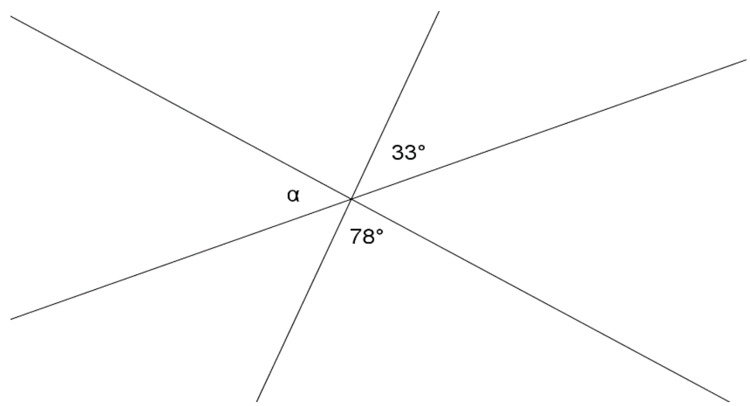
2016 Regional Mathematics Contest
Geometry Test

In each of the following, choose the BEST answer and record your choice on the answer sheet provided. To ensure correct scoring, be sure to make all erasures completely. The tiebreaker questions at the end of the exam will be used to resolve ties in first, second, and/or third place. They will be used in the order given. Complete the 25 multiple choice questions before attempting the tiebreaker questions. Figures are not necessarily drawn to scale.

1. Find the measure of α.
   a. $45°$
   b. $78°$
   c. $33°$
   d. $69°$
   e. Not enough information



2. If the angles of a triangle are $42°, 73°,$ and $65°$, which of the following would be an exterior angle of the triangle.
   a. $115°$
   b. $65°$
   c. $86°$
   d. $117°$
   e. None of these

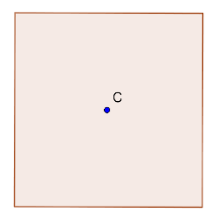
3. To inscribe a circle in a triangle, which of the following must be constructed?
   a. The perpendicular bisectors of two sides.
   b. The bisectors of two angles.
   c. The altitudes of the triangle.
   d. The diameter of the circle.
   e. None of these.

4. Which of the following are properties of parallelograms?

   I. The opposite angles are congruent.
   II. The diagonals are perpendicular.
   III. The adjacent angles are supplementary.
   IV. The diagonals bisect each other.

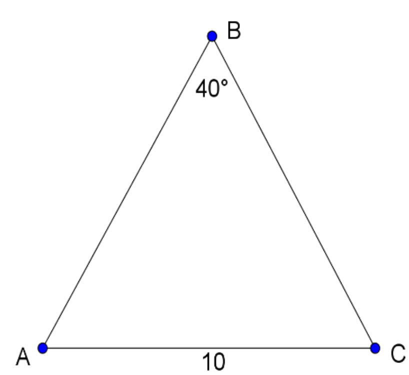
   a. I and II only
   b. I, II and III
   c. II and III only
   d. I, III and IV
   e. All are properties of parallelograms.

5. Which of the following transformations will **not** map the square onto itself?
   a. A rotation of $180°$about the center.
   b. A rotation of $270°$about the center.
   c. A reflection about the perpendicular bisector of a side.
   d. A rotation of $45°$ about the center.
   e. A reflection about a diagonal.

