Arkansas Council of Teachers of Mathematics

2017 Regional Exam

Pre-Calculus

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tie breaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreakers.

1. Which complex number has a modulus of $\sqrt{29}$ and an argument of approximately 201.8°?

    A. $-2-5i$
    B. $-5-2i$
    C. $-2+5i$
    D. $-5+2i$
    E. None of these.

2. Find the equation of the circle whose center is at $(4, -5)$ and contains the point $(8, -1)$.

    A. $(x-4)^2 +(y+5)^2 = 52$
    B. $(x+4)^2 +(y-5)^2 = 52$
    C. $(x-4)^2 -(y+5)^2 = 52$
    D. $(x-4)^2 +(y+5)^2 = 32$
    E. $(x+4)^2 +(y-5)^2 = 32$

3. Find all possible solutions, in radians, for: $\cos 2x + \cos x = 0$

    A. $\pi + 2n\pi$
    B. $\dfrac{5\pi}{3} + 2n\pi$
    C. $\dfrac{\pi}{3} + 2n\pi$
    D. All of these
    E. None of these

4. Determine the value of the 10th term that will appear in the given sequence $\{1,3,6,10,15,\ldots\}$

    A. 65
    B. 25
    C. 58
    D. 45
    E. None of these

5. On what domain does the function, $f(x) = \dfrac{x^2}{x^2+1}$, have an inverse.

    A. $(-\infty, \infty)$

    B. $(0, \infty)$

    C. $(-2, 2)$

    D. F always has an inverse except when $x = i$

    E. F never has an inverse

6. Find the directrix of the parabola given by $y = \dfrac{1}{4}(x-2)^2 + 3$.

    A. $x = 0$

    B. $y = 0$

    C. $y = 2$

    D. $y = 4$

    E. $x = 2$

7. The number of trees (N) that can be found per acre is a function of the diameter (S) of the trees. The function that describes this is approximated by $N = 50 \cdot 10^{-0.03S}$. Find a function that describes the diameter of the trees (S) as a function of the number of trees per acre (N).

    A. $S = \dfrac{\log N - 5}{-.03}$

    B. $S = \dfrac{\sqrt[10]{\dfrac{N}{50}}}{-.03}$

    C. $S = 50 \cdot 10^{\frac{-100}{3}N}$

    D. $S = \dfrac{-100}{3}(\log N - \log 50)$

    E. This function does not have an inverse.

8. Determine the coefficient in front of x^{12} for the polynomial given by $(x+2)^{15}$

    A. 455

    B. 8

    C. 3,640

    D. 21,840

    E. 1,760

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9. What is the range of the arcsin function, in radians?

    A. $-2\pi \le y \le 2\pi$

    B. $-\infty \le y \le \infty$

    C. $0 < y < \pi$

    D. $0 \le y < \dfrac{\pi}{2}$

    E. $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

10. A rock is dropped from a tall tower. In the first second, the rock falls 16 feet. In the second second, the rock falls 48 feet. In the third second it falls 80 feet. In the fourth second, it falls 112 feet. How long will it take for the rock to hit the ground if it is dropped from the top of a 520 foot building?

    A. 5 seconds
    B. 16.5 seconds
    C. 10 seconds
    D. 5.7 seconds
    E. 4.5 seconds

11. A new strain of the bird flu was found in an Arkansas town. The town has a population of 5000 people. The CDC (Center for Disease Control) will put the town under quarantine if more than 50% of the population develops the disease. After how many days will the town be under quarantine if the spread of the virus is found to be given by:

$n = \dfrac{5000}{1+4500e^{-0.6t}}$ where n is the number of people with the disease and t is the number of days since the outbreak.

    A. -1.33 days
    B. 6.4 days
    C. 140 days
    D. 14 days
    E. None of these

12. Two vectors act on each other to produce a resultant of 32.9N at 68.3°. One of the vectors is 20N at 40°. What is the other vector?

    A. 18N, $\theta = 160°$
    B. 18N, $\theta = 100°$
   C. 21.8N, $\theta = 174.3°$
    D. 49N, $\theta = 68.3°$
    E. None of these

13. A house sits on a steep hill. The side of the house makes an angle of 110° with the ground going down the hill. A ladder is set up so that the base of the ladder is 5 feet from the wall of the house when measured along the ground. The ladder needs to go up to a height of 12 feet on the wall. What angle will the ladder make with the ground?

