

ACTM Regional Pre-Calculus/Trigonometry Exam
March 5, 2016

Mark your answer choice on the answer sheet provided. If you have time, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages.

1. Simplify the expression: $(2x^2y)(3x^{-2}y^2)$.

a.) $6xy^2$ b.) $6y^3$ c.) $5xy^2$ d.) $5y^3$

2. Simplify as much as possible: $\log_b a \cdot \log_a b$.

a.) $\frac{a}{b}$ b.) ab c.) 0 d.) 1 e.) none of these

3. Write the following expression in standard form $a + bi$: $-2(\cos 120^\circ + i \sin 120^\circ)$.

a.) $1 - i\sqrt{3}$ b.) $1 + i\sqrt{3}$ c.) $-1 - i\sqrt{3}$ d.) $-1 + i\sqrt{3}$

4. Evaluate: $\frac{100!}{2!98!}$.

a.) 0.5 b.) 485100 c.) 4950 d.) 9900 e.) none of these

5. A Ferris wheel with a diameter of 120 feet makes a complete revolution in 3 minutes. Which of the following is closest to the linear speed of a passenger in feet per second?

a.) 0.5 ft/sec b.) 1 ft/sec c.) 2 ft/sec d.) 3 ft/sec

6. Evaluate: $\log_{e^2} e$.

a.) 0.5 b.) 1 c.) 2 d.) -1 e.) 0

7. The solution set for $\sqrt{\cos^2 \theta + \sin \theta + 1} = \sin \theta + 1$ where $0 \leq \theta < 2\pi$ is

a.) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$ b.) $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}\right\}$ c.) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}\right\}$ d.) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

8. Find the exact value of $\sin \left[\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right]$.
- a.) $-\frac{1}{2}$ b.) $\frac{1}{2}$ c.) 0 d.) $\frac{\sqrt{3}}{2}$
9. A radioactive substance has a half-life of 100 hours. If a sample originally contained 100 grams, how long would it take to decay to 20 grams? Use the exponential model $A = A_0 e^{kt}$. Choose the closest answer.
- a.) 8 days b.) 10 days c.) 25 days d.) 230 days e.) none of these
10. How many solutions does $a^x = -ax$ for $a > 1$ have?
- a.) infinitely many b.) 3 c.) 2 d.) 1 e.) 0
11. Find the exact value of $\frac{\cos \theta}{\sin(90^\circ - \theta)}$.
- a.) 1 b.) -1 c.) 0 d.) undefined e.) none of these
12. Find the distance between complex numbers $-3i$ and 4.
- a.) 25 b.) 1 c.) 5 d.) $1i$
13. Determine the solution set of the equation $\log_2(3x + 4) - \log_2(x + 1) = 1$.
- a.) $\{-2, -4/3\}$ b.) $\{-2\}$ c.) $\{-4/3, -1\}$ d.) no solution
14. What is the domain of $f(x) = \sec^{-1} x$?
- a.) $\{x : |x| \geq 1\}$ b.) $\{x : |x| \leq 1\}$ c.) $\{x : |x| > 1\}$ d.) $\{x : |x| < 1\}$
15. Which of the following is equivalent to $\tan[\cos^{-1}(x)]$ for $0 < x < 1$?
- a.) $\sqrt{1 - x^2}$ b.) $\sqrt{x^2 - 1}$ c.) $\sqrt{1 + x^2}$ d.) $\sqrt{1/x^2 - 1}$
16. The world population on January 1, 2016 was approximately 7.4 billion and growing 1.1% annually. Assuming the current growth rate continues, during which year will the world population surpass 9 billion?
- a.) 2035 b.) 2034 c.) 2033 d.) 2032

17. Determine the exact value of $\sum_{n=0}^{\infty} \sin^n\left(\frac{\pi}{12}\right) \cos^n\left(\frac{\pi}{12}\right).$

- a.) $4/3$ b.) $1/4$ c.) $\sqrt{3}/2$ d.) $1/2$ e.) $2\sqrt{3}/3$

18. Find the equation of an ellipse centered at the origin that has y-intercepts of ± 3 and a minor axis with a length of 4.

a.) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ b.) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ c.) $\frac{x^2}{9} + \frac{y^2}{16} = 1$ d.) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

19. Find the smallest angle of a triangle with side lengths of $a = 5, b = 6, c = 10$. Round to the nearest degree.

- a.) 22° b.) 23° c.) 27° d.) 131°

20. Solve the equation $\cos(2\theta) + 6 \sin^2 \theta = 4$ for $0 \leq \theta < 2\pi$.

a.) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ b.) $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{11\pi}{6}\right\}$

c.) $\left\{\frac{\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}\right\}$ d.) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}\right\}$

21. The complex number $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{15}$ in $a + bi$ form is

- a.) $-i$ b.) i c.) 1 d.) $\frac{15\sqrt{3}}{2} + \frac{15i}{2}$

22. For the function $f(x) = \frac{1-x}{2+x}$ find $f^{-1}(x)$.

a.) $f^{-1}(x) = \frac{2+x}{x+1}$ b.) $f^{-1}(x) = \frac{1-2x}{x-1}$

c.) $f^{-1}(x) = \frac{2-x}{1-x}$ d.) $f^{-1}(x) = \frac{1-2x}{x+1}$

23. Find the exact value of $\tan\left[2 \sin^{-1}\left(\frac{1}{2}\right)\right]$.

- a.) $-\sqrt{3}$ b.) $-1/\sqrt{3}$ c.) $\sqrt{3}$ d.) $1/\sqrt{3}$

24. Find the rectangular representation of the polar equation $r = \frac{1}{2+\sin \theta}$.

a.) $2x^2 + 2y^2 + y - 1 = 0$ b.) $2x^2 + y^2 + 2y - 1 = 0$

c.) $4x^2 + 4y^2 + 2y - 1 = 0$ d.) $4x^2 + 3y^2 + 2y - 1 = 0$

25. To measure the width of a river, a surveyor measures the distance between points A and B on one bank to be 150 ft. The surveyor then measures the angles made between the riverbank and the line of sight to a tree (point C) on the other side of the river from points A and B. The angles are found to be $m\angle CAB = 105^\circ$ and $m\angle ABC = 40^\circ$. Calculate the width of the river to the nearest foot.

- a.) 125 ft b.) 162 ft c.) 103 ft d.) 101 ft

TIEBREAKERS

Name : _____

Show all your work to receive maximum credit.

1. Find the exact solution for $3^{2x+1} = 5^{1-x}$.

2. Establish the identity $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = -\cos(2\theta)$.

3. Find all cube roots of the complex number $4\sqrt{2} + 4i\sqrt{2}$. Write them in standard form: $a + bi$.

ANSWERS

1. B
2. D
3. A
4. C
5. C
6. A
7. D
8. B
9. B
10. D
11. A
12. C
13. D
14. A
15. D
16. C
17. A
18. B
19. A
20. A
21. B
22. D
23. C
24. D
25. B

Tiebreakers:

$$1. x = \frac{\ln(5/3)}{\ln 45}$$

$$2. \frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = \frac{\frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}} = \sin^2 \theta - \cos^2 \theta = 1 - 2 \cos^2 \theta = -\cos(2\theta)$$

$$3. \frac{\sqrt{6}+\sqrt{2}}{2} + i \frac{\sqrt{6}-\sqrt{2}}{2}, -\sqrt{2} + i\sqrt{2}, -\frac{\sqrt{6}-\sqrt{2}}{2} - i \frac{\sqrt{6}+\sqrt{2}}{2}$$