of the line passing through the point $(4, 5)$ perpendicular to the line passing through the points $(-1, 3)$ and $(2, 9)$.
   a. $y = -2x + 13$
   b. $y = \frac{1}{2}x + 3$
   c. $y = -\frac{1}{2}x + \frac{13}{2}$
   d. $y = -\frac{1}{2}x + 7$
   e. $y = 2x - 3$

3. The domain of the function $f(x) = (x - 9x^{-1})^{-1}$ is the following:
   a. $(-\infty, 0) \cup (0,3) \cup (3, \infty)$
   b. $(-\infty, -3) \cup (-3, 0) \cup (0,3) \cup (3, \infty)$
   c. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
   d. $(-3, 3)$
   e. $(-3, 0) \cup (0,3)$

4. The inverse of $f(x) = \frac{5x-3}{2x+1}$ is:
   a. $f^{-1}(x) = \frac{x+3}{-2x+5}$
   b. $f^{-1}(x) = \frac{2x+1}{5x-3}$
   c. $f^{-1}(x) = \frac{2x-3}{5x+1}$
   d. $f^{-1}(x) = \frac{2}{5}x - 3$
   e. $f^{-1}(x) = \frac{x}{5} - \frac{3}{2x}$

5. The solutions to $12 = (m^2 - 5m)^2 + (m^2 - 5m)$ are:
   a. 3 and -4
   b. 1 and 4
   c. 1, -1, 4, and -4
   d. 1, 4, $\frac{5+\sqrt{37}}{2}$, and $\frac{5-\sqrt{37}}{2}$
   e. 3, -4, 1, and 4

6. The line of symmetry of $f(x) = -3x^2 - 3x + 1$ is:
   a. $x = \frac{1}{2}$
   b. $x = \frac{7}{4}$
   c. $x = -\frac{1}{2}$
   d. $x = -\frac{7}{4}$
   e. $x = 1$

7. Simplify the expression: $\frac{1+i}{(1-i)^2}$
   a. $-\frac{1}{2} + \frac{1}{2}i$
   b. $1 + \frac{1}{2}i$
   c. $-\frac{1}{2} + i$
   d. $\frac{1}{2} + \frac{1}{2}i$
   e. $1 + 2i$

8. For $f(x) = 4x - 3x^2,\ g(x) = 2x - 4$, find $(f \circ g)(x)$.
   a. $(f \circ g)(x) = -12x^2 + 56x - 64$
   b. $(f \circ g)(x) = -6x^3 + 20x^2 - 16x$
   c. $(f \circ g)(x) = -6x^2 + 8x - 4$
   d. $(f \circ g)(x) = -3x^2 + 6x - 4$
   e. $(f \circ g)(x) = -3x^2 + 8x - 16$

9. Suppose you select, without looking, two marbles from a bag containing 4 red marbles and 10 green marbles. What is the probability of selecting, without replacement, two red marbles? Simplify your results.
   a. $\frac{3}{49}$
   b. $\frac{1}{2}$
   c. $\frac{6}{91}$
   d. $\frac{47}{91}$
   e. $\frac{1}{7}$

10. Solve: $\frac{(e^{3x+1})^2}{e^4} = e^{10x}$
   a. $x = \frac{2\pm\sqrt{31}}{9}$
   b. $x = \frac{1}{2}$
   c. $x = -\frac{1}{2}$
   d. $x = \frac{1}{17}$
   e. No solution

11. Simplify $\log_{10} 11 \cdot \log_{11} 12 \cdot \log_{12} 13 \cdot \ldots \cdot \log_{999} 1000$:
    a. 1
    b. 3
    c. 1000
    d. 100
    e. None of the above

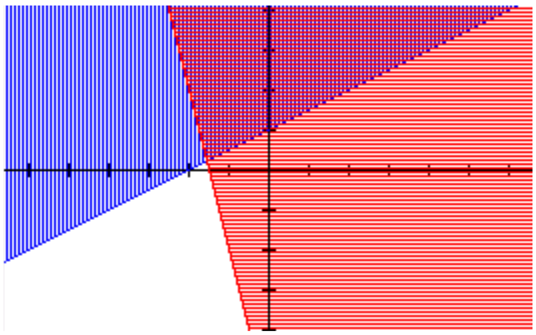
12. Solve $|3x - 1| > 5x - 2$
    a. $\left(\frac{3}{8}, \frac{1}{2}\right)$
    b. $\left(-\infty, \frac{1}{2}\right)$
    c. $\left(\frac{3}{8}, \infty\right)$
    d. $(-\infty, \infty)$
    e. $\left(-\infty, \frac{3}{8}\right) \cup \left(\frac{3}{8}, \frac{1}{2}\right)$