    A. 14.5°
    B. 41.6°
    C. 17°
    D. 18.9°
    E. 51°

110°
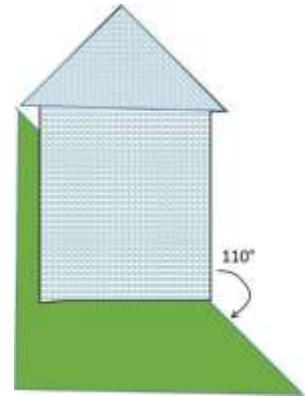
14. Find the $\sin\left(\arccos\left(\dfrac{2}{x}\right)\right)$

A. $\dfrac{\sqrt{4-x^2}}{2}$ B. $\dfrac{x}{\sqrt{x^2+4}}$ C. $\dfrac{\sqrt{x^2-4}}{x}$ D. $\dfrac{\sqrt{x^2+4}}{x}$ E. Not possible

15. A sheet of paper is moving at x ft/sec on a printing machine. If a sensor detects a problem, then the distance the sheet travels before the signal reaches the control center is given by $R(x)=\dfrac{1}{3}x$. Once the signal reaches the control center, the distance the sheet continues to travel before the machine can stop is given by $S(x)=\dfrac{1}{10}x^2$. Find a function, F(x), that gives the total distance the sheet of paper travels once the problem is detected by the sensor.

    A. $F(x)=\dfrac{1}{10}x^2+\dfrac{1}{3}x$

    B. $F(x)=\dfrac{3}{10}x, \ x\neq 0$

    C. $F(x)=\dfrac{1}{30}x^3$

    D. $F(x)=\dfrac{1}{10}x^2+x$

    E. None of these

16. What is the sum of the coefficients when you expand $(x+1)^7$?
    A. 64
    B. 49
    C. 81
    D. 128
    E. 62

17. Determine a formula that will compute the nth term for the given sequence: $\{1,3,6,10,15,...\}$

A. $a_n = \dfrac{n(n+1)}{2}$

B. $a_n = a_1 + (n-1)2$

C. $a_n = a_1 + (n+1)2$

D. $a_n = (n+1)(n-1) - n$

E. None of these

18. A plane is heading due north at 500 km/hr. A wind of 48 km/hr is blowing from the southeast at 28° west of north. What is the resultant speed of the plane?

    A. 543 km/hr   B. 548 km/hr   C. 458 km/hr   D. 479 km/hr   E. 434 km/hr

19. Find the asymptotes of the hyperbola defined by the equation:
$$9x^2 - 4y^2 - 18x + 32y - 91 = 0$$

A. $y = \pm\dfrac{3}{2}x$

B. $y = -\dfrac{3}{2}x + 6$ and $y = \dfrac{3}{2}x$

C. $y = \pm x$

D. $y = -\dfrac{3}{2}x + \dfrac{11}{2}$ and $y = \dfrac{3}{2}x + \dfrac{5}{2}$

E. None of these

20. Given that the $\sin s = -\dfrac{2}{3}$ with s in quadrant III, find the $\tan s$.

    A. $-\dfrac{\sqrt{5}}{2}$ B. $\dfrac{2\sqrt{5}}{5}$ C. $-\dfrac{3}{2}$ D. $-\dfrac{2\sqrt{13}}{13}$ E. $\dfrac{\sqrt{5}}{2}$

21. Find the equation in standard form of the conic section given by $4x^2 - 32x - y^2 + 6y = -19$.

A. $\dfrac{4(x-1)^2}{83} - \dfrac{(y-3)^2}{83} = 1$

B. $\dfrac{(x-16)^2}{249} - \dfrac{(y-3)^2}{996} = 0$

C. $\dfrac{(x-4)^2}{9} - \dfrac{(y-3)^2}{36} = 1$

D. $\dfrac{(x-16)^2}{249} - \dfrac{(y-3)^2}{996} = 1$

E. None of these

22. Which of the following functions has a period of $a \cdot \pi$ and an amplitude of b?

A. $y = b \sin\left(\dfrac{2}{a}x\right)$

B. $y = \dfrac{1}{b}\sin\left(\dfrac{2}{a}x\right)$

C. $y = b\sin(ax)$

D. $y = a\sin\left(\dfrac{2}{b}x\right)$

E. $y = \dfrac{1}{a}\sin(bx)$

23. Determine a formula to compute the nth term for the recursive relation defined as
$a_{n+1} = 2a_n + 1$ where $a_1 = 1$.

