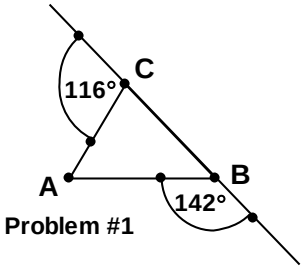
ACTM State Exam
2011 – Geometry

In each of the following select the best answer and mark the corresponding letter on the answer sheet. You should complete the first 25 questions before attempting the Tie Breaker problems since they will only be used to break ties for 1st, 2nd, and/or 3rd place. Please note, the pictures are not necessarily drawn to scale.

1. The measure of ∠BAC in the accompanying figure is

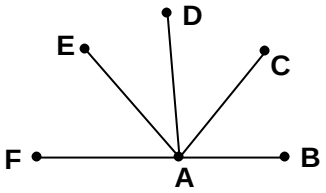


Problem #1

A. 54° **B. 88°** **C. 78°**

D. 38° **E. None of these**

2. A linear pair of angles in the accompanying figure is

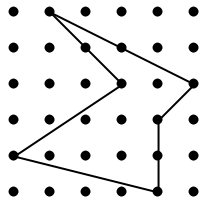


Problem #2

A. ∠BAC, ∠FAD **B. ∠BAE, ∠FAD**

C. ∠BAD, ∠FAE **D. ∠BAD, ∠EAD** **E. None of these**

3. The area of the polygon on the grid is


Problem #3

A. 9½ **B. 11** **C. 10½**

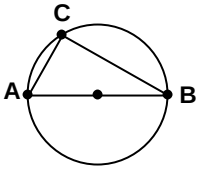
D. 10 **E. None of these**

4. A circle is drawn inside a rectangle so that it is tangent to three sides of the rectangle. If the dimensions of the rectangle are 2 and 3 the ratio of the circumference of the circle to the perimeter of the rectangle is

A. π : 5 **B. π : 6** **C. π : 10**

D. Not enough information **E. None of these**

5. The diameter, $\overline{AB}$ of the circle is 10. What are the possible lengths of the chords $\overline{AC}$ and $\overline{BC}$?



Problem #5

A. 6, $4\sqrt{5}$ **B. 5, $5\sqrt{3}$** **C. 4, $3\sqrt{21}$**

D. 3, $2\sqrt{26}$ **E. None of these**

6. A tangent to a circle is always perpendicular to the radius of the circle drawn to the point of tangency. What is the slope of the tangent to the circle $(x - 2)^2 + (y - 1)^2 = 25$ at the point (6,4)?

A. $-\frac{4}{3}$ **B.** $\frac{4}{3}$ **C.** $\frac{3}{4}$ **D.** $-\frac{3}{4}$

E. None of these

7. The equation of the line that is parallel to $2x - 3y = 6$, with a y-intercept that is four more than the y-intercept of the given line, is

A. $2x - 3y = 10$ **B.** $2x - 3y = 24$ **C.** $2x - 3y = -6$

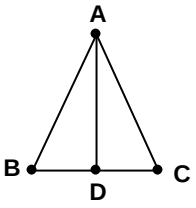
D. $3x - 2y = -4$ **E.** None of these

8. The points A = (–1, –3), B = (4, –1), C = (7, 3), and D = (2,1) are the vertices of a parallelogram. The point of intersection of the diagonals of ABCD is

A. (4, 0) **B.** (0, 4) **C.** (0, 3) **D.** (3, 0)

E. None of these

9. $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$. Which of the following statements is/are true?



Problem #9

P: $\overline{AB} \cong \overline{AC}$

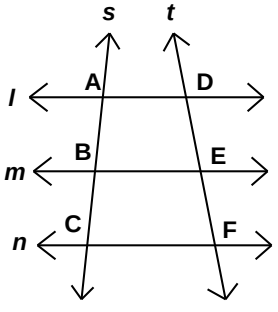
Q: $\Delta ABD \cong \Delta ADC$

R: $\angle BAD \cong \angle DAC$

A. P and Q **B.** P and R **C.** Q and R

D. P, Q, and R **E.** None of these

10. In the figure $l \parallel m \parallel n$ and s and t are transversals. If AB = 2x + 3, BC = 6x + 1, DE = x + 4 and EF = 3x + 2, then the ratio of AB to AC is



Problem #10

A. 2 : 5 **B.** 3 : 5 **C.** 5 : 8

D. 8 : 5 **E.** None of these

11. Consecutive angles of the quadrilateral PQRS are supplementary. Which of the following best describes PQRS?

A. PQRS is a rectangle **B. PQRS is an isosceles trapezoid**

C. PQRS is a kite **D. PQRS is a parallelogram**

E. None of these

12. A quadrilateral ABCD is inscribed in a circle. Which of the following statements is always true about ABCD?

A. ABCD is a parallelogram
B. Opposite pairs of angles are supplementary
C. ABCD is an isosceles trapezoid
D. ABCD is a kite
E. None of these

