

2017-ACTM Regional Mathematics Contest

Geometry

Answer each of the multiple-choice questions and mark your answers on that answer sheet provided. When finished with the multiple-choice items, then answer the tie breaker items in the order provided. Be sure your name is on each page of the tie-breaker items.

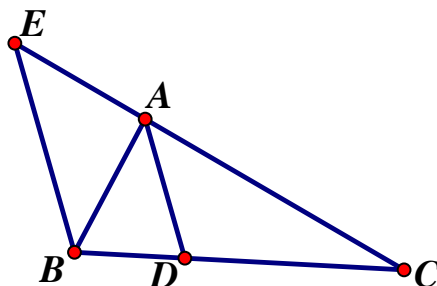
- A sewing club is making a quilt consisting of 25 squares. Each of the squares has a diagonal measuring $30\sqrt{2}$ cm. If the quilt has five rows and five columns, what is the perimeter of the quilt?

a) 150 cm b) 300 cm c) 600 cm d) 900 cm
- Point M is plotted in the coordinate plane at $(-3, 4)$ and is reflected over the line $y = -x + 4$. What are the coordinates of M' after the reflection?

a) $(-4, 3)$
 b) $(0, 7)$
 c) $(4, -3)$
 d) $(7, 0)$
- If a cylindrical barrel measures 22 inches in diameter, how many inches will it roll in eight revolutions along a smooth surface?

a. 121π b. 168π c. 176π d. 228π
- In the coordinate plane, $\triangle ABC$ has vertices at $A(1, -2)$, $B(1, 0.5)$, and $C(2, 1)$ and $\triangle DEF$ has vertices at $D(4, -3)$, $E(4, 2)$ and $F(6, 3)$. $\triangle DEF$ is the image of $\triangle ABC$ under a dilation of the plane. What is the scale factor and center of dilation?

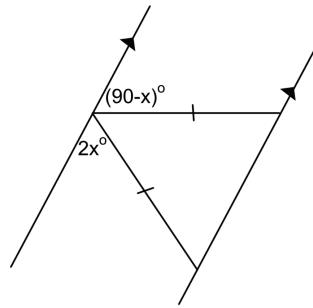
a) Center of dilation $(0, 0)$ and scale factor 2
 b) Center of dilation $(-1, -2)$ and scale factor 2
 c) Center of dilation $(-2, -1)$ and scale factor 4
 d) Center of dilation $(-2, -1)$ and scale factor 2
- In the figure below, \overline{AD} is the angle bisector of $\angle BAC$ and $\overline{EB} \parallel \overline{AD}$. Which triangle is isosceles?



- a) $\triangle ABE$
 b) $\triangle ABD$
 c) $\triangle ADC$
 d) $\triangle ABC$
 e) None of a - d

6. Determine the value of x in the figure below. The lines are marked as parallel and congruent segments are marked.

- a) 90
- b) 60
- c) 30
- d) 15

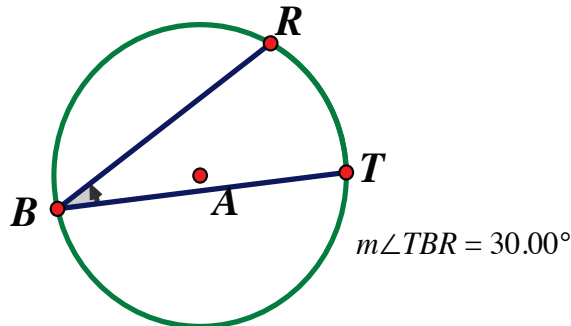


7. Given coordinates of the vertices of a quadrilateral, determine the most specific class to which it belongs: square, rectangle, parallelogram. Find the perimeter of the quadrilateral.

$A(4, 3)$, $B(11, 3)$, $C(14, 7)$, $D(7, 7)$

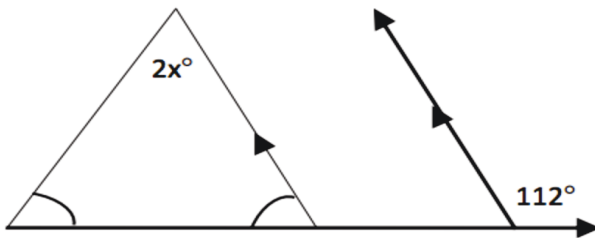
- a) Parallelogram with a perimeter of 20 units
- b) Parallelogram with a perimeter of 22 units
- c) Parallelogram with a perimeter of 24 units
- d) None of these

8. The radius is 18 ft. The length of \widehat{RT} is _____



- a) 3π ft.
- b) 6π ft.
- c) 18π ft.
- d) 36π ft

9. Find the value of x in the figure below:



- a) 56
- b) 22
- c) 30
- d) 44

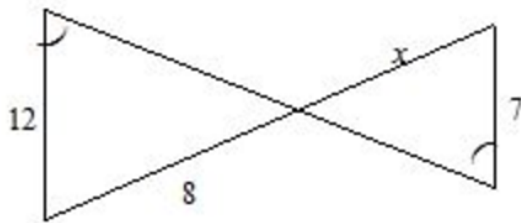
10. Which of the following is **not** true for a parallelogram?