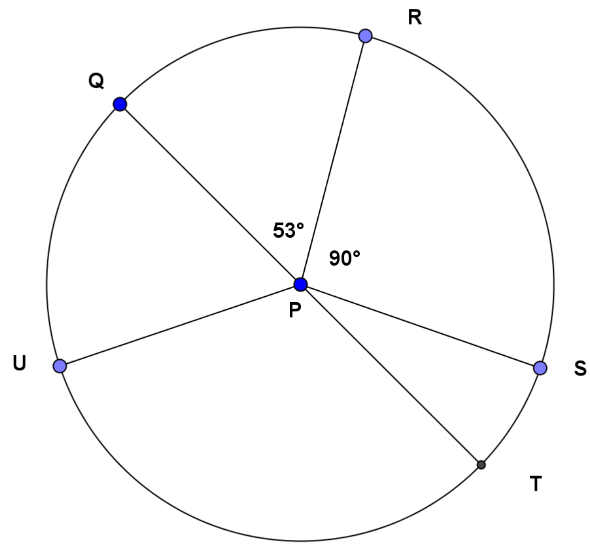


6. Write the equation of the circle with center at $(-4, 3)$ passing through the origin.
   a. $(x-4)^2 + (y+3)^2 = 25$
   b. $(x-4)^2 + (y+3)^2 = 16$
   c. $(x+4)^2 + (y-3)^2 = 9$
   d. $(x+4)^2 + (y-3)^2 = 25$
   e. $(x+4)^2 + (y-3)^2 = 16$

7. Find the area of the isosceles triangle ABC.
   a. 64.3
   b. 68.7
   c. 34.4
   d. 73.1
   e. None of these



8. The circle with center P is divided into sectors. The segment $\overline{QT}$ is a diameter of circle P. What is the measure of arc $\widehat{QTS}$?
   a. $143°$
   b. $233°$
   c. $217°$
   d. $57°$
   e. Not enough information



9. In the above figure, if $m\angle QPU = 57°$, and $QT = 4$, what is the length of arc $\widehat{RQU}$?
   a. $\frac{57\pi}{90}$
   b. $\frac{11\pi}{9}$
   c. $\frac{22\pi}{9}$
   d. $\frac{11\pi}{18}$
   e. $\frac{7\pi}{3}$

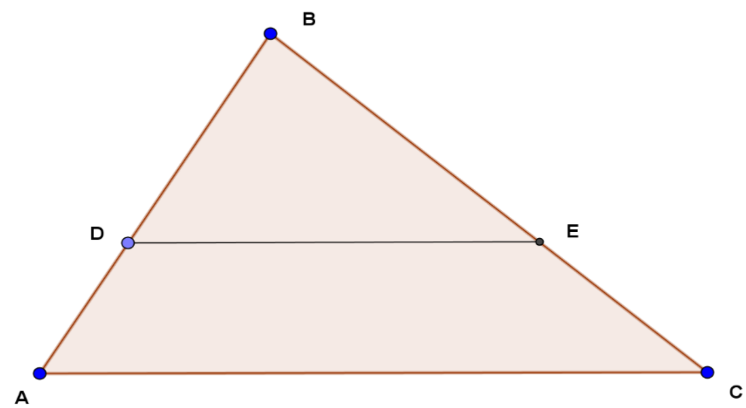
10. If a regular octagon has a side length of 6, what is the radius, to the nearest tenth, of the circle circumscribing the octagon?
    a. 4.2
    b. 7.8
    c. 7.2
    d. 6.0
    e. 8.4

11. For the octagon in Question 13, to the nearest tenth, what is the radius if the circle inscribed in the octagon?
    a. 4.2
    b. 7.8
    c. 7.2
    d. 6.0
    e. 8.4

Use the information provided to answer questions 12 – 14.

Given: ΔABC with $\overleftrightarrow{AC} \parallel \overleftrightarrow{DE}$

Prove: $\frac{AD}{AB} = \frac{CE}{CB}$



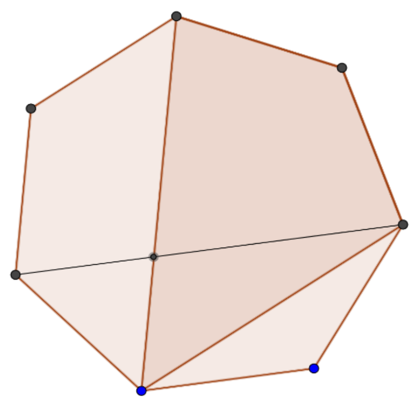
Statements	Reasons
1. $\overleftrightarrow{AC} \parallel \overleftrightarrow{DE}$	1. Given
2. $\angle BDE \cong \angle BAC, \angle BED \cong \angle BCA$	2. ?
3. $\Delta ABC \sim \Delta DBE$	3. If two pairs of corresponding angles are congruent, the triangles are similar.
4. $\frac{DB}{AB} = \frac{EB}{CB}$	4. The lengths of corresponding sides of similar triangles are proportional.
5. $\frac{AB-DB}{AB} = \frac{CB-EB}{CB}$	5. A property of proportions
6. $AB = AD + DB,\ CB = CE + EB$	6. ?
7. $AD = AB - DB, CE = CB - EB$	7. Subtraction Property of Equality
8. $\frac{AD}{AB} = \frac{CE}{CB}$	8. Substitution