A. $a_n = 2^n - \left(1 - 2^n\right)$

B. $a_n = 2^{n+1} - \left(2n + 1\right)$

C. $a_n = 2^{n-1} + \left(2n - 3\right)$

D. $a_n = 2^n - 1$

E. All of these

24. Two trucks are pulling a stump out of the ground. Before the stump is pulled out, the one truck exerted 1800 lbs of force and the other truck exerted 2200 lbs of force. The angle between the chains connecting each truck to the stump is 28.5°. What is the combined force acting on the stump?

    A. 4000 lbs
    B. 2843 lbs
    C. 3878 lbs
    D. 2200 lbs
    E. None of these

25. Let $f(x) = \dfrac{1}{8}x + 4$ and let $g(x) = x^3$. Evaluate $\left(f^{-1} \circ g^{-1}\right)(8)$

A. $4\dfrac{1}{4}$

B. 4,064

C. 2

D. $-\dfrac{1}{2}$

E. -16

Name:_____

Suppose that a survey has been conducted to investigate the annual population movement between a city and its surrounding suburbs. It was found that 95% of the people who live in the city remained in the city the following year, while only 5% left for the suburbs. It was also found that 97% of those who live in the suburbs remain in the suburbs, but 3% left to live in the city.

a) If these trends continue, then what is the population that civil engineers should expect for the city and the suburbs in 2 years, if the initial population for the city is 1,000 and the suburbs is 1,000?
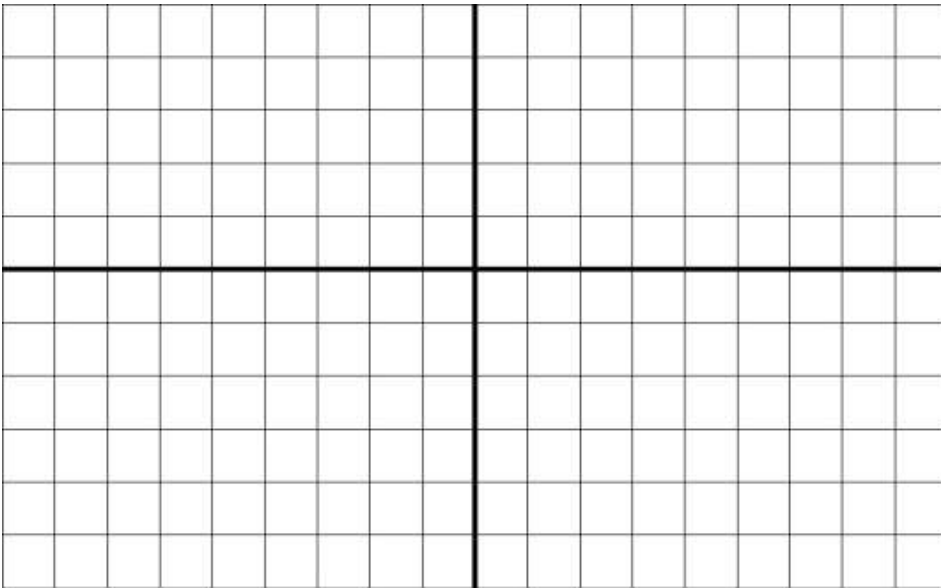
b) If these trends continue, then what is the population that civil engineers should expect for the city and the suburbs in 2 years, if the initial population of the city is 5,000 and the suburbs is 1,000?

Name:_____

An ellipse has foci at (1,2) and (7,2). The major axis has a length of 8 units. Find the standard equation for this conic section.
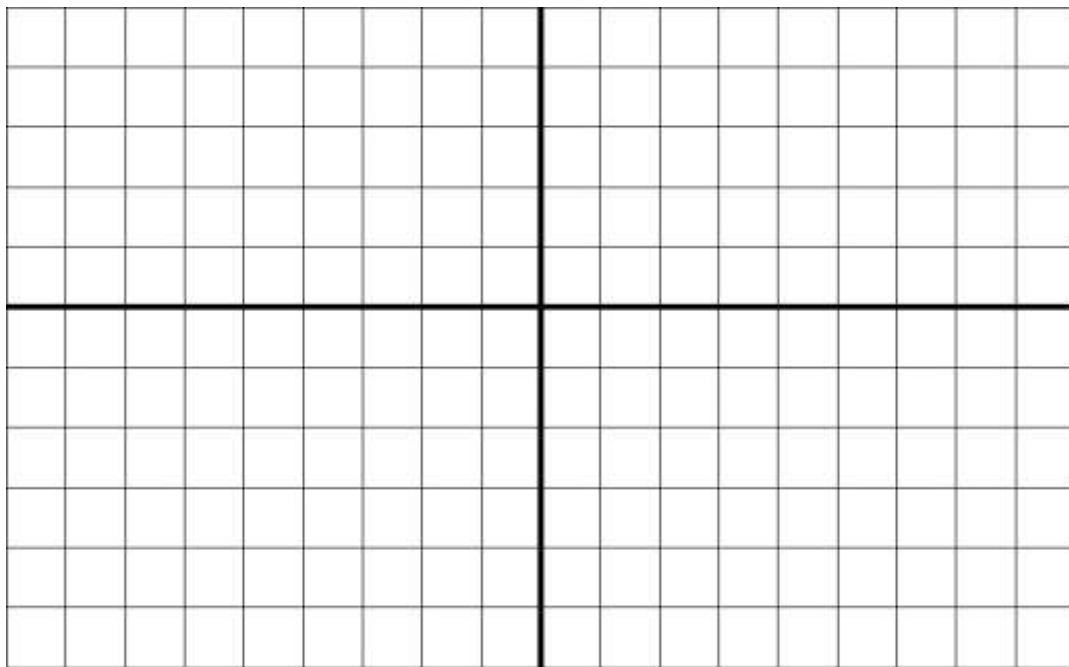
Graph the ellipse given in this problem.