- a) Opposite angles are congruent.
- b) Opposite sides are congruent.
- c) Diagonals bisect opposite angles.
- d) Diagonals bisect each other.

11. Find the perimeter of an equilateral triangle with a height of $7\sqrt{3}$.

- a) 7
- b) $21\sqrt{2}$
- c) $21\sqrt{3}$
- d) 42

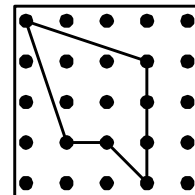
12. Explain why the triangles below are similar and find the value of x .



Not drawn to scale

- a) SSS Theorem $x=10\frac{1}{2}$
- b) AA Theorem, $x=10\frac{1}{2}$
- c) SAS Theorem, $x=4\frac{2}{3}$
- d) AA Theorem, $x=4\frac{2}{3}$

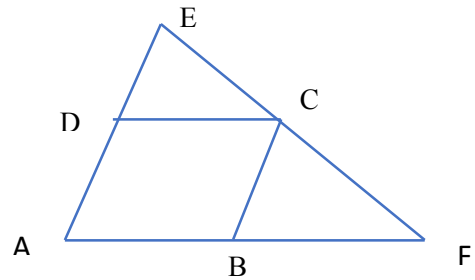
13. The figure in the drawing to the right is sketched on a geoboard in which the horizontal and vertical distance between any two dots is 1 unit. Determine the perimeter of the sketched figure.



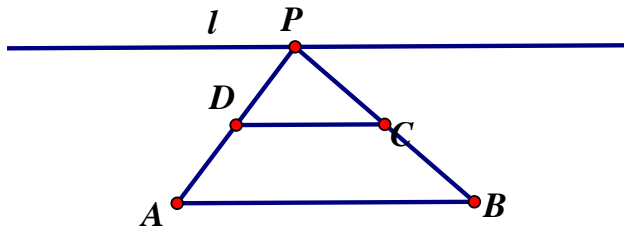
- a. $2\sqrt{10} + 4 + \sqrt{2}$ units
- b. $2\sqrt{10} + 5$ units
- c. 6.5 units
- d. None of the above

14. In the diagram below ABCD is a parallelogram. Angle DCB is 75° . Angle EFA is 40° . Which of the following is the measure of Angle DEC ?

- a. 35° b. 65° c. 75° d. 105°



15. In the figure below, line l is parallel to segment AB. Segment AB is fixed. Point P moves from its current position to right on line l , as P moves to the right which of the following measures is always increasing?



- a) The distance PB
 b) The distance DC
 c) The perimeter of triangle PAB
 d) The area of triangle PAB
 e) None of the above

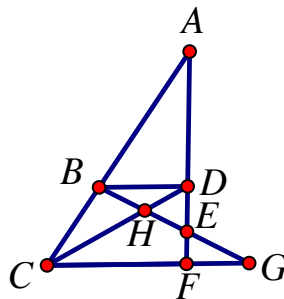
16. In the figure below:

$AB = CG$ and $\overline{BD} \parallel \overline{CG}$.

Consider these statements:

- I. $\triangle ABD \sim \triangle ACF$ II. $\triangle ABE \sim \triangle GFE$ III. $\triangle BDH \sim \triangle GHC$

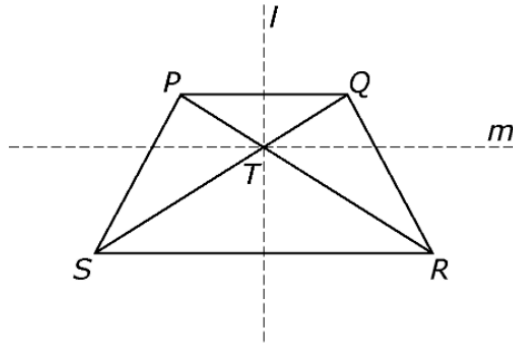
- a) I and II
 b) II and III
 c) I and III
 d) All the statements.
 e) None of the statements.



