ACTM Regional Calculus Competition
2017

Begin by removing the three tie breaker sheets at the end of the exam and writing your name on all three pages. Work the multiple-choice questions first, choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Make sure you attempt the tie-breaker questions at the end of the test starting with tie breaker 1, then 2, and then 3 if you have time. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple-choice questions.

1. $\lim_{x\to 2}\left(\frac{x^3-2x^2}{x-2}\right)=4$. By the definition of a limit, there is a positive real number δ such that $\left|\frac{x^3-2x^2}{x-2}-4\right|<0.41$ if $0<|x-2|<\delta$. The largest valid value of δ is
   A. 0.02
   B. 0.05
   C. 0.1
   D. 0.2
   E. 0.5

2. $\lim_{x\to 0}\left(\frac{\cot(5x)}{\cot(7x)}\right)=$
   A. 0
   B. 1
   C. $\frac{5}{7}$
   D. $\frac{7}{5}$
   E. This limit does not exist.

3. Suppose that $\lim_{x\to 4}\left(f(x)\right)=\infty$. Which of the following must occur?
   A. The graph of f has a vertical asymptote at $x = 4$.
   B. The graph of f has a removable discontinuity at $x = 4$.
   C. The graph of f has a horizontal asymptote at $y = 4$.
   D. The graph of f has a point at $(4, \infty)$.
   E. Each of the other answers is incorrect.

4. The following table gives the temperature of an object T in °C as a function of time t since noon in minutes.

t	0	15	30	45	60
T	45	48	53	60	70

   Using this data what is the best estimate of the instantaneous rate of change in the temperature at 12:45?
   A. 0.77
   B. 0.67
   C. 0.57
   D. 0.47
   E. 0.20

5. Let P be the price of a certain stock. Suppose we know that the price is currently increasing, but it will be topping out in the near future. Which of the following must be true. t = time.

A. $P'(t) > 0, P''(t) > 0$
B. $P'(t) > 0, P''(t) = 0$
C. $P'(t) > 0, P''(t) < 0$
D. $P'(t) < 0, P''(t) < 0$
E. $P'(t) < 0, P''(t) > 0$

6. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$

A. Does not exist
B. 0
C. $f(x)$
D. $f'(x)$
E. $f''(x)$

7. $\frac{d}{dx}\left(\frac{7}{x} - 4\sqrt{x} + 3\right) =$

A. $7\ln(x) - 2\sqrt{x} + 3x$
B. $-\frac{7}{x^2} - 2\sqrt{x}$
C. $-\frac{7}{x^2} - \frac{2}{\sqrt{x}}$
D. $7x - 4\sqrt{x}$
E. Each of the other answers is incorrect.

8. A freezer depreciates at a rate of 15% per year. It was purchased new for \$875. How fast is it depreciating when it is exactly 5 years old?

A. \$19.69 per year
B. \$42.50 per year
C. \$62.00 per year
D. \$63.10 per year
E. Each of the other answers is incorrect.

9. Approximate $\frac{d}{dx}\left(x^{4x}\right)\Big|_{x=2}$ to the nearest integer.

A. 1,888
B. 710
C. 256
D. 1,024
E. 1,734

10. $\frac{d}{dx}\left(\sin\left(\ln\left(x\right)\right)\right) =$

A. $\dfrac{\cos\left(\ln\left(x\right)\right)}{x}$

B. $\cos\left(\ln\left(x\right)\right)$

C. $\cos\left(\frac{1}{x}\right)$

D. $\sin\left(\frac{1}{x}\right)$

E. $\dfrac{1}{\sin\left(\ln\left(x\right)\right)}$

11. What is the equation of the tangent line to $f\left(x\right) = \dfrac{3x+2}{4x-7}$ at $x = 1$?

A. $y = -\frac{29}{9}x + \frac{38}{9}$

B. $y = \frac{29}{9}x - \frac{44}{9}$

C. $y = -\frac{29}{9}x + \frac{14}{9}$

D. $y = \frac{3}{4}x - \frac{29}{12}$

E. Each of the other answers is incorrect.

12. $f\left(x\right) = 2e^x \sin\left(4x\right)$ Evaluate the first and second derivatives when $x = 0$.