12. Which reason justifies the statement in step 2 of the proof?
    a. If two parallel lines are cut by a transversal, the alternate interior angles are congruent.
    b. If two parallel lines are cut by a transversal, the corresponding interior angles are congruent.
    c. Vertical angles are congruent.
    d. If two parallel lines are cut by a transversal, the alternate exterior angles are congruent.
    e. None of the above.

13. Which reason justifies the statement instep 6 of the proof?
    a. Addition Property of Equality
    b. Definition of congruence.
    c. Definition of betweenness.
    d. Definition of midpoint.
    e. None of the above.

14. If $AC = 12, DE = 8, and\ BD = 6$, then $AD =$
    a. 9
    b. 4
    c. 16
    d. 3
    e. Not enough information

15. What is the sum of the measures of all interior angles of the figure below?
    a. $900°$
    b. $1260°$
    c. $1080°$
    d. $1440°$
    e. None of these



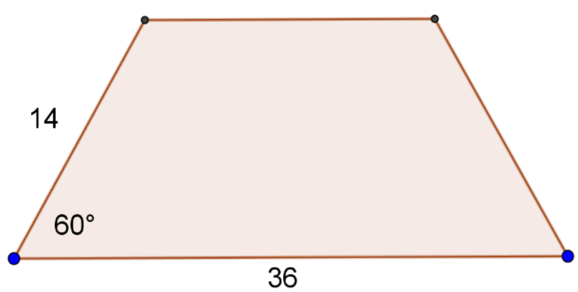
16. Which of the following is not used in the construction of the perpendicular bisector of the segment $\overline{AB}$?
    a. A circle of radius AB with center A
    b. A circle of radius AB with center B
    c. The intersection points of the circles of radius AB with centers A and B.
    d. The midpoint of the segment $\overline{AB}$.
    e. All of these are used.

17. In two similar right triangles, the lengths of the corresponding sides are in the ratio of 5:3. If the larger triangle has a hypotenuse of length 17 and the length of the shortest side is 8, what is the area of the smaller triangle?
    a. 72
    b. 40.8
    c. 21.6
    d. 36
    e. None of these

18. Given the points $A(2,9), B(-3,1)$ and $C(10,-2)$, Which of the following points could be the fourth vertex of parallelogram $ABCD$?
    a. $(8,-11)$
    b. $(5,8)$
    c. $(15,6)$
    d. $(5,-6)$
    e. None of these.

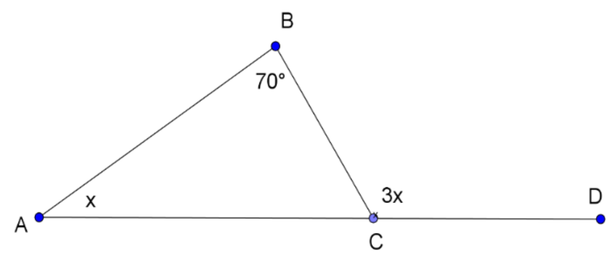
19. Which of the following could be done to quadruple the volume of a right circular cone?
    a. Quadrupling the height.
    b. Quadrupling the radius.
    c. Doubling the radius.
    d. Quadrupling the height and doubling the radius.
    e. Quadrupling the height or doubling the radius.