17. A cone that holds ice cream at Baskin Robins has a radius of 3 cm at the top and is 6 cm tall. It is filled completely with ice cream. Also, there is half a spherical dip of ice cream on top of the cone. The sphere has a diameter of 8 cm. Which of the following values gives the total volume of ice cream on and in the cone?

- a. 18π b. 42.6π c. 60.6π d. 85.3π

18. Which of the following transformations of $\triangle PTS$ could be used to show that $\triangle PTS \cong \triangle QTR$?



- a) A reflection over \overline{QS}
 b) A reflection over \overline{PR}
 c) A reflection over line m
 d) A reflection over line l

19. Which of the following is a correct explanation of how to rotate the plane 30° counterclockwise about point X to find the image of point A, which will be named point A'.

• A

• X

- a) Draw a circle with center A and radius XA. Move point X counterclockwise around the circle to point A' such that the measure of $\angle AXA'$ is 30° .
 b) Draw a circle with center X and radius XA. Move point X counterclockwise around the circle to point A' such that the measure of $\angle AXA'$ is 30° .
 c) Draw a circle with \overline{XA} as a diameter. Move point X counterclockwise around the circle to point A' such that the measure of $\angle AXA'$ is 30° .
 d) Draw a circle with \overline{XA} as a diameter. Move point A counterclockwise around the circle to point A' such that the measure of $\angle AXA'$ is 30° .

20. In an equilateral triangle, if the legs have length x , then what is the length of the altitude?

- a) x^2
- b) $\frac{x}{2}\sqrt{2}$
- c) $\frac{x}{2}\sqrt{3}$
- d) $\frac{3x^2}{4}$

21. How far does the tip of a minute hand on a clock travel in 48 minutes, if the distance from the center to the tip is 8 cm.

- a) 4π cm
- b) 6π cm
- c) 6.4π cm
- d) 12.8π cm

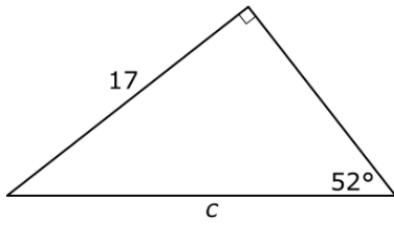
22. A student has two straws. One is 6 cm long and the other is 10 cm. She picks a third straw at random from a group of 4 straws. The lengths are 3 cm, 4 cm, 5 cm, and 6 cm. What is the probability that the straw she picks will allow her to form a triangle?

- a) 100%
- b) 75%
- c) 50%
- d) 25%

23. The bases of a prism are triangles having sides of measure 9 cm, 12 cm, and 15 cm. If the height is h , which of the following expressions will result in the measure of the surface area of the prism?

- a) $108 + (9h + 12h + 15h) \text{ cm}^3$
- b) $108 + h(9 + 12 + 15) \text{ cm}^2$
- c) $54 + 36h \text{ cm}^2$
- d) $108 + 36h^3 \text{ cm}^2$

24. In the figure below the $\sin(52^\circ) = \frac{17}{c}$. Based on the figure, which of the following equations is also true?



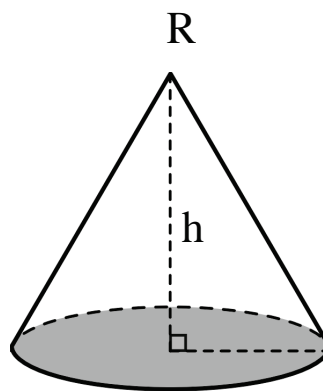
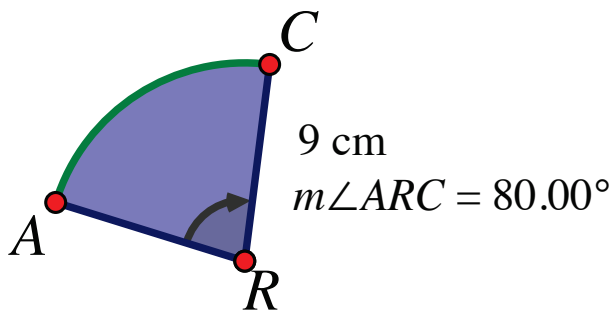
- a) $\sin(38^\circ) = \frac{c}{17}$
- b) $\cos(38^\circ) = \frac{17}{c}$
- c) $\cos(52^\circ) = \frac{17}{c}$
- d) $\tan(52^\circ) = \frac{c}{17}$