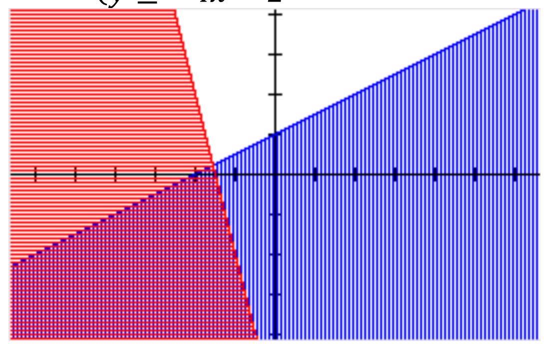
13. The product of the digits of a two-digit number plus the sum of the digits is equal to the two-digit number itself. What is the units digit of the number?
    a. 1
    b. 3
    c. 5
    d. 7
    e. 9

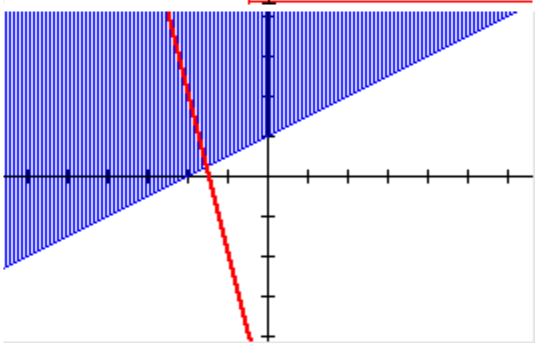
14. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 \\ 2 & -1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, find $BA + 2C$.
    a. $\begin{bmatrix} 19 & 7 \\ -4 & 3 \end{bmatrix}$
    b. $\begin{bmatrix} 3 & 1 \\ -8 & 19 \end{bmatrix}$
    c. $\begin{bmatrix} 9 & 1 \\ -2 & 7 \end{bmatrix}$
    d. $\begin{bmatrix} 36 & 16 \\ -6 & 4 \end{bmatrix}$
    e. None of the above

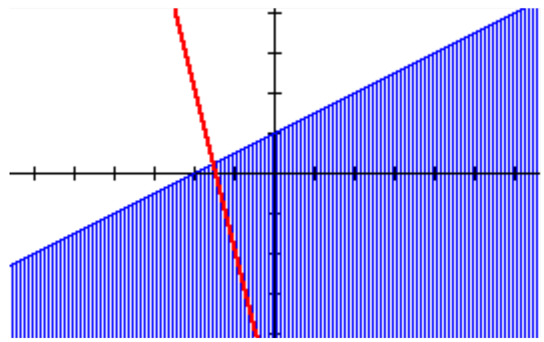
15. Find the vertices of the ellipse with the equation $16x^2 + 9y^2 = 144$.
    a. $(0, 3)$ and $(0, -3)$
    b. $(0, 3)$ and $(0, 4)$
    c. $(0, 4)$ and $(0, -4)$
    d. $(0, -3)$ and $(0, -4)$
    e. $(-3, 0)$ and $(3, 0)$

16. Graph the solution of the following system of inequalities: $\begin{cases} y \le \frac{1}{2}x + 5 \\ y \ge -4x - 2 \end{cases}$

a. 

b. 

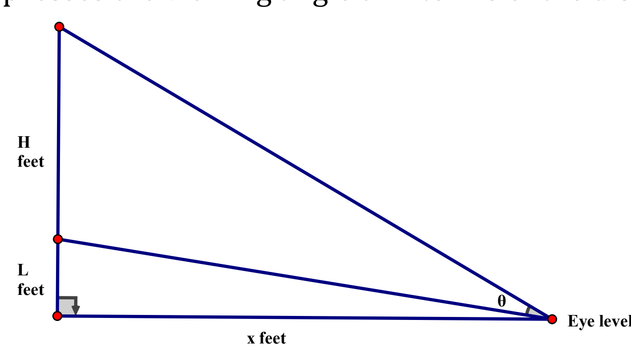
c. 

d. 