13. PQRSTUVWX is a regular polygon. The measure of the angle, $\angle$PQR is

A. 140° **B. 40°** **C. 20°** **D. 36°**

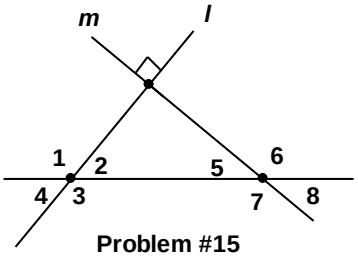
E. None of these

14. An equilateral triangle and a regular hexagon are inscribed in a circle. The ratio of the perimeter of the hexagon to the perimeter of the triangle is

A. 2 : 1 **B. 2 : $\sqrt{3}$** **C. 3 : $\sqrt{2}$**

D. Not enough information **E. None of these**

15. In the figure *l* $\perp$ *m*. A pair of complementary angles is



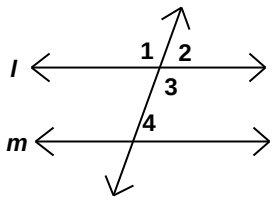
Problem #15

A. $\angle$1 and $\angle$5 **B. $\angle$2 and $\angle$3**

C. $\angle$3 and $\angle$7 **D. $\angle$4 and $\angle$8**

E. None of these

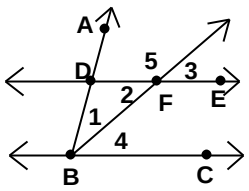
16. In the figure if *l* II *m*, m∠2 = x°, m∠3 = y° and m∠4 = (8x – 3y)°, then x =



Problem #16

A. 54 B. 135 C. 45
D. 108 E. None of these

17. Given that $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ and ∠1 ≅ ∠3. Which of the following statements is/are true?



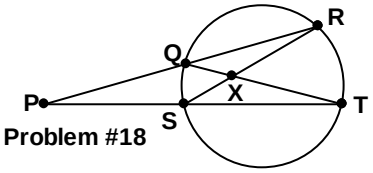
Problem #17

P: DF = DB
Q: $\overrightarrow{BF}$ bisects ∠ABC
R: ∠4 and ∠5 are supplementary

A. P and Q B. P and R C. Q and R

D. P, Q, and R E. None of these

18. In the figure m∠QPS = 12° and m∠PRS = 20°. Then m∠QXS =

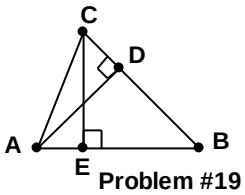


Problem #18

A. 52° B. 20^0 C. 64°

D. 40° E. None of these

19. In ΔABC, $\overline{AD}$ and $\overline{CE}$ are altitudes. If AB = 40, BC = 37, and CE = 12, then to the nearest hundredth AD =

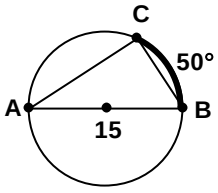


Problem #19

A. 6.49 B. 12.97 C. 11.10

D. 5.55 E. None of these

20. In the circle $\overline{AB}$ is a diameter, AB = 15 and the measure of the arc BC is 50° To the nearest hundredth, BC =

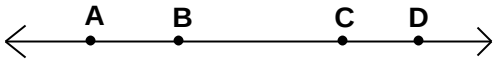


Problem #20

A. 9.64 B. 6.34 C. 13.59

D. 11.49 E. None of these

21. A, B, C and D are four distinct points on a line. Using these 4 points, how many segments can be named?



Problem #21

A. 12 **B.** 8 **C.** 3 **D.** 9

E. None of these

22. The number of regular solids that have triangular faces is

A. 5 **B.** 4 **C.** 3 **D.** 2

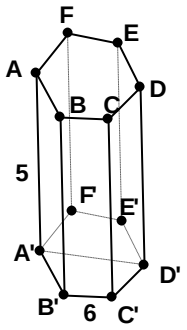
E. None of these

23. A prism and a pyramid have the same number of edges. If the base of the prism is an octagon, then the number of vertices in the pyramid is

A. 12 **B.** 13 **C.** 14 **D.** 16

E. None of these

24. The right prism in the figure has a regular hexagonal base. If the side of the hexagon is 6, the height of of the prism is 5, then AD' =

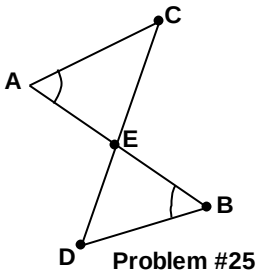


Problem #24

A. 13 **B.** 12 **C.** $\sqrt{61}$

D. Not enough information **E.** None of these

25. In the figure $\angle CAB \cong \angle DBA$, DB = 10, AC = 20, and AB = 36, then AE =



Problem #25

A. 20 **B.** 16 **C.** 12

D. 24 **E.** None of these

When you have answered the 25 questions on the exam, proceed to the tie breaker problems. They will be used to break ties for first, second or third place. They will be used in the order they are given.

