ACTM State Math Contest
Pre-Calculus/Trigonometry 2009

Select the best answer for each of the following questions and mark it on the answer sheet provided. Be sure to read all of the answer choices before making your selection. When you are finished with the multiple-choice, attempt the tiebreaker questions.

1. The average airspeed of a single engine aircraft is 150 miles per hour. If the aircraft flew the same distance in 2 hours with the wind as it flew in 3 hours against the wind, what was the wind speed?

a. 20 mph b. 30 mph c. 40 mph d. 25 mph

2. Convert $32^{o}20'35''$ to decimal degrees. Round answer to two decimal places

a. 32.25^{o} b. 32.24^{o} c. 32.44^{o} d. 32.34^{o}

3. Rationalize the denominator: $\dfrac{3}{5+\sqrt{3}}$

a. $\dfrac{15-3\sqrt{3}}{22}$ b. $\dfrac{15+3\sqrt{3}}{22}$ c. $\dfrac{15+3\sqrt{3}}{28}$ d. $\dfrac{15-3\sqrt{3}}{28}$

4. Callaway Golf Company has determined that the marginal cost of manufacturing Big Bertha golf clubs (*x*) may be expressed by the quadratic function $C(x) = 4.9x^2 - 617.4x + 19,600$. How many clubs should be manufactured to minimize the marginal cost?

a. 53 clubs b. 73 clubs c. 63 clubs d. 43 clubs

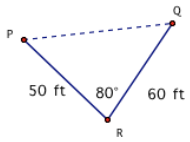
5. Find the exact value of $\tan\left[\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right]$

a. $\sqrt{3}$ b. $-\sqrt{3}$ c. $\dfrac{1}{-\sqrt{3}}$ d. $\dfrac{1}{\sqrt{3}}$

6. A straight trail with a uniform inclination leads from a hotel, elevation 5000 feet, to a lake in the valley, elevation 4100 feet. The length of the trail is 4100 feet. What is the inclination grade of the trail?

a. 9.7^{o} b. 12.7^{o} c. 13.7^{o} d. 14.7^{o}

7. Two homes are located on opposite sides of a small hill at points P and Q. To measure the distance between them, a surveyor walks the distance of 50 feet from house P to point R, uses a transit to measure $\angle PRQ$, which is found to be 80^o, and then walks to house Q, a distance of 60 feet. How far apart are the houses?



a. 71.12 ft b. 72.11 ft c. 81.12 ft d. 82.11 ft

8. Perform the indicated operations and simplify: $\frac{4x+8}{9x^3} \div \frac{x^2-4}{3x}$

a. $\frac{4(x+2)^2(x-2)}{27x^4}$ b. $\frac{4}{3x^2(x-2)}$ c. $\frac{4x}{x-2}$ d. $\frac{4}{x^2(x-2)}$

9. Write in terms of simplest logarithmic form: $\log_b\left(\frac{x^4y^2}{z^5}\right)$

a. $\frac{(4\log_b x)(2\log_b y)}{(5\log_b z)}$ b. $4\log_b x - 2\log_b y + 5\log_b z$

c. $\log_b x^4 + \log_b y^2 - \log_b z^5$ d. $P_1 = \left(3, \sqrt{2}\right)$

10. Find $\begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & -4 & 1 \end{vmatrix}$

a. 2 b. 9 c. 6 d. -3

11. Find the work done by a force of 5 pounds acting in the direction 60^o to the horizontal in moving an object 20 feet from (0,0) to (20,0)

a. 70 ft-lb b. 60 ft-lb c. 40 ft-lb d. 50 ft-lb

12. Write the following expression in standard form a + bi: $\left[3\left(\cos 20^{o}+i\sin 20^{o}\right)\right]^{3}$

a. $\frac{27}{2}+\frac{27\sqrt{3}}{2}i$ b. $\frac{27}{2}-\frac{27\sqrt{3}}{2}i$ c. $\frac{27\sqrt{3}}{2}-\frac{27}{2}i$ d. $\frac{27\sqrt{3}}{2}+\frac{27}{2}i$

13. A bridge is built in the shape of a semi-elliptical arch. The bridge has a span of 60 feet and a maximum height of 20 feet. Find the height of the arch at a distance of 10 feet from the center.

a. 19.72 ft b.18.86 ft c. 14.91 ft d. 20.67 ft

14. Change $r=\frac{2}{3+2\cos\theta}$ to a rectangular equation.

a. $9x^{2}+5y^{2}+8x-4=0$ b. $9x^{2}-5y^{2}-8x-4=0$

c. $9x^{2}-5y^{2}-8x+4=0$ d. $5x^{2}+9y^{2}+8x-4=0$

15. A brick staircase has a total of 25 steps. The bottom step requires 80 bricks. Each successive step requires three less bricks than the prior step. How many bricks are required for the top step?

