Arkansas Council of Teachers of Mathematics

2013 Regional Exam

Pre-Calculus/Trig

For questions 1 through 23, mark your answer choice on the answer sheet provided. After completing items 1 through 23, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages.

1. Find the reference angle for an angle of 193°.

A) – 167° B) 167 C) 13° D) None of these

2. Find the values of x that satisfies the equation below on the interval $\left[0, \frac{\pi}{2}\right]$

$$\tan^2(3x) = 3$$

A) $\frac{\pi}{9}, \frac{4\pi}{9}$ B) $\frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$ C) $\frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}$ D) $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}$

3. Convert -240° to radians and write as the least possible positive coterminal angle.

A) $-\frac{8\pi}{3}$ B) $-\frac{4\pi}{3}$ C) $\frac{2\pi}{3}$ D) $\frac{4\pi}{3}$

4. Evaluate: $3\log_4 15$ to the nearest tenth.

A) 5.9 B) 2.0 C) 1.5 D) 3.5

5. If the point (3, – 5) is on the terminal side of an angle, θ, in standard position , then find the $\cos\theta$

A) $\frac{3\sqrt{34}}{34}$ B) $\frac{3}{4}$ C) $\frac{-5}{3}$ D) None of these

6. A rational function has an oblique asymptote of $y = x - 2$. If the numerator of the rational function is $x^2 - x$, then which of the following is (are) a possible denominator(s)?

A) $x^3 - 3x^2 + 2x$ B) $x - 3$ C) $x + 1$

D) none of these E) both B and C

7. A ship is heading due south and spots a light house off to the west. The angle between due south and the lighthouse is $26°$. The ship continues to head due south at 25 mph for 35 minutes. Now the lighthouse is seen at $20°$ between due north and the light house. How far is the ship from the lighthouse at the last reading?

A) 533 miles B) 12.6 miles C) 8.9 miles D) None of these

8. For what value of *n* is the following true? $\sum_{k=1}^{n} 2^k = 62$

A) n = 8 B) n = 5 C) n= 4 D) None of these

9. The minute hand of a clock is 3 cm long. How long is the hour hand if the tangent of the angle between the hands at 4:30 PM is 1 .

A) 3 cm B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{3\sqrt{2}}{2}$

10. Convert the following polar equation to rectangular form:

$$r = 3\csc\left(\frac{1}{2}\theta\right)\sec\left(\frac{1}{2}\theta\right)$$

A) x = 3 B) y = 3 C) x = 6 D) y = 6 E) none of these

11. Evaluate: $\sum_{i=0}^{\infty} 8\left(\frac{1}{2}\right)^i$

A) 16 B) ∞ C) 15 D) None of these

12. Convert $\left(6, -\frac{2\pi}{3}\right)$ from polar coordinates to rectangular coordinates.

A) $\left(-2\sqrt{3}, -4\right)$ B) $\left(-4, -2\sqrt{3}\right)$ C) $\left(8, -120\right)$ D) $(-3, -3\sqrt{3})$

13. Find the phase shift for the function: $y = \frac{1}{2}\sin\left(x - \frac{\pi}{3}\right)$.

A) $\frac{1}{2}$ B) $-\frac{\pi}{3}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{6}$

14. What is (are) the asymptote(s) for the rational function $f(x) = \frac{x^2 + x - 6}{\left(x^2 - 4\right)\left(x + 3\right)}$

A) x=2, x = – 2, x = – 3, y = 0 B) x=2, x = – 2, y = 0

C) x=-2, y = 0 D) None of these

15. A passenger on an airplane notices that the angle of depression to one city is $34°$ while the angle of depression to another city is $58°$. She asks the steward and finds out that the plane is at an elevation of 28,000 feet. What is the distance between the two cities?