20. Find the area of the isosceles trapezoid below.
    a. 504
    b. $203\sqrt{3}$
    c. $406\sqrt{3}$
    d. 873
    e. None of these

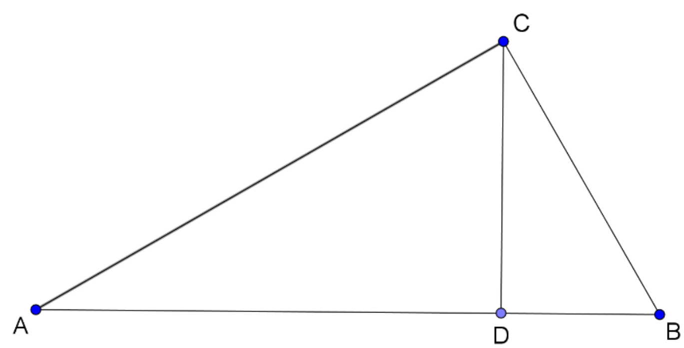


21. If $A(1,-1)$ is the midpoint of $\overline{BC}$ with $B(7,2)$, what are the coordinates of the midpoint of $\overline{AC}$?
   a. $(-2,-2.5)$
   b. $(3,1.5)$
   c. $(-5,-4)$
   d. $(4,\ 0.5)$
   e. None of these

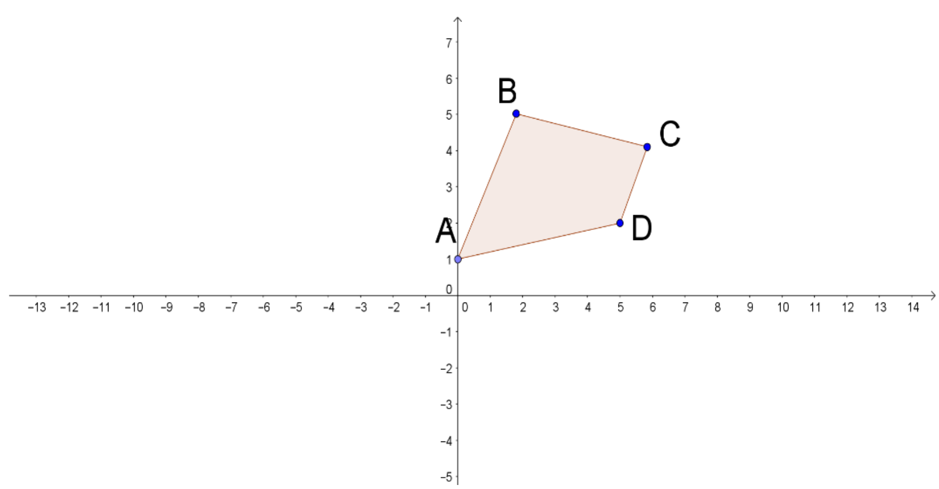
22. Find the measure of $\measuredangle ACB$.
   a. $35°$
   b. $75°$
   c. $92.5°$
   d. $70°$
   e. Not enough information



23. In the right triangle ABC below, if $BC = 6$ and $AC = 8$, find AD.
   a. 10
   b. 3.6
   c. 4.8
   d. 6.4
   e. None of these



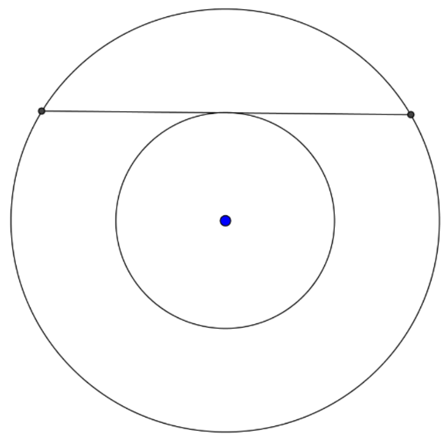
24. If quadrilateral $A'B'C'D'$ is the reflection of quadrilateral $ABCD$ about the line $y = -x$, what are the coordinates of B' ?
   a. $(6,-1)$
   b. $(-2,-1)$
   c. $(-5,-2)$
   d. $(-2,-5)$
   e. $(-1,-2)$



25. The Eiffel Tower is approximately 1,063 feet tall. A tourist whose eyes are 5 feet above the ground observes the tower from 1000 feet away. To the nearest hundredth, what is the angle of elevation from her eyes to the top of the tower?
    a. $46.75°$
    b. $43.39°$
    c. $46.61°$
    d. $43.25°$
    e. $44.89°$