Find all x-intercepts and y-intercepts.

Name:_____

Graph the function: $f(x) = \dfrac{x^3 - x^2 - 5x - 3}{2x^2 - 5x - 3}$

Find all asymptotes and undefined points:

Find all x-intercepts:

Find all y-intercepts:

Describe the end behavior of this graph:

Answer Key:

1. B	14. C
2. D	15. A
3. D	16. D
4. E	17. A
5. B	18. A
6. C	19. D
7. D	20. B
8. C	21. C
9. E	22. A
10. D	23. D
11. D	24. C
12. B	25. E
13. E	

Tie Breaker 1:

a) Year 1: The population of the city is given by: 1000(.95)+1000(.03) = 980 people and the population of the suburbs is given by 2000 – [1000(.95)+1000(.03)] = 1020 people.

In other words, the city would lose 5% (or 50 people) but they would gain 3% of the suburbs population (or 30 people). That would put the population of the city at 980 and the population of the suburbs at 1020 people.

Year 2: The population of the city is given by 1000(.95+.03)(.95)+(.03)[(1000)(.97)+(1000)(.05)] = 961.6 or about 962 people. The population of the suburbs can be found by 6000 – (1000(.95+.03)(.95)+(.03)[(1000)(.97)+(1000)(.05)])= 1038 people.
In other words, the city would lose another 5% (or 49 people) but they would gain 3% of the suburbs population (or 30.6 people). That would mean that 961.6 (or 962) people would live in the city and 1038 people would then live in the suburbs.

b) Year 1: The population of the city is found by 5000(.95)+1000(.03)=4780 people. The population of the suburbs can be found by 6000 – [5000(.95)+1000(.03)]=1220 people

In other words, the city would lose 5% (or 250 people) but they would gain 3% of the suburbs population (or 30 people). That would put the population of the city at 4780 and the population of the suburbs at 1220 people.

Year 2: The population of the city can be found by .95[5000(.95)+1000(.03)]+.03[1000(.97)+5000(.05)]=4577.6 or about 4578 people. That would give the population of the suburbs to be .97[1000(.97)+5000(.05)]+.05[5000(.95)+1000(.03)]= 1422.4 or 1422 people.
In other words, the city would lose another 5% (or 239 people) but they would gain 3% of the suburbs population (or 36.6 people). That would put the population of the city at 4,577.6 or 4,578 people and the population of the suburbs would be 1,422 people.
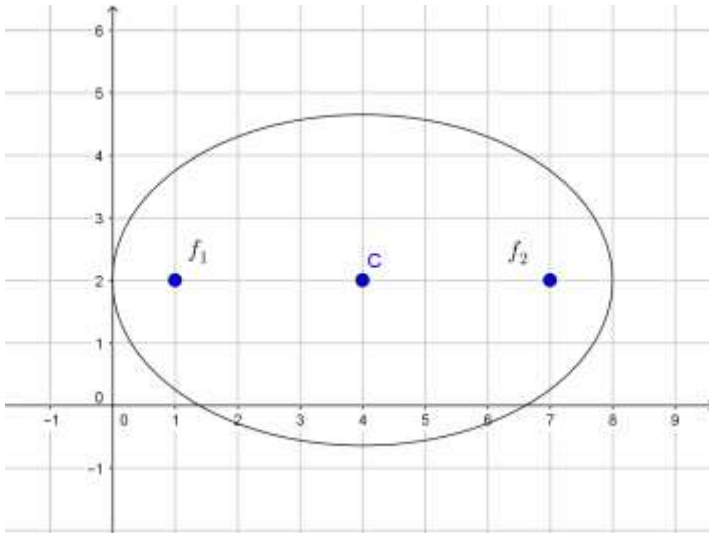
Tie Breaker 2:

The foci are at (1,2) and (7,2) so the center is at $\left(\dfrac{7+1}{2},\dfrac{2+2}{2}\right)=(4,2)$.