A) 4.5 miles B) 11.9 miles C) 4.9 miles D) 9.1 miles

16. The area of a triangle whose sides are 250 ft, 218 ft, and 180 ft is approximately

A) $19{,}130 ft^2$ B) $2{,}250 ft^2$ C) $3{,}132 ft^2$ D) $227{,}800 ft^2$

17. Find the equation of the ellipse whose vertices are at $(0,\pm 6)$ and passes through the point (3, 2).

A) $\frac{x^2}{36}+\frac{y^2}{36}=1$ B) $\frac{x^2}{-6}+\frac{y^2}{6}=1$ C) $x^2+y^2=36$ D) $\frac{8x^2}{81}+\frac{y^2}{36}=1$

18. Find the inverse of $f(x)=\log(x^3)$

A) $f^{-1}(x)=3(10)^x$ B) $f^{-1}(x)=(10)^{3x}$ C) $f^{-1}(x)=3\ln(x)$ D) None of these

19. Solve the equation: $\log_2 x+\log_2(x-6)=4$

A) -2 and 8 B) 11 C) 6 D) 8

20. A population of bacteria grows at a rate of $P(t)=100e^{0.234t}$, where t is in hours. In how many hours will the population reach 12,000?

A) 512 hours B) 20.5 hours C) 1.1 hours D) 28.1 hours

21. Find the summation notation that describes the sum of the infinite geometric series: $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$

A) $\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{-n}$ B) $\sum_{k=1}^{\infty}\left(\frac{1}{2}k+\frac{1}{2}\right)$ C) $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n}$ D) $\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^{n}$

22. A vertical cell phone tower sits on a hill that makes a 7° angle with the horizontal. Two guy wires are attached to the top of the tower so that one is anchored 100 feet directly up the hill and the other guy wire is anchored 100 feet directly down the hill. If the tower is 120 feet tall, then how long is each guy wire?

A) 156ft & 178ft B) 165ft & 147ft C) both are 119ft D) None of these

23. Find the exact value of $sin\frac{5\pi}{12}$.

A) 0.966 B) $\frac{1}{2}\sqrt{2+\sqrt{3}}$ C) $\frac{1}{2}\sqrt{1+\sqrt{3}}$ D) $\frac{1}{2}\sqrt{2-\sqrt{3}}$

Name:_____

Tie Breaker #1:

Find the value of a such that the following rational function has a discontinuity at x = 2 and asymptotes of x = 3 and y = x + 3.

$$f(x) = \frac{x^3 - 2x^2 + 4x + a}{x^2 - 5x + 6}$$

Name:_____

Tie Breaker #2

An asteroid orbits the sun in an elliptical path with eccentricity of $e = 0.795$. The closest that this asteroid approaches the sun is 0.485 AU (Astronomical Units). What is the maximum distance between the sun and this asteroid to the nearest 0.1 AU?

Name:_____

Tie Breaker #3

Find a quadratic function that passes through the points (– 1, 3), (1, 1) and (2, 6).

Answers:

1. C
2. B
3. C
4. A
5. A
6. C
7. C
8. B
9. D
10. D
11. A
12. D
13. C
14. C
15. A
16. A
17. D
18. D
19. D
20. B
21. D
22. B
23. B

Tie Breaker 1:

a = – 8

Tie Breaker 2:

If $e = \frac{c}{a}$, then $a - c = 0.485$ or $a = 0.485 + c$

Since $e = \frac{c}{a} = 0.795 \Rightarrow c = 0.795a$

This gives $c = 0.795(0.485 + c)$

Which gives $c - 0.795c = .385575 \Rightarrow 0.205c = .385575$

Solving for c gives c = 1.9AU

This means that a = 2.4AU

The maximum distance between the sun and the asteroid is $a + c$ = 4.3 AU

Tie Breaker 3:

Let $ax^2 + bx + c = y$

Substituting yields:

$a(-1)^2 + b(-1) + c = 3$ $\qquad$ $1a - 1b + c = 3$

$a(1)^2 + b(1) + c = 1$ $\qquad$ which becomes $1a + 1b + c = 1$

$a(2)^2 + b(2) + c = 6$ $\qquad$ $4a + 2b + c = 6$

Solving this system of equations yields a = 2, b = -1 and c = 0.

The quadratic therefore is $y = 2x^2 - x$