2017-ACTM Regional Mathematics Contest

Geometry
Tie-Breakers

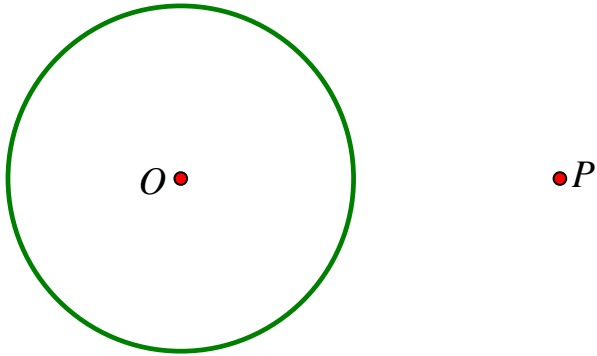
Name: _____

1. Sector ARC has a radius of 9 cm and an angle that measures 80° . When sector ARC is cut out and AR and RC are taped together, they form a cone. The length of AC becomes the circumference of the base of the cone. What is the height, h , of the cone?



Name: _____

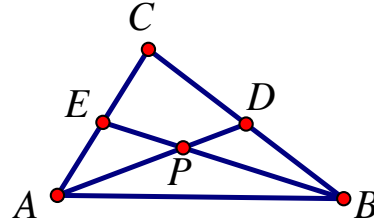
2. a) Given Circle with center at O and a point P outside of the circle, construct the two tangent lines through P to the circle with center at O . Label the lines as m and k .
- b) How do you know that lines m and k are the tangent lines to circle with center O through point P ?



Name: _____

3. In Triangle ABC, \overline{AD} and \overline{BE} are medians intersecting at P.
Determine the value of the ratio, $\frac{\text{Area } \triangle PAB}{\text{Area } \triangle PDE}$

Justify your conclusion.

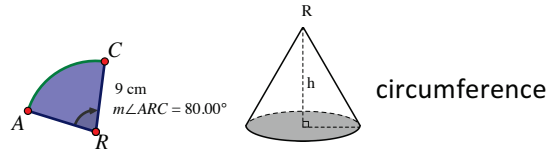


Answers to Multiple Choice Items

1. C
2. B
3. C
4. D
5. A
6. C
7. C
8. B
9. B
- 10.C
- 11.D
- 12.D
- 13.A
- 14.B
- 15.C
- 16.C
- 17.C
- 18.D
- 19.D
- 20.C
- 21.D
- 22.C
- 23.B
- 24.B

Solution to Tie Breaker 1

If sector R were drawn to be a full circle, then the Circle R would be 18π .



Thus, the length of arc AC becomes $\frac{80}{360} = \frac{x}{18\pi}$; $x = 4\pi$ (which is the circumference of the base of the cone.)

So: $4\pi = 2\pi r$ where r is the radius of the base of the cone.

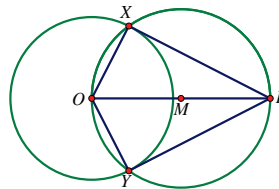
Then $r = 2$, which is the radius of the base of the cone.

Using the Pythagorean theorem, then $2^2 + h^2 = 9^2$

$$\text{Then } h = \sqrt{77}$$

Solution to Tie Breaker 2

Given: Circle with center O
Point P outside circle O



Proof: Construct line segment OP

Find midpoint of OP and label M

Use point M and construct circle with center M, which goes through points P and O.

Circles O and M intersect in two points, X and Y

Segments OX and OY are radii of Circle O

Draw line segments PX and PY

$PX \perp OX$ and $PY \perp OY$ because OP is the diameter of circle M

Angles formed by OXP and OYP are right angles because they intersect semicircles formed by diameter OP.

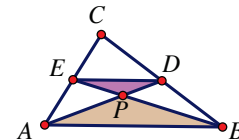
Therefore, PX and PY are tangent to circle O.

Solution to Tie Breaker 3

In Triangle ABC, \overline{AD} and \overline{BE} are medians intersecting at P.

Determine the value of the ratio,

$$\frac{\text{Area } \triangle PAB}{\text{Area } \triangle PDE}$$



$$\frac{\text{Area } \triangle PAB}{\text{Area } \triangle PDE} = 4.00$$

Draw line segment ED.

Since E and D are medians of $\triangle ABC$, then ED is parallel to AB and $ED = \frac{1}{2}AB$

By AA theorem, $\triangle ABP \sim \triangle PDE$

Therefore ratio of areas is 4:1