e. None of the above

17. Admission to the Children's Museum costs twice as much for adults as for children. Admission to the museum for 4 adults and 16 children cost \$72. Find the cost of each adult's admission and each child's admission.
   a. Adult admission: \$12; Child admission: \$6
   b. Adult admission: \$3; Child admission: \$6
   c. Adult admission: \$6; Child admission: \$3
   d. Adult admission: \$7; Child admission: \$3.5
   e. Not enough information to determine solution

18. A movie theater has a screen of height H feet, and the bottom of the screen is located L feet above your eye level. Which of the following expresses the viewing angle θ in terms of the distance x that you sit from the screen?
   a. $\sin^{-1}\left(\frac{H}{\sqrt{x^2+(H+L)^2}}\right)$
   b. $\tan^{-1}\left(\frac{H+L}{x}\right) - \tan^{-1}\left(\frac{L}{x}\right)$
   c. $\tan^{-1}\left(\frac{H}{\sqrt{x^2+L^2}}\right)$
   d. $\tan^{-1}\left(\frac{H-L}{x}\right)$
   e. $\tan^{-1}\left(\frac{H}{x}\right) - \tan^{-1}\left(\frac{L}{x}\right)$



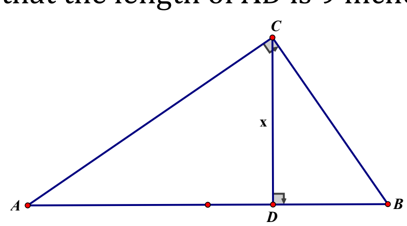
19. The graph of the function $f(x) = ax^2 + bx + c$ passes through the points $(1,8), (2,0), (3,16)$. What is $f(x)$?
    a. $f(x) = 2x^2 + 3x - 14$
    b. $f(x) = 3x^2 - 11x + 10$
    c. $f(x) = x^2 - 4x + 4$
    d. $f(x) = 12x^2 - 44x + 40$
    e. $f(x) = 6x^2 + 22x - 10$

20. An equilateral triangle of side length 3 centimeters is inscribed in a circle. Find the total area of the region that lies inside the circle but outside the triangle.
    a. $3\pi - \frac{9\sqrt{3}}{4}$
    b. $2\pi - \frac{\sqrt{2}}{3}$
    c. $3\pi - \frac{7\sqrt{3}}{4}$
    d. $4\pi - \frac{7\sqrt{3}}{2}$
    e. $4\pi - \frac{5\sqrt{2}}{3}$

21. Which of the following is true of the graph $f(x) = -a \sin(\pi - bx)$ for any positive a and b?
    a. The amplitude is a, the period is π, and the x-intercepts are spaced at $\frac{\pi}{a}$ unit intervals.
    b. The amplitude is a, the period is $\frac{\pi}{b}$, and the x-intercepts are spaced at $\frac{\pi}{2b}$ unit intervals.
    c. The amplitude is a, the period is $\frac{2\pi}{b}$, and the x-intercepts are spaced at $\frac{\pi}{b}$ unit intervals.
    d. The amplitude is $-a$, the period is $\frac{\pi}{b}$, and the x-intercepts are spaced at $\frac{\pi}{2b}$ unit intervals.
    e. Unable to determine with the given information.

22. The graphs of the equations $y = ax$ and $y = \frac{b}{x}$ have two intersection points, one of which is $(-3,-2)$. What is the other intersection point?
    a. $(2,3)$
    b. $(2,-3)$
    c. $(-2,3)$
    d. $(3,2)$
    e. $(3,-2)$

23. In the figure below, suppose we know that the length of $\overline{\text{AD}}$ is 9 inches and the length of $\overline{\text{DB}}$ is 1 inch. What is the length of x?
    a. $\sqrt{3}$
    b. $\sqrt{5}$
    c. $\frac{5}{2}$
    d. 3
    e. 4



24. Simplify the following: $a + \frac{1}{b + \frac{1}{a+b}}$
    a. $\frac{2a+b}{b+1}$
    b. $\frac{a^2+ab^2-ab}{a^2+ab+1}$
    c. $\frac{a^2b+ab^2+2a+b}{ab+b^2+1}$
    d. $\frac{a^3+3a^2b+3ab^2+1}{a^2+ab+b^2+1}$
    e. $\frac{a^2b+ab+b}{a^2b+ab^2+a+b+1}$

25. Solve for x: $\cos(2x) - \cos x = 0$
    a. $\frac{\pi}{2} + k \cdot \pi$, where k is any integer
    b. $\frac{2\pi}{3} \cdot k$, where k is any integer
    c. $\frac{2\pi}{3} \cdot k + \pi$, where k is any integer
    d. $2\pi \cdot k$, where k is any integer
    e. $\pi \cdot k$ or $\frac{2\pi}{3} \cdot k$, where k is any integer