a. 8 bricks b. 3 bricks c. 5 bricks d. 7 bricks

16. A manufacturer of skateboards makes a Youth model and a Super Deluxe model. Each Youth board requires 2 labor-hours for fabricating and 3 labor-hours for finishing. Each Super Deluxe board requires 5 labor-hours for fabricating and 4 labor-hours for finishing. The maximum labor-hours available per week in the fabricating and finishing departments are 120 and 75, respectively. Since the company makes a profit of $20 each Youth board and $65 on each Super Deluxe board, they want to know how many boards of each type should be manufactured each week in order to maximize total weekly profit. Let $P(x,y)=20x+65y$ Let X = number of Youth boards and Y= number of Super Deluxe boards. Determine the constraints for the manufacture of the boards.

a. $\begin{array}{l}2x+3y\le 120\\3x+4y\le 75\\x\ge 0,y\ge 0\end{array}$ b. $\begin{array}{l}2x+5y\le 120\\3x+4y\le 75\\x\ge 0,y\ge 0\end{array}$ c. $\begin{array}{l}2x+5y\le 120\\3x+4y\le 75\end{array}$ d. $\begin{array}{l}2x+3y\le 120\\3x+4y\le 75\end{array}$

17. Find the exact value of $\frac{\sin 50^{o}}{\cos 40^{o}}$

a. $\sqrt{2}$ b. $\sqrt{3}$ c. $-\sqrt{2}$ d. 1

18. Given $f(x)=\frac{2}{1+2x^2}$ and $g(x)=3x$. Find $(f\circ g)(2)$

a. $\frac{73}{2}$ b. $\frac{2}{3}$ c. $\frac{2}{73}$ d. $\frac{3}{2}$

19. A special window in the shape of a rectangle with two semi-circles at each end is to be constructed so that the outside dimensions are 100 feet in length. Find the dimensions of the rectangle that maximizes its area.

a. 50 ft x $\frac{25}{\pi}$ b. 50 ft x $\frac{\pi}{25}$ c. 25 ft x $\frac{\pi}{50}$ d. 25 ft x $\frac{50}{\pi}$ ft

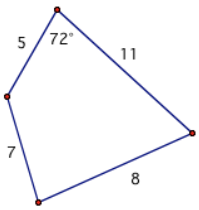
20. Solve the inequality and write your final answer in interval notation: $\frac{x+5}{x-3}\le 0$

a. $(-\infty,-5]\cup[3,\infty)$ b. $(-\infty,-5]\cup(3,\infty)$ c. [-5, 3) d. [-5, 3]

21. The electromotive force E, in volts, in a certain ac circuit obeys the equation $E=120\sin(120\pi t)$. What is the maximum value of E?

a. 120 volts b. 220 volts c. 100 volts d. 200 volts

22. Find the area of the given quadrilateral.



a. 54.15 units^2 b. 44.15 units^2 c. 64.15 units^2 d. 52.15 units^2

23. Given $\frac{17}{20}$ and $P_2=\left(8\sqrt{2},2\sqrt{2}\right)$, find $\|v\|$

a. 14 b. 12 c. 8 d. 10

24. A pond currently has 2000 trout in it. A fish hatchery decides to add an additional 20 trout each month. In addition, it is known that the trout population is growing 3% per month. The size of the population after n months is given by the recursively defined sequence $p_0 = 2000$ and $p_n = 1.03p_{n-1} + 20$. How many trout are in the pond after two months?

a. 2120 b. 2162 c. 2142 d. 2130

25. Find all vertical asymptotes for the graph of *f* if $f(x) = \frac{x+5}{x^2+2x-63}$

a. x = -9, x = 7 b. y = -9, y = 7 c. x = 9, x = -7 d. y = 9, y = -7

ACTM State PreCalculus/Trigonometry

Name: _____

Tiebreaker #1

The distance from the Sun to Earth is approximately 149,600,000 km. The distance from the Sun to Venus is approximately 108,200,000 km. The elongation angle α is the angle formed between the line of sight from Earth to the Sun and the line of sight from Earth to Venus. Suppose that the elongation angle for Venus is 10^{o}. Find the possible distance from Earth to Venus.

Name: _____

Tiebreaker #2

A golf ball is selected at random from a container. If the container has 9 white balls, 8 green balls, and 3 orange balls, find the probability that a white ball or green ball is chosen.

Name: _____

Tiebreaker #3

Graph at least one period for $y = 2\sin\left(\pi x^2 + \frac{1}{2}\right)$

Solutions for ACTM Regional PreCalculus/Trigonometry

1. b
2. d
3. a
4. a
5. a
6. b
7. b
8. b
9. c
10. a
11. d
12. a
13. b
14. d
15. a
16. b
17. d
18. c
19. d
20. c
21. a
22. a
23. d
24. b
25. a

Tiebreakers

1. 42,300,000 km or 252,400,000 km

2. $\frac{17}{20}$

3.