A. $f'(0) = 8$, $f''(0) = -16$

B. $f'(0) = 8$, $f''(0) = 16$

C. $f'(0) = -8$, $f''(0) = 8$

D. $f'(0) = 0$, $f''(0) = 8$

E. Each of the other answers is incorrect.

13. A manufacturer is building a box in the shape of a right rectangular prism. The bottom and top of the box is to be a square and the sides are rectangles. The box is to hold 1000 cm^3. How wide should the square bottom be to minimize the amount of material needed for the box?

A. 5 cm

B. 100 cm

C. 25 cm

D. 10 cm

E. Each of the other answers is incorrect.

14. An oil spill is in the shape of a circular disk. At time t = 3 hours after the spill the radius of the spill is 100 meters. At this time the radius is increasing at a rate of 2 meters per hour. How fast is the area of the spill increasing at this time?

A. 628.32 m/hr

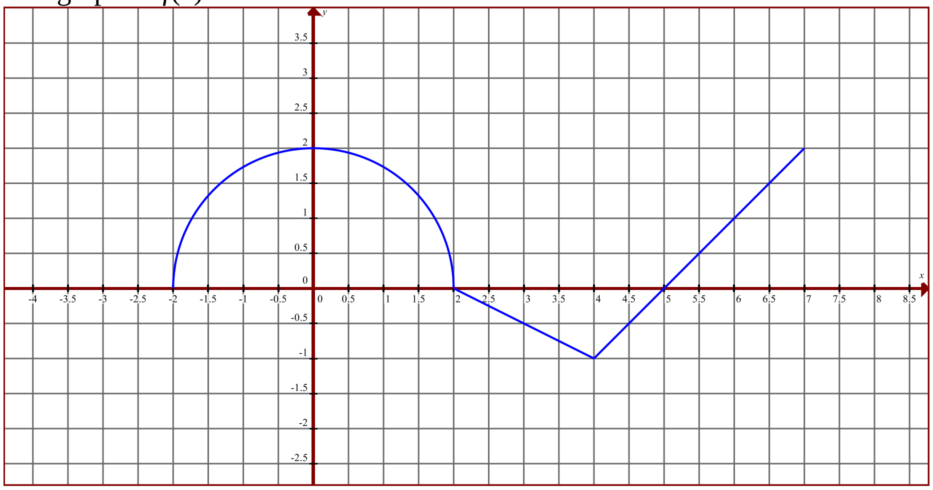
B. 1256.64 m/hr

C. 6283.19 m/hr

D. 31,415.93 m/hr

E. Each of the other answers is incorrect.

15. The graph of $f(x)$:



Correct to two decimal places

$$\int_0^5 f(x)dx \approx$$

A. 1.64
B. 4.64
C. 4.78
D. 6.78
E. Each of the other answers is incorrect.

16. A region is bounded by the curves $x = -2$, $x = 1$, $y = x^3$, and $y = e^x$. Compute the area of the region. Round your answer to two decimal places.

A. 4.22
B. 6.33
C. 8.24
D. 9.11
E. Each of the other answers is incorrect.

17. Applying L'Hopital's rule, $\lim_{x\to 0^+}\left(\tan(x)\ln(x)\right) =$

A. $\lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{-\csc^2(x)}\right)$

B. $\lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{\sec^2(x)}\right)$

C. $\lim_{x\to 0^+}\left(\frac{\sec^2(x)}{x}\right)$

D. $\lim_{x\to 0^+}\left(\sec^2(x)\ln(x)\right)$

E. Each of the other answers is incorrect.

18. $\frac{d}{dx}\int_2^x \cos(t)dt =$

A. $\sin(x)$

B. $\sin(x)-\sin(2)$

C. $\cos(x)-\cos(2)$

D. $\cos(x)$

E. Each of the other answers is incorrect.

19. $f(x)=\cos\left(x^3\right)$. The second derivative $f''(x)$ =

A. $-\cos(6x)$

B. $-3x^2\sin\left(x^3\right)$

C. $-9x^4\cos\left(x^3\right)-6x\sin\left(x^3\right)$

D. $-18x^3\cos\left(x^3\right)$

E. Each of the other answers is incorrect.

20. Approximate $\int_0^6 e^{-x^2}\,dx$ using the Trapezoidal Rule with 3 divisions.

A. 0.037
B. 0.577
C. 0.886
D. 1.037
E. 2.037

21. The number *N* of items sold by a company in the two-week period following the Super Bowl is a function of *c* the number of times their commercial is aired during the Super Bowl. Which of the following is a correct interpretation of $N'(4) = 50{,}000$?