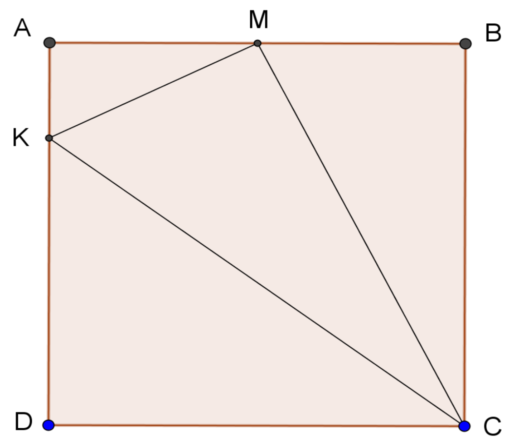
Tie Breaker #1

The two circles below are concentric. The length of the chord tangent to the inner circle is 30 mm. What is the area between the two circles?



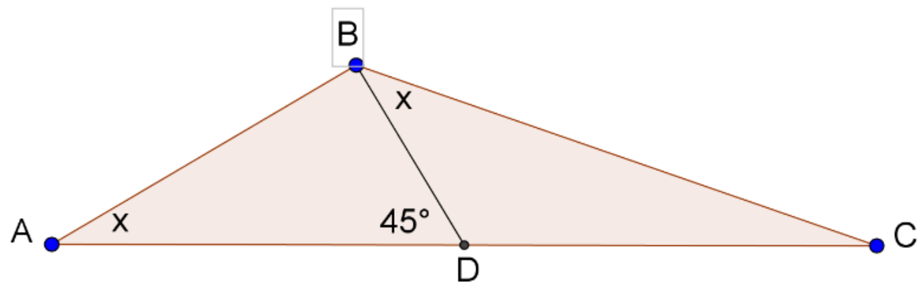
Tie Breaker #2

In the square $ABCD$, M is the midpoint of $\overline{AB}$. A line perpendicular to $\overleftrightarrow{MC}$ intersects $\overline{AD}$ at K. Prove that $\angle BCM \cong \angle KCM$.



Tie Breaker #3

In triangle ABC, median $\overline{BD}$ is such that $\angle A \cong \angle DBC$. If $m\angle ADB = 45°$, prove that $m\angle A = 30°$.



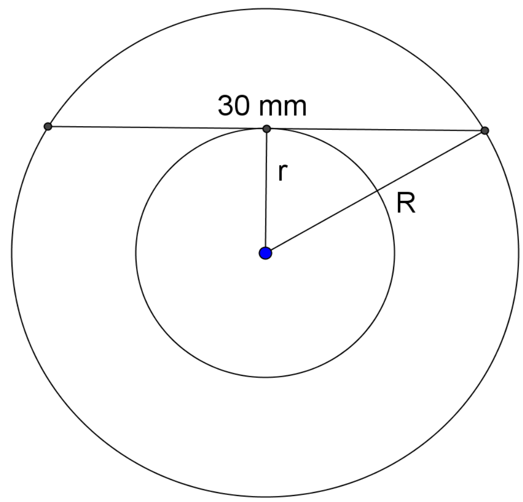
Answer Key

2016 Regional Geometry Exam

1. D
2. A
3. B
4. D
5. D
6. D
7. B
8. C
9. B
10. B
11. C
12. B
13. C
14. D
15. B
16. D
17. C
18. C
19. E
20. B
21. A
22. B
23. D
24. C
25. C

Tie Breaker #1

The two circles below are concentric. The length of the chord tangent to the inner circle is 30 mm. What is the area between the two circles?