Since the center is at (4,2) and one of the foci is at (1,2) that means that c=3.

Since the major axis is 8, we know that 2a=8 so a=4.

Since $a^2=b^2+c^2$, we get $16=b^2+9$ or $b^2=7$

That gives the standard form of the ellipse as $\dfrac{(x-4)^2}{16}+\dfrac{(y-2)^2}{7}=1$

Finding the zeros: Let x = 0 and we get $\dfrac{(-4)^2}{16}+\dfrac{(y-2)^2}{7}=1 \Rightarrow 1+\dfrac{(y-2)^2}{7}=1 \Rightarrow \dfrac{(y-2)^2}{7}=0 \Rightarrow y-2=0$

so $y=2$. The y-intercept is at (0, 2).

To find the x-intercept, let y=0 and we get $\dfrac{(x-4)^2}{16}+\dfrac{(y-2)^2}{7}=1 \Rightarrow \dfrac{(x-4)^2}{16}+\dfrac{(-2)^2}{7}=1$

$\Rightarrow \dfrac{(x-4)^2}{16}+\dfrac{4}{7}=1 \Rightarrow \dfrac{(x-4)^2}{16}=\dfrac{3}{7} \Rightarrow (x-4)^2=\dfrac{48}{7} \Rightarrow (x-4)=\pm\sqrt{\dfrac{48}{7}} \Rightarrow x=4\pm\sqrt{\dfrac{48}{7}}$. From this we

can find that $x=4\pm\dfrac{\sqrt{48}}{\sqrt{7}}$ and this gives us that $x=4\pm\dfrac{\sqrt{336}}{7}$ which is approximately 1.4 and 6.6 for the x-

coordinates. Therefore the x-intercepts are $\left(4+\dfrac{\sqrt{336}}{7},0\right)$ and $\left(4-\dfrac{\sqrt{336}}{7},0\right)$.

Tie Breaker 3:

Graph:
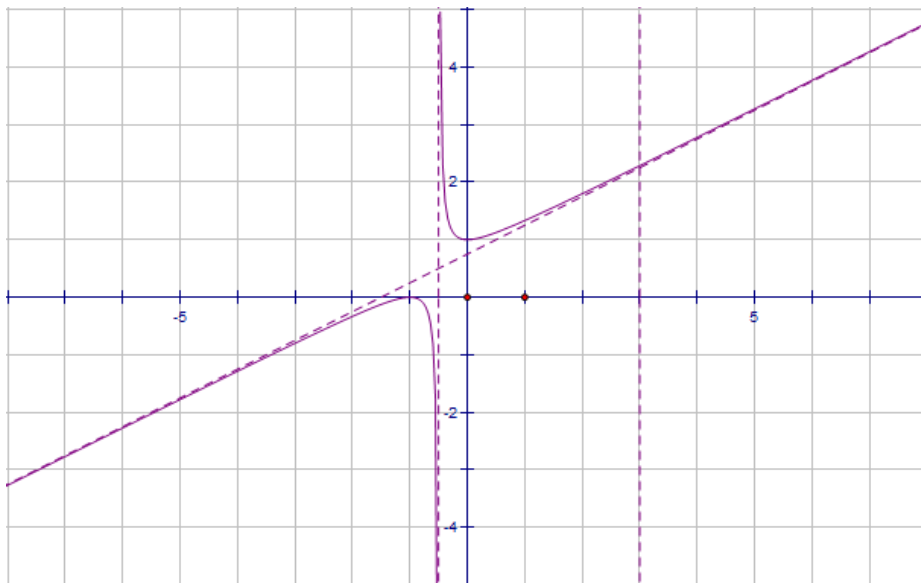
Find all asymptotes and undefined points: $y = 0$, $\quad y = \dfrac{1}{2}x + \dfrac{3}{4}$ and a hole in the graph at x=3

Find all x-intercepts: no x-intercepts

Find all y-intercepts: (0,1)

Describe the end behavior of this graph: as x approaches infinity the graph goes to infinity. As x approaches negative infinity, the graph approaches negative infinity.