A. If they were to increase the number of times the commercial was aired during the Super Bowl from 4 to 5, then they would sell approximately 50,000 more units during the two weeks following the Super Bowl.
B. If they air the commercial 4 times during the Super Bowl, then they will sell approximately 50,000 units during the two weeks following the Super Bowl.
C. Increasing the number of commercial airings by 4 during the Super Bowl will result in approximately 50,000 more units being sold during the two weeks following the Super Bowl.
D. Airing the commercial 50,000 times will result in 4 more units being sold during the two weeks following the Super Bowl.
E. Each of the other answers is incorrect.

22. $\frac{d}{d\theta}\left(\sec^2(\theta)\right) =$

A. $\tan(\theta)$

B. $2\sec(\theta)$

C. $2\sec^2(\theta)\tan(\theta)$

D. $1+\tan^2(\theta)$

E. Each of the other answers is incorrect.

For questions 23-25 let *R* be the region bounded by the curves $y = 1+\cos\left(\frac{\pi}{2}x\right)$ and $y = 2-(x-2)^2$.

23. Approximate the area of the region *R*. Round your answer to four decimal places.
A. 2.5000
B. 2.6066
C. 3.2264
D. 3.7522
E. Each of the other answers is incorrect.

24. Approximate the volume of the solid formed by revolving the region *R* about the *x*-axis. Round your answer to four decimal places.
A. 14.2547
B. 16.5870
C. 18.2236
D. 28.2567
E. Each of the other answers is incorrect.

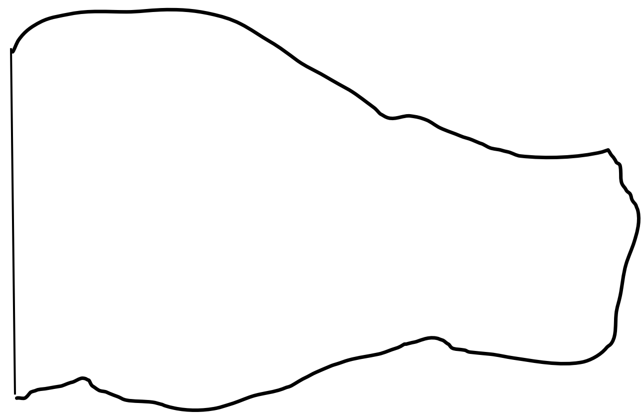
25. The region *R* is the base of a solid *S*. Slices through *S* perpendicular to the *x*-axis form rectangles with height equal to ½ the base length. Approximate the volume of the solid *S*. Round your answer to four decimal places.
A. 2.0654
B. 4.1308
C. 8.2935
D. 12.9772
E. Each of the other answers is incorrect.

Name ________________________________

Tie Breaker 1:

A line tangent to $f(x) = x^2 - 2x - 2$ passes through the point (5,9). Find the point(s) of tangency.

Tie Breaker 2:
A surveyor needs to approximate the area of a small lake.



The lake is bounded on the west by a dam that runs North/South. The surveyor has equipment that allows him to measure the width of the lake along North/South lines parallel to the dam. Unfortunately, because of the terrain, he is unable to keep the East/West distance constant between readings. He makes the following table of readings:

Distance East from dam (meters)	0	5	17	30	32
Width of Lake (meters)	17	16	12	10	0

Approximate the area of the lake using a trapezoidal approximation.

Name ________________________________

Tie Breaker 3:

The familiar formula $y = -4.9t^2 + v_0 t + y_0$ for the height *y* of a projectile after *t* seconds is based on an assumption of a constant gravitational acceleration $a \approx -9.8$ m/s^2. However, in most cases acceleration is not constant, due to factors such as wind resistance.

a) A ball is dropped (initial velocity 0) from a very great height. The acceleration *a* after *t* seconds varies over time according to the function

$$a(t) = \frac{-9.8}{(t+1)^2} \text{ m/s}^2$$

Find the function $v(t)$ for the velocity *v* of the ball after *t* seconds.

b) Find the distance traveled by the ball between $t = 2$ and $t = 5$ seconds.