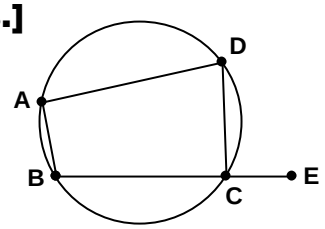
Tie Breaker Questions

Name_____ **School**_____

[Please Print] **[Please Print]**

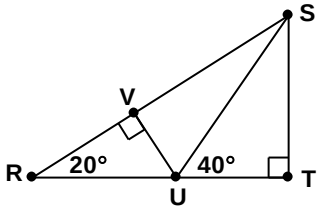
The tie breaker questions will be used to break ties for first, second and/or third place. They will be used in the order that they are given. Complete the 25 questions before attempting the tie breaker questions.

1. ABCD is an inscribed quadrilateral. B, C and E are collinear points. Prove: $\angle DAB \cong \angle DCE$
[Be sure to supply reasons for your conclusions.]



Tie Breaker #1

2. In the figure ΔRST is a right triangle, $m\angle SUT = 40°$, $m\angle SRT = 20°$, and ST = 45. If $\overline{UV} \perp \overline{RS}$, find UV.



Tie Breaker #2

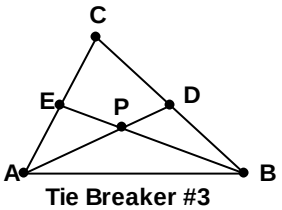
Name_____ **School**_____

[Please Print] **[Please Print]**

3. In ΔABC $\overline{AD}$ and $\overline{BE}$ are medians intersecting at P.

Determine the value of the ratio $\frac{\text{Area}(\Delta PED)}{\text{Area}(\Delta PAB)}$.

Justify your conclusion.



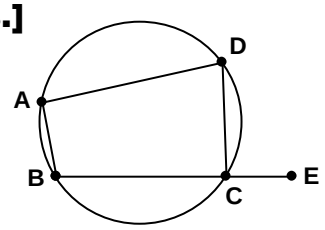
Tie Breaker #3

Key – Geometry 2011

1. C
2. E
3. A
4. A
5. B
6. A
7. C
8. D
9. B
10. C
11. D
12. B
13. A
14. B
15. D
16. A
17. D
18. A
19. B
20. B
21. E
22. C
23. B
24. A
25. D

Tie Breaker Questions –

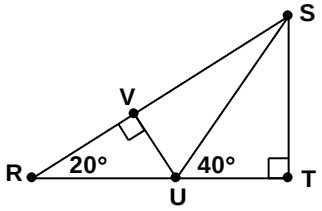
1. ABCD is an inscribed quadrilateral. B, C and E are collinear points. Prove: $\angle DAB \cong \angle DCE$ [Be sure to supply reasons for your conclusions.]



Tie Breaker #1

$\angle DAB$ and $\angle DCB$ are opposite angles of a cyclic quadrilateral and are supplementary angles. $\angle DCB$ and $\angle DCE$ are a linear pair of angles and are supplementary. Therefore, $\angle DAB \cong \angle DCE$. [Note: The student may elect to use inscribed angles to arrive at the fact that $\angle DAB$ and $\angle DCB$ are supplementary angles.]

2. In the figure ΔRST is a right triangle, $m\angle SUT = 40°$, $m\angle SRT = 20°$, and ST = 45. If $\overline{UV} \perp \overline{RS}$, find UV.



Tie Breaker #2

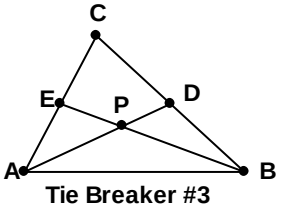
tan(40°) = $\frac{45}{UT}$**. UT =** $\frac{45}{\tan(40^\circ)}$

tan(20°) = $\frac{45}{RT}$**. RT =** $\frac{45}{\tan(20^\circ)}$

RU = RT – UT = $\frac{45}{\tan(20^\circ)} - \frac{45}{\tan(40^\circ)}$

UV = RU(sin(20°) = $\left(\frac{45}{\tan(20^\circ)} - \frac{45}{\tan(40^\circ)}\right)\sin(20^\circ) = 23.9439\ldots$

3. In ΔABC $\overline{AD}$ and $\overline{BE}$ are medians intersecting at P. Determine the value of the ratio $\frac{\text{Area}(\Delta PED)}{\text{Area}(\Delta PAB)}$**. Justify your conclusion.**



Tie Breaker #3

Medians of a triangle intersect at a point that is two-thirds of the distance from the vertex to the midpoint of the opposite side. Therefore, AP = 2PD, and BP = 2PE. $\angle APB \cong \angle DPE$ since they are a vertical pair of angles. Therefore, $\Delta APB \sim \Delta DPE$. The constant of proportionality is 2. Therefore $\frac{\text{Area}(\Delta PED)}{\text{Area}(\Delta PAB)} = \frac{1}{2^2} = \frac{1}{4}$**.**