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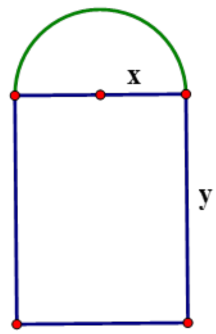
Tie Breaker #1

Name: ___________________
School: __________________

A Norman window has the shape of a rectangle with a semicircle on top. Sky Blue Windows is designing a Norman window with 24 feet of trim (perimeter). What are the dimensions that allow the maximum amount of light to enter a house? What is the area yielded by these dimensions?



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Tie Breaker #2

Name: ___________________
School: __________________

Martin and Eva pool their money to buy snacks after school. One day, they spend \$1.85 on 1 carton of milk, 2 donuts, and 1 bottle of juice. The next day, they spent \$2.30 on 3 donuts and 2 bottles of juice. The third day, they bought 1 carton of milk, 1 donut, and 2 bottles of juice and spent \$1.75. On the fourth day, they have a total of \$1.80 left. Is this enough to buy 2 cartons of milk and 2 donuts?

Tie Breaker #3

Name: ___________________
School: __________________

Eratosthenes, an astronomer who lived in Greece in the third century B.C., is credited with providing the first accurate measure of Earth's circumference. He found that at noon on the same day of the summer solstice, the sun was directly over the city of Syene. At the same time, in Alexandria, which is north of Syene, the sun was $7°12'$ south of being directly overhead. If the distance between the two cities was 5000 stadia, find Eratosthenes' measure of the circumference of Earth.

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Answer Key

1. C
2. D
3. B
4. A
5. D
6. C
7. A
8. A
9. C
10. C
11. B
12. B
13. E
14. A
15. C
16. E
17. C
18. B
19. D
20. A
21. C
22. D
23. D
24. C
25. B

<u>Tie Breaker #1:</u>

$\frac{1}{2} \cdot 2\pi x + 2x + 2y = 24$, so $y = 12 - \frac{\pi x}{2} - x$

$A(x) = \frac{1}{2}\pi x^2 + 2x\left(12 - \frac{\pi x}{2} - x\right)$

$A(x) = \frac{1}{2}\pi x^2 + 24x - \pi x^2 - 2x^2$

$A(x) = 24x - \frac{1}{2}\pi x^2 - 2x^2$ or $A(x) = 24x - (\frac{1}{2}\pi + 2)x^2$

Since $A(x)$ is a quadratic function with $a = -\left(\frac{1}{2}\pi + 2\right) < 0$,the maximum function value occurs at the vertex of the graph of $A(x)$.

The first coordinate of the vertex is

$$\frac{-b}{2a} = -\frac{24}{2\left[-\left(\frac{1}{2}\pi + 2\right)\right]} = \frac{24}{\pi + 4}$$

When $x = \frac{24}{\pi+4}$, then $y = \frac{24}{\pi+4}$.

Thus, the maximum amount of light will enter when the dimensions of the rectangular part of the window are $2x$ by y, or $\frac{48}{\pi+4}$ft by $\frac{24}{\pi+4}$ft, or approximately 6.72 ft by 3.36 ft. The resulting area is approx. 40.33ft^2.

Tie Breaker #2:

Let x, y, and z represent the prices of a carton of milk, a donut, and a bottle of juice, respectively.
Need to solve:

$$x + 2y + z = 1.85$$
$$3y + 2z = 2.30$$
$$x + y + 2z = 1.75$$

Solution: $x = \$0.45, y = \$0.50, z = \$0.40$.
The two cartons of milk and 2 donuts will cost $2(\$0.45) + 2(\$0.50)$, or \$1.90. They will NOT have enough money.

Tie Breaker #3:

Since there are 360° in a full rotation around Earth, the following proportion can be written.

$$\frac{360°}{7°12'} = \frac{c}{5000}$$

Solving for c, we have

$$(7°12')c = (360°)5000$$
$$c = \frac{(360°)5000}{7°12'}$$
$$c = \frac{(360°)5000}{7.2°} \text{ since } 7°12' = 7\,\frac{12°}{60}$$
$$c = 250{,}000$$

Eratosthenes' measure of Earth's circumference was 250,000 stadia.

Note that other methods could be used, such as using the arclength formula ($S = \theta \cdot r$) to find r, then calculate the circumference with that r.