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Key

1.	**C**
2.	**D**
3.	**A**
4.	**C**
5.	**C**
6.	**D**
7.	**C**
8.	**D**
9.	**E**
10.	**A**
11.	**C**
12.	**B**
13.	**D**
14.	**B**
15.	**A**
16.	**B**
17.	**A**
18.	**D**
19.	**C**
20.	**D**
21.	**A**
22.	**C**
23.	**B**
24.	**B**
25.	**A**

Tie Breaker 1:

A line tangent to $f(x) = x^2 - 2x - 2$ passes through the point (5,9). Find the point(s) of tangency.

Solution

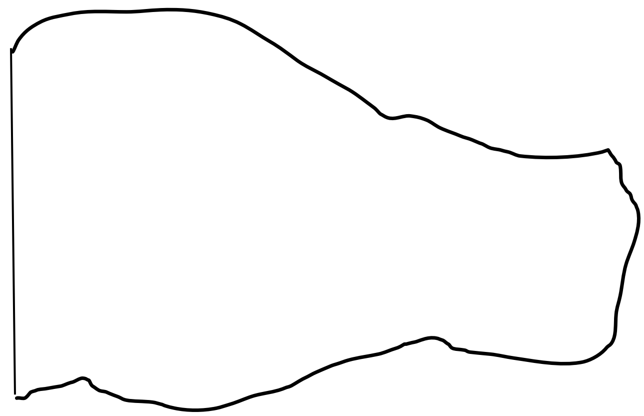
Let a be the value of x at the point of tangency, the point is $(a, f(a)) = (a, a^2 - 2a - 2)$. The slope of the tangent line is $f'(a) = 2a - 2$. Substituting in $y - y_1 = m(x - x_1)$ becomes

$$9 - (a^2 - 2a - 2) = (2a - 2)(5 - a)$$
$$-a^2 + 2a + 11 = -2a^2 + 12a - 10$$
$$a^2 - 10a + 21 = 0$$
$$(a - 3)(a - 7) = 0$$

So, $a = 3$ or $a = 7$

Points of tangency are (3,1) and (7,33).

Tie Breaker 2:
A surveyor needs to approximate the area of a small lake.



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Approximate the area of the lake using a trapezoidal approximation.

Solution

$$A \approx \frac{1}{2}(5)\left(17+16\right)+\frac{1}{2}(12)\left(16+12\right)+\frac{1}{2}(13)\left(12+10\right)+\frac{1}{2}(2)\left(10+0\right)= \ 403.5 \text{ m}^2.$$

<u>Tie Breaker 3</u>:

The familiar formula $y = -4.9t^2 + v_0 t + y_0$ for the height y of a projectile after t seconds is based on an assumption of a constant gravitational acceleration $a \approx -9.8$ m/s^2. However, in most cases acceleration is not constant, due to such factors as wind resistance.

a) A ball is dropped (initial velocity 0) from a very great height. The acceleration a after t seconds varies over time according to the function

$$a(t) = \frac{-9.8}{(t+1)^2} \text{ m/s}^2$$

Find the function $v(t)$ for the velocity v of the ball after t seconds.

b) Find the distance traveled by the ball between t = 2 and t = 5 seconds.

Solution

a) $v(t) = \int a(t)dt = \int \frac{-9.8}{(t+1)^2}dt = \frac{9.8}{t+1} + C$

$$v(0) = \frac{9.8}{0+1} + C = 0 \Rightarrow C = -9.8$$

$$v(t) = -9.8 + \frac{9.8}{t+1} \text{ m/s}$$

$$\Delta y = \int_2^5 v(t)dt = \int_2^5 \left(-9.8 + \frac{9.8}{t+1}\right)dt = \left(-9.8t + 9.8\ln(t+1)\right)\Big|_2^5$$

b) $= \left(-49 + 9.8\ln(6)\right) - \left(-19.6 + 9.8\ln(3)\right)$

$$= -29.4 + 9.8\ln(2)$$

$$\approx -22.607 \text{ m}$$

So the distance traveled is approximately 22.607 meters.