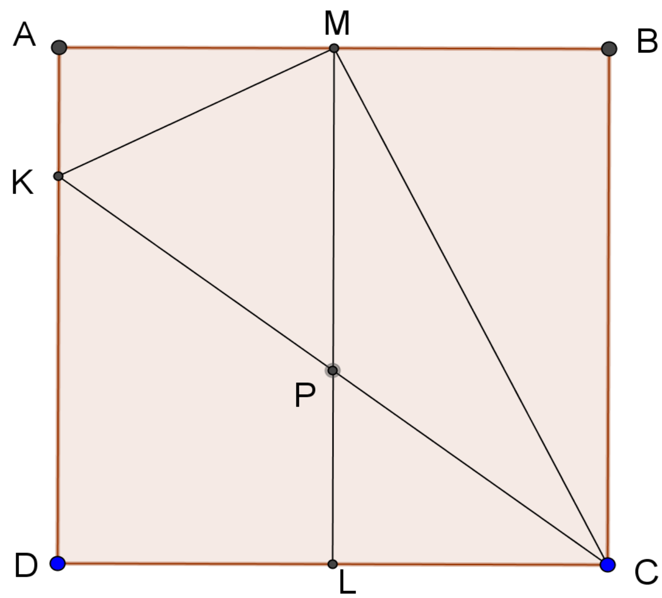


Solution: By Pythagorean Theorem, $R^2 - r^2 = 225$. The area between the circles is the difference of the areas of the larger and smaller circles:

$$A = \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 225\pi \text{ mm}^2$$

Tie Breaker #2

In the square $ABCD$, M is the midpoint of $\overline{AB}$. A line perpendicular to $\overleftrightarrow{MC}$ intersects $\overline{AD}$ at K. Prove that $\angle BCM \cong \angle KCM$.

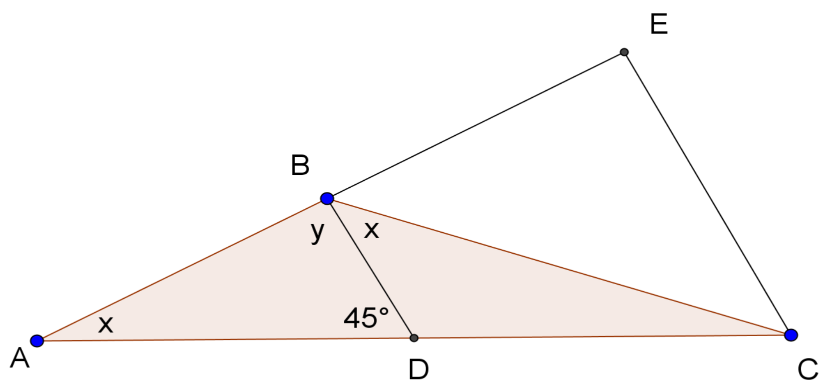


Solution: Draw $\overleftrightarrow{ML} \parallel \overleftrightarrow{AD}$. Then since $\overline{AM} \cong \overline{MB}$, $\overline{KP} \cong \overline{PC}$. This means $\overline{MP}$ is the median of right triangle KMC to the hypotenuse. Therefore, triangle MPC is an isosceles triangle.

Hence $\angle PCM \cong \angle PMC$. By the Alternate Interior Angles Theorem, we also have $\angle BCM \cong \angle LMC$. Therefore, by transitivity, we have $\angle BCM \cong \angle LMC \cong \angle PMC \cong \angle KMC$.

Tie Breaker #3

In triangle ABC, median $\overline{BD}$ is such that $\angle A \cong \angle DBC$. If $m\angle ADB = 45°$, prove that $m\angle A = 30°$.



Solution: Since $\angle A \cong \angle DBC$ and $\angle BCD$ is a common angle, triangle ABC is similar to triangle BDC.

Draw the perpendicular from C to $\overleftrightarrow{AB}$ and extend $\overline{AB}$ to intersect the perpendicular at E.

Let $y = m\angle ABD$. By angle sum of triangles, $x + y + 45° = 180°$, or equivalently, $x + y = 135°$.

Since $\angle EBC$ is the supplement of $\angle ABC$, and $m\angle ABC = x + y$, $m\angle EBC = 45°$. Since triangle BCE is a right triangle, $m\angle BCE = 45°$. Let $p = CE$ Pythagorean Theorem,

$$BC^2 = CE^2 + BE^2 = 2CE^2 = 2p^2$$

By similarity, $\frac{AC}{BC} = \frac{BC}{DC}$, or equivalently,

$$BC^2 = AC \cdot DC = (2DC)DC = 2DC^2$$

Combining these equations, we get that $DC = p$, so $AC = 2p$. Therefore, since the hypotenuse is twice the length of the shortest side, the right triangle ACE is a

$30° - 60° - 90°$ triangle, and since CE is the shortest side, $m\angle